The rise at the eighth year here is almost precisely the same as in the twenty offices—the ratios at the seventh, eighth, and ninth years in the total experience of both being respectively ·026 : ·038 : ·020 and ·025 : ·038 : ·019.

Professor Bartlett, of the Mutual Life, wrote me that he could give no satisfactory explanation. If, as I have suggested, we assume that the half-credit system is the root of the matter as regards the English statistics, what are we to think of the American ones? We have the option of two conclusions. We have here either a very remarkable coincidence or the indication of the working of some law. Which is it? We incline to the former opinion. Perhaps some of your readers will give us their views.

Yours respectfully,

T. B. MACAULAY.

Montreal,
164 *St. James' Street,*
May 8*th* 1879.

ON THE RELATION BETWEEN THE VALUE OF A POLICY AND THE RATE OF INTEREST.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In vol. xvii of the *Journal of the Institute*, both Mr. Macfadyen and Mr. Sutton prove satisfactorily that, under certain limitations, the value of a policy by a higher rate of interest is less than the value by a lower rate. As the following demonstration is possibly more simple than what these gentlemen give, it may not be uninteresting to some of your readers. In the cases referred to, the proof proceeds from the formula

$$1-\frac{1+a_{x+1}}{1+a_x},$$

but without reference to the duration of the policy. The value may be also expressed by

$$\frac{A_{x+n}-A_x}{1-A_x}.$$

Now, assuming the dash to indicate values relating to a higher rate of interest, we have

$${}_nV_x > {}_nV'_x$$

when $$\frac{A_{x+n}-A_n}{1-A_x} > \frac{A'_{x+n}-A'_x}{1-A'_x};$$

that is, when $$\frac{A_{x+n}-A_x}{A'_{x+n}-A'_x} > \frac{1-A_x}{1-A'_x}.$$

But $$A_{x+n}-A_x = v\left(\frac{d_{x+n}}{l_{x+n}}-\frac{d_x}{l_x}\right)+v^2\left(\frac{d_{x+n+1}}{l_{x+n}}-\frac{d_{x+1}}{l_x}\right)+\ldots\ldots$$

and $$A'_{x+n}-A'_x = v'\left(\frac{d_{x+n}}{l_{x+n}}-\frac{d_x}{l_x}\right)+v'^2\left(\frac{d_{x+n+1}}{l_{x+n}}-\frac{d_{x+1}}{l_x}\right)+\ldots\ldots$$

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therefore, as $v' < v$, we have each term of the series for $A_{x+n} - A_x$ greater than the corresponding term of the series for $A'_{x+n} - A'_x$. But in the event of some of the terms being negative, which might easily be shown to be the case according to various mortality tables, we could not state which series is the greater. If the value of $A_{x+n} - A_x$ be written

$$v(q_{x+n} - q_x) + v^2(p_{x+n}q_{x+n+1} - p_x q_{x+1}) + \dots\dots$$

it becomes apparent that, when the mortality increases with the age,

$$A_{x+n} - A_x > A'_{x+n} - A'_x,$$

and consequently $$\frac{A_{x+n} - A_x}{A'_{x+n} - A'_x} > \text{unity}.$$

But as $$A_x > A'_x, \quad \frac{1 - A_x}{1 - A'_x} < \text{unity}, \quad \therefore \; {}_nV_x > {}_nV'_x.$$

In both of the demonstrations above referred to, the proof is founded on the assumption that $a_x > a_{x+1}$, and > the annuity at all succeeding ages. Mr. Sprague, in vol. xxi, p. 94, referring to the theorem, investigates under what conditions $a_x < a_{x+1}$, and there proves that when such is the case we must have

$$vq_x > \varpi_{x+1}.$$

That is, the premium for a single year greater than the whole-life premium at the next higher age.

I am, Sir,
Your obedient servant,
WM. G. WALTON.

Scottish Amicable Life Society,
Glasgow, 16 *May* 1879.

MR. GRAY'S METHODS OF CONSTRUCTING LIFE TABLES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The example appended to my letter in your last number contained some errors (for which I am to blame), and it was not, as I now find, sufficiently explained.

Believing, as I do, that the process which I then sought to elucidate deserves more attention than it has hitherto gained, I should feel obliged by your permitting me to give a further illustration of it. For this purpose I will use the Institute H^M Table, which is accessible to all readers of the *Journal*.

The problem is to construct $\log N_x$, the log of D_x being given. This is Problem XXII, page 124 of Gray's *Tables and Formulæ*. Mr. Gray showed that the work brought out $\log a_x$ and $\log(1 + a_x)$ as well as $\log N_x$; but he did not specially notice the remarkable fact that, excepting the datum $\log D_x$, not a figure besides appears in the process, and he did not mention any further uses of these quantities.

On inspection of the following example it will be seen that there