Fourier Methods for Field and Phase-shift Calculations of Long-range Electromagnetic Fields

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Recently, by analyzing the problem of the observation of superconducting fluxons by transmission electron microscopy, it has been found that the calculation of the electron optical phase shift can been carried out successfully by a new approach[1,2]. First the vector potential is decomposed into its Fourier components and then the phase shift is calculated for each component separately. In this way, once the problem of finding the vector potential in analytical form has been solved, the Fourier transform of the phase shift is immediately obtained, and can be inverted either analytically or numerically. The main advantages of this approach are that the case of a periodic array of fluxon can be easily analyzed [1], a troublesome problem in the former real space approach owing to the long-range behaviour of the fluxon magnetic field, and that new superconducting structures, like pancake vortices present in high-T_C materials [2,3], which were beyond the scope of the flux tube model and its implementations, can be successfully investigated. In this way, it is possible to interpret recently-obtained experimental results by means of the new 1-MV holographic microscope relative to fluxons pinned at tilted columnar defects [4,5] and to correlate the image features to the anisotropy of the underlying structure, as also shown in other contributions [6].

In this work it will be outlined how this approach can be profitably extended also to other cases. Mansuripur [7] introduced Fourier methods for the numerical calculation of the magnetic field, its vector potential and the corresponding phase shift, under the assumption that the distribution of the magnetization is doubly periodic so that the Fast Fourier Tranform algorithm can be safely applied. These requirements are not met by the case of antiparallel magnetic stripe domains, lying in a semi-infinite specimen, where the question is how the presence of the edge and of the associated fringing field influences the field and corresponding phase shift. It can be shown that by means of our Fourier approach this problem can be solved analytically, obtaining the solution in closed form when the domain wall width is negligible [8].

In the case of electrostatic fields, Fourier methods were employed by Vanzi [9] in his investigation of electric fields to prove important relations in the real space. Let us focus our attention on the Fourier space. The general solution of the Laplace equation, e.g. in the vacuum region above the specimen, $z > 0$, can be written as

$$V(x, y, z > 0) = \frac{1}{4\pi^2} \iint \tilde{V}_+(k_x, k_y) \, e^{i k_x x + i k_y y - k_\perp z} \, d k_x \, dk_y$$

where $k_\perp = \sqrt{k_x^2 + k_y^2}$ and the Fourier transform $\tilde{V}_+(k_x, k_y)$ refers to the potential distribution at the upper specimen surface. From this expression it is easy to ascertain that the corresponding contribution to the phase shift is given by

$$\varphi_+(x, y) = \frac{\pi}{\lambda E} \int_0^\infty V(x, y, z) \, dz = \frac{1}{4\pi\lambda E} \iint \frac{\tilde{V}_+(k_x, k_y)}{k_\perp} \, e^{i k_x x + i k_y y} \, d k_x \, dk_y$$

The former equation allows us to extract a simple and significative relation in the Fourier space between the Fourier transform of the phase shift and potential:

$$\tilde{\varphi}_+(k_x, k_y) = \frac{1}{4\pi\lambda E} \frac{\tilde{V}_+(k_x, k_y)}{k_\perp}$$

These considerations can be extended also to the vacuum region below the specimen, emphasizing that the calculation of the external phase shift in the Fourier space, at least formally, is a very simple matter, once the potential distribution on the two surfaces of the specimen is known.

These results have been applied to the analytical model for the electric field associated to a periodic array of alternating p- and n-doped stripes lying in a half-plane, tilted with respect to the specimen edges [10]. The solution of this problem has been found in the real space, by exploiting the striking similarity with the well-known optical problem of the diffraction of an inclined plane wave by a perfectly conducting half-plane [11]. As it is possible to calculate the Fourier transform of the potential at the specimen surface, it turns out that the the time-consuming integration along z of the potential (whose expression in the whole space, although analytical, is much more complicated with respect to its value on the specimen plane) is replaced by the division by $k_\perp$. The inverse Fourier transform can be subsequently carried out by a mixed analytical-numerical method which allows a substantial reduction of the computation time for the phase shift.

My coworkers, M. Beleggia and P.F. Fazzini, and I are presently exploiting the capabilities of Fourier methods when applied to the investigation of long range electromagnetic fields and our results [12] seem to indicate that this is a powerful approach to find the solution of otherwise unmanageable problems and even when the solution of the problem is known by real space methods, it can offer a useful different perspective or at least lead to computational benefits.

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