Complex Planetary Systems II: Latest Methods for an Interdisciplinary Approach
Proceedings IAU Symposium No. 382, 2024
A. Lemaitre & A.-S. Libert, eds.
doi:10.1017/S1743921323003915

On the long-term orbital evolution of a satellite revolving around an oblate body

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Abstract. In the context of a perturbed two body problem, in which the Keplerian motion of the small object (the satellite) is perturbed by the oblateness of the central body (the asteroid) and the attraction of a third body (the Sun), we discuss the long-term evolution of the orbital elements of a satellite orbiting an oblate body, with a particular focus on the behavior of the inclination and the longitude of the ascending node. We derive analytically the position of the Laplace plane as a function of several parameters and use this solution to analyse the long-term evolution of distant circular orbits. The analytical study is complemented by numerical tests, performed in the context of both Cartesian and Hamiltonian frameworks. The results give a description of the orbital dynamical environment of asteroids and reveal the parameters that play a key role in the long-term stability of distant circular orbits.

Keywords. Laplace plane, obliquity, Hamiltonian approach, Cartesian approach

1. Introduction

Recent and future missions have the purpose of orbiting and landing on the surface of small bodies, like asteroids, comets and planetary satellites. Some of them are sample return missions (e. g. OSIRIS-REx mission). Such missions rely on a good knowledge of the dynamics around the explored object since a detailed study of the motion on various time scales can provide optimal trajectories for the spacecraft, give stable orbits or even describe the space environment. Indeed, a cartographic study of the dynamics around minor bodies might assess the possibility of existence of accompanying swarms of particles. On the other hand, post mission stability analysis can provide valuable information on the body system, opening windows into the past of the Solar system and creating opportunities for future human explorations.

In this work we are using both the Cartesian equations of motion and the Hamiltonian formalism to investigate the long–term evolution of the orbits of a particle orbiting an asteroid within the context of a model taking into account the attraction of the central body (the asteroid), including the perturbations induced by the J_2, J_3 and J_4 harmonic terms, and the attraction of the Sun. In particular, we determine the location of the Laplace plane, also called the invariable plane (see Allan & Cook (1964)), whose normal vector is located in the plane determined by the normal vector of the equatorial plane of the asteroid and the normal vector of the Sun's orbital plane, and use this solution to analyse the long-term evolution of distant initially circular orbits.

We perform a parametric study to identify those parameters whose variation leads to large amplitude librational motions of the inclination and the longitude of the ascending node of initially circular orbits. In this sense, we show that the obliquity of the minor body is a key parameter that plays a major role in the long-term evolution of the inclination and the longitude of the ascending node. We illustrate this aspect by analysing two cases, namely the dynamics around Vesta (the obliquity of Vesta has a moderate value $\varepsilon = 15.66^o$, according to Bills & Nimmo (2011)) and, respectively, around Eros (the obliquity of Eros has the extreme value of $\varepsilon = 89^o$, see Souchay et al. (2003)).

We also reveal the role played by the secular resonances and the J_3 harmonics terms in the long-term evolution of the eccentricity of initially circular orbits and based on this analysis we discuss the conditions in which the asteroids might have accompanying satellites or swarms of particles.

2. Models and methods

We consider an infinitesimal particle (natural satellite or spacecraft) orbiting an oblate minor body (asteroid or comet) which is uniformly rotating around a fixed axis, aligned with the body axis (the principal axis with the highest moment of inertia), and revolves around the Sun on an elliptical orbit.

In *Cartesian coordinates*, the motion of the particle, referred to a centred body system, is described by:

$$\ddot{\mathbf{r}} = R_3(-\theta)\ \nabla V(\mathbf{r}) - Gm_S\Big(\frac{\mathbf{r}-\mathbf{r}_S}{|\mathbf{r}-\mathbf{r}_S|^3} + \frac{\mathbf{r}_S}{|\mathbf{r}_S|^3}\Big)\ , \tag{2.1}$$

where $\mathbf{r}$ is the position vector of the particle, $\mathbf{r}_S$ is the position vector of the Sun, V is the gravitational potential of the oblate body (see Kaula (1961)), m_S is the Sun's mass, G is the gravitational constant, ∇ is the gradient operator, θ is an angle describing the rotation of the minor body and $R_3(\theta)$ is the rotation matrix of angle θ around the third axis.

Using the action–angle *Delaunay variables* $(L, G, H, M, \omega, \Omega)$, which are related to the orbital elements $(a, e, i, M, \omega, \Omega)$ by $L = \sqrt{\mu a}$, $G = L\sqrt{1-e^2}$, $H = G\cos i$, where $\mu = Gm$, m being mass of the minor oblate body, the dynamics is described by *the Hamiltonian*

$$\mathcal{H} = -\frac{\mu^2}{2L^2} - \mathcal{R}_A - \mathcal{R}_S\ , \tag{2.2}$$

where $\mathcal{R}_A$ (see Kaula (1961)) and $\mathcal{R}_S$ (see Kaula (1962)) are the disturbing functions due to the minor body and respectively the Sun.

Kaula (1961) gives a Fourier expansion of $\mathcal{R}_A$ in terms of the orbital elements and rotation angle θ:

$$\mathcal{R}_A = -\frac{\mu_A}{a}\sum_{n=2}^{\infty}\sum_{m=0}^{n}\Big(\frac{R_A}{a}\Big)^n J_{nm}\sum_{p=0}^{n}F_{nmp}(i)\sum_{q=-\infty}^{\infty}G_{npq}(e)\,cs_{nm}\Big(\Psi_{nmpq}(M,\omega,\Omega,\theta)\Big)\,, \tag{2.3}$$

where R_A is the reference radius of the minor body, J_{nm} are the harmonic coefficients, F_{nmp}, G_{npq} are the inclination and eccentricity functions, cs_{nm} is the cosine (sine) function if $n-m$ is even (odd) and

$$\Psi_{nmpq}(M,\omega,\Omega,\theta) = (n-2p)\omega + (n-2p+q)M + m(\Omega-\theta) - m\lambda_{nm}.$$

Kaula (1962) derived the expansion of the third body disturbing function, as a function of the orbital elements of both the perturbed and perturbing bodies, in the form:

$$\mathcal{R}_S = \sum \mathcal{A}^S_{k_1k_2k_3k_4k_5k_6}(a/a^*_S, e, i, e^*_S, i^*_S)\cos(k_1M + k_2M^*_S + k_3\omega + k_4\omega^*_S + k_5\Omega + k_6\Omega^*_S),$$

respectively, with k_j integers, where the elements of the satellite and the third-body are referred to the celestial equator. Since the motion of the minor body is assumed to be elliptic, it follows that, for the Sun, the angles ω^*_S and Ω^*_S are constant and $\dot{M}^*_S$ varies linearly in time.

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Focusing on the study of distant circular orbits (see also Allan & Cook (1964)), we consider a double averaged model that includes the effects of J_2 and the attraction of the Sun. Expressed in *the Milankovitch elements* $\mathbf{e} = e\mathbf{P}$ and $\mathbf{h} = (1-e)^2\mathbf{R}$, where $\mathbf{P}$ is a unit vector in the orbital plane and directed toward pericentre, while $\mathbf{R}$ is the unit vector along the positive normal to the orbital plane, the equations of motion has the form:

$$\dot{\mathbf{h}} = \mathbf{h} \times \frac{\partial V^*}{\partial \mathbf{h}} + \mathbf{e} \times \frac{\partial V^*}{\partial \mathbf{e}}\,, \qquad \dot{\mathbf{e}} = \mathbf{e} \times \frac{\partial V^*}{\partial \mathbf{h}} + \mathbf{h} \times \frac{\partial V^*}{\partial \mathbf{e}}\,, \tag{2.4}$$

where V^* is the potential averaged over the mean anomalies.

3. Results

The second equation of the system (2.4) is identically satisfied with $e = 0$. So, assuming that the orbit is circular, the first equation of (2.4) becomes:

$$\dot{\mathbf{R}} = -\omega_0(\mathbf{R} \cdot \mathbf{R}_0)(\mathbf{R}_0 \times \mathbf{R}) - \omega_1(\mathbf{R} \cdot \mathbf{R}_1)(\mathbf{R}_1 \times \mathbf{R}), \tag{3.1}$$

where $\mathbf{R_0}$ is the unit vector along the normal to the equatorial plane of the asteroid, $\mathbf{R_1}$ is the unit vector along the normal to the orbital plane of the Sun, ω_0 is a constant depending on J_2 and ω_1 a constant depending on the mass of the Sun.

The identification of the equilibrium points of the dynamical problem (3.1) is effectively a problem of eigenvalues and eigenvectors (see Allan & Cook (1964)):

$$(\omega_0\mathbf{R}_0\mathbf{R}_0 + \omega_1\mathbf{R}_1\mathbf{R}_1)\mathbf{V} = \lambda\mathbf{V},$$

where $\mathbf{AB}$ is the dyadic product of the vectors $\mathbf{A}$ and $\mathbf{B}$. The eigenvalues λ are obtained by finding the roots of the following equation

$$det(\omega_0\mathbf{R}_0\mathbf{R}_0 + \omega_1\mathbf{R}_1\mathbf{R}_1 - \lambda\underline{I}) = 0,$$

where $\underline{I}$ is the idemtensor. It is easy to see that one eigenvalue is null, let us denote it by λ_1, $\lambda_1 = 0$, while the other two, namely λ_2 and λ_3 might be expressed in terms of the constants ω_0 and ω_1. By computing the corresponding eigenvectors, we determine, in fact, the equilibrium positions for the considered dynamical system. The inclination and the longitude of the ascending node of the equilibrium points are deduced from the Cartesian coordinates of the unit vectors associated to the eigenvectors.

In view of the above remarks, it follows that the equation (3.1) admits 3 equilibrium points in the domain $i \in [0^0, 90^0]$, $\Omega \in [0^0, 180^0]$, two stable and one unstable. The position of the equilibria can be determined by drawing the level sets as in Figure 1, where two cases have been depicted, namely the dynamics around Vesta (top panels) and around Eros (bottom panels).

In the (i, Ω) plane, there are possible two types of motions, namely librational and rotational, and since the vectorial equation (3.1) can be integrated (it can be reduced to a set of equations similar to the Euler equations for the motion of a rigid body) the position of the equilibria as well as the amplitude and the period of librations are provided by analytical formulas.

For instance, the position of the stable equlibrium point located at $\Omega = 0^o$, which, in facts, gives the position of the Laplace plane (the angle between the Laplace plane and the equatorial plane of the minor body), is shown in Figure 2 as a function of the distance (expressed in equivalent radii) from the central body. The amplitude of the librational island depends on many parameters, such as the orbital elements of the Sun, the value of J_2, in fact - the mass of the minor body and its obliquity. We identified the obliquity as a key parameter in the sense that the larger the value of the obliquity is, the larger the amplitude of the libration should be (see Figure 1).

The results are validated by numerical studies obtained by integrating the Cartesian equations of motion (2.1) and the canonical equations associated to the Hamiltonian

