On the continuum damage mechanics approach to modeling of polar ice fracture: a reply

1. BACKGROUND

Fracture of ice is an important process in ice-sheet dynamics, leading to the detachment of icebergs from glