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Solar Metal Abundance Inferred from Helioseismology

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Abstract. In our previous work (Takata & Shibahashi 1998), we constructed a solar model called the seismic solar model, which has the consistent profile of sound speed as well as the consistent depth of the convection zone with helioseismology. The profile of the heavy element abundance, however, had to be assumed to be constant for feasibility. Here we try to constrain the distribution of the heavy element abundance as well by the solar oscillation frequencies, adopting all of the basic equations which govern the solar structure.

1. Introduction

Helioseismology provides us with invaluable information about the solar interior. Recent observations from space or by the ground-based networks give us the extremely high precision data of solar oscillations. At present we can fairly accurately infer the sound speed profile or the rotation rate profile of the Sun as well as the depth of the convection zone and the surface helium abundance. The helium abundance of the Sun has been estimated by helioseismology in many works whose recent consensus seems to be that the main source of the uncertainty in the solar helium abundance determined by helioseismology comes from the uncertainties in the equation of state. In other words, we know the surface helium abundance of the Sun with an error as small as that in the equation of state. The next target for helioseismology would be the heavy element abundance, which we discuss in this paper. The direct motivation of this work comes from another place; in a previous work (Takata & Shibahashi 1998), we had to assume that the heavy elements were distributed uniformly to construct the solar model consistent with helioseismology, because we did not have any other condition to fix the whole profile of the heavy element abundance. The constant distribution of the heavy element abundance is, however, not consistent with the diffusion process, which must operate inside the Sun during its evolution. We therefore consider the possibility of direct inversion for the heavy element abundance. In this paper we do not distinguish each element of the heavy elements but treat them as a whole using their mass fraction Z just like in the theory of stellar structure.

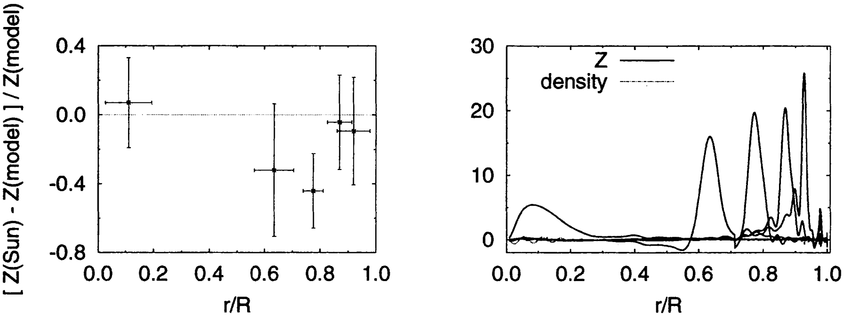

Figure 1. Z inversions (left) and the averaging kernels (right).

2. Z inversion

We perform the optimally localized averaging inversion for the heavy element abundance. This method is based on the integral equations, which relate the difference in a set of the structure variables between the Sun and the reference model to the frequency difference of each mode. The main problem of the current work is to obtain the appropriate kernels of the heavy element abundance in these integral equations by converting the kernels of sound speed and density, which are already known. We follow the general method of the kernel conversion given by Gough, Thompson & Kosovichev (2000, in preparation). We adopt the heavy element abundance and density as the variables of the converted kernels. The reader should keep in mind that we have to consider the energy equation and the transfer equation as well as the nuclear reaction rate, the opacity and the equation of state when we convert the kernels. The adopted microphysics, which are needed in converting kernels, are as follows: we use the opacity calculated by the OPAL group (Iglesias & Rogers 1996); the OPAL equation of state (Rogers, Swenson & Iglesias 1996) is used for consistency with the opacity; we refer to the cross sections of the nuclear reaction rates described in Adelberger et al. (1999). We stress that we need these microphysics only to calculate the derivatives of these quantities since the equilibrium quantities are already given in the reference model. Note that the total luminosity constraint is adopted in the inversion process as well as the ordinary frequency relations and the total mass constraint. This is simply because the inversion results should be consistent with the observed total luminosity. Model S by Christensen-Dalsgaard et al. (1996) is adopted as the reference model of the inversion. We use the frequency data obtained by the *Michelson Doppler Imager* instrument on board the *Solar Heliospheric Observatory* spacecraft (Schou et al. 1998). We actually use only a part of the 360-day frequency data set between 1 mHz and 4 mHz.

3. Results and Discussion

The results of the heavy element inversions together with the corresponding averaging kernels are shown in Figure 1. We do not see any point between $0.2R_\odot$ and $0.5R_\odot$ in the left panel of this figure because we have only poor averaging kernels in this region, which have multiple peaks and/or very broad widths. We physically interpret this fact in the following. Since we adopt density as a counterpart variable of the heavy element, the Z kernels are related to the response of the frequencies at fixed density. When the density profile is fixed, the pressure profile is also fixed because of the hydrostatic equilibrium and the mass conservation. Then the heavy element abundance can affect the oscillation frequencies only through the adiabatic index Γ_1, which is directly related to sound speed. While the partial derivative $(\partial\Gamma_1/\partial Z)_{\rho,T,X}$ is generally small in the case relevant to the solar interior because the solar matter is nearly an ideal gas, the derivative is even smaller in the inner core than in the outer envelope since more elements are ionized in the envelope than in the core. This behaviour of the sensitivity of Γ_1 to Z explains why we have the good averaging kernels of Z in the outer 40% region in radius (the right panel of Figure 1). On the other hand, the point in the core in the left panel of Figure 1 can be explained from another point of view. We find a peak around $r \approx 0.1R_\odot$ in all of the Z kernels, which are superposed to make the averaging kernels. In fact, we see that peak in both of the mode conditions, which relate the differences in Z and density to the eigenfrequency differences, and the total luminosity constraint, which is not imposed when we derive the kernels of the mode conditions. It is this peak that enables us to have a well-localized averaging kernel in the core. The reason for the peak in the Z kernel of the total luminosity constraint is that the total luminosity is sensitive to the temperature at the central region where nuclear reactions take place and the temperature gradient depends on the opacity, which is significantly influenced by Z. Note that the position of the peak coincides with that of the maximum energy generation by the pp chain. As we see in the left panel of Figure 1, the Z inversions still have significant uncertainties. In fact, all of the points except one at $0.77R_\odot$ suggest that the reference model has the consistent Z abundance with the real Sun. On the other hand, if we pay attention to the outer 3 points, all of which should indicate a single value because they are located in the convection zone, we find that they are consistent with a value of Z which is lower than the model by 20 or 30%. In conclusion, this preliminary study suggests that the heavy element abundance in the convection zone could be 20 or 30% smaller in the real Sun than in the standard solar model though we do not estimate the effect of the uncertainties in the equation of state, the nuclear reaction rates and the opacity.

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