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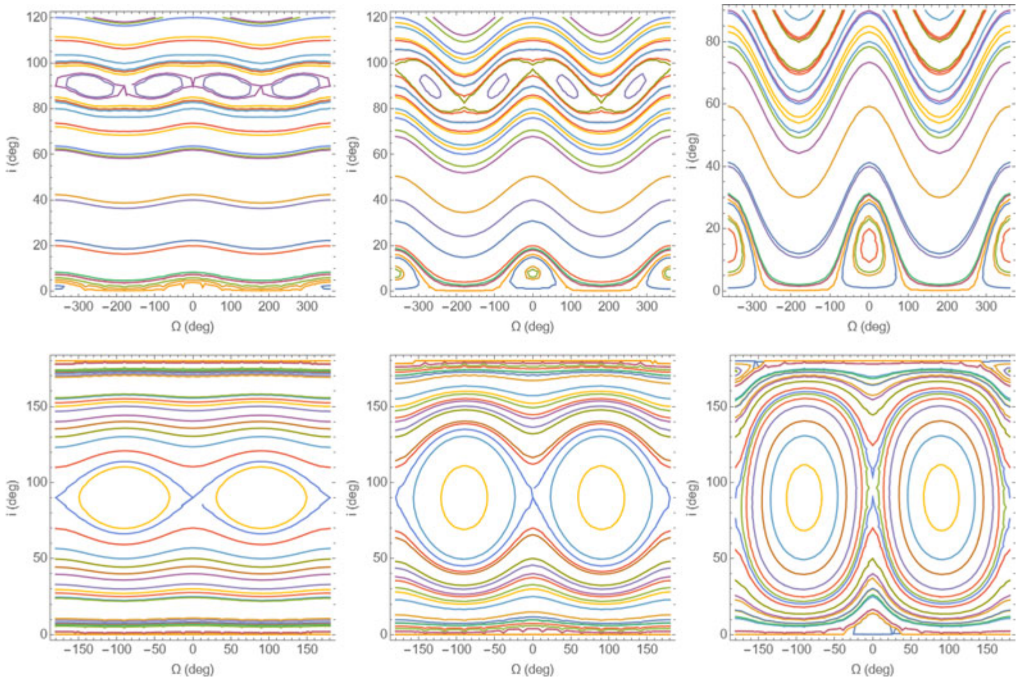

Figure 1. The level sets for the case of Vesta: $a = 6000$ km (top left), $a = 9000$ km (top middle), $a = 15000$ km (top right) and for the case of Eros: $a = 200$ km (bottom left), $a = 250$ km (bottom middle), $a = 280$ km (bottom right).

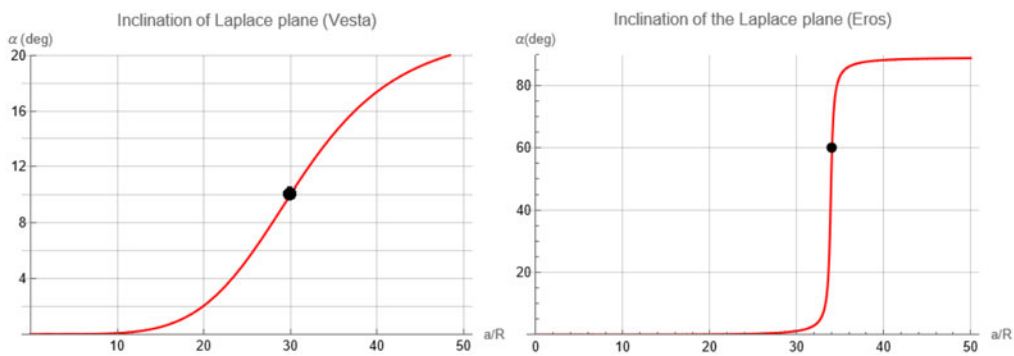

Figure 2. The location of the Laplace plane in the cases of Vesta (left) and Eros (right).

(2.2) and by considering a more complete model that includes all zonal harmonic terms up to degree $n = 4$.

Although the inclination and the longitude of the ascending node have the behaviour predicted by the above analytical arguments, the eccentricity does not remain constant. As effect of J_3, which induces a slight variation of the eccentricity (even if the initial eccentricity is zero), and of the secular solar resonances, which might induce very large variations of eccentricity for non–circular orbits, the long-term behaviour of the orbits is complex. Indeed, if an initially circular orbit suffers a variation in inclination large enough to cross a secular solar resonance, then, as effect of J_3, the eccentricity becomes larger than zero making possible for the orbit to be captured in the libration region of the secular resonances, which induces large excursions in eccentricity.

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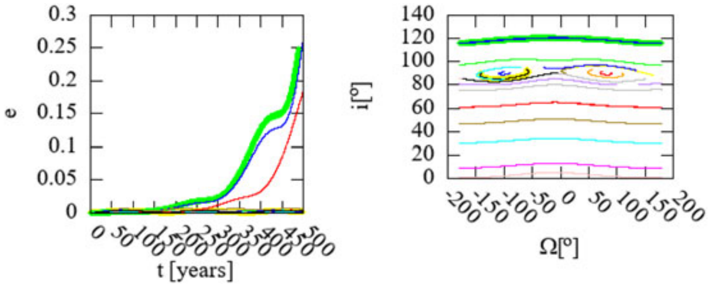

Figure 3. Evolution of the orbital elements for the case of particles orbiting Vesta, located at the distance 6000 km, with the initial eccentricity $e = 10^{-4}$. The green and yellow thick lines represent orbits propagated using the Cartesian approach and the thin lines are obtained using the Hamiltonian formalism. In both frameworks, the effects of the J_2, J_3 and J_4 harmonic terms is taken into account.

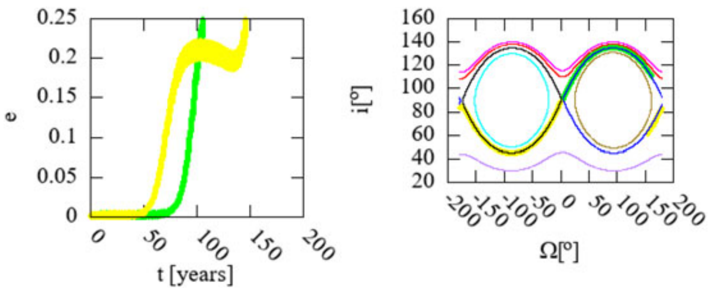

Figure 4. Evolution of the orbital elements for the case of particles orbiting Eros, located at the distance 250 km, with the initial eccentricity $e = 10^{-6}$. The green and yellow thick lines represent orbits propagated using the Cartesian approach (considering all 3 harmonic terms) and the thin lines are obtained by using the Hamiltonian formalism (considering only the J_2 term).

For relatively small inclinations (the secular resonances are absent), the circular orbits around Vesta are stable (remain almost circular) even at appreciable distances (see Figure 3). At $i = 63.4^o$ and $i = 63.4^o$ and $i = 116.4^o$ are located the critical inclination resonances. The curves in green, blue and red (Figure 3) cross one of these resonances, and as described above the eccentricity increases considerably.

On the other hand, the initially circular orbits around Eros do not share the same properties, due to the large obliquity of Eros, about 89^o. At distances, let say larger than 250 km, the inclinations of the orbits perform large excursions (see Figure 1, bottom panels) and as a consequence, they cross the regions where secular regions are located. As effect of the J_3 harmonic term the eccentricity becomes larger than zero, and the secular resonances induces a large increase in eccentricity (see Figure 4).

4. Implications

In this work we have done an analysis of the equilibrium points (in the (Ω, i) plane) and we have obtained some preliminary results: the position of the equilibrium points and the amplitude of the libration islands depend on a variety of parameters (the semi-major axis of the asteroid, its eccentricity, its obliquity and so on). The simulations show that a key parameter is the obliquity of the minor body. The results are important for the determination of the orbital dynamical environment of asteroids and, in particular, in the analysis of the long-term stability of circular orbits. A preliminary conclusion is the following: Vesta might possess satellites even at appreciable distances, provided the inclination is less than 45^0. However, in the case of Eros, for altitudes larger than 250 km, the large Eros' obliquity of 89° (see Souchay et al. (2003)) induces large excursions in inclinations that cross multiple secular resonances. Eros cannot have satellites on circular orbits at distances larger than 250 km.

Acknowledgements. This work was co-funded by the European Social Fund, through Operational Programme Human Capital 2014-2020, project number POCU/993/6/13/153322, project title "Educational and training support for PhD students and young researchers in preparation for insertion into the labor market". Acknowledgment is given to infrastructure support from the Operational Program Competitiveness 2014-2020, Axis 1, under POC/448/1/1 Research infrastructure projects for public R&D institutions/Sections F 2018, through the Research Center with Integrated Techniques for Atmospheric Aerosol Investigation in Romania (RECENT AIR) project, under grant agreement MySMIS no.127324.

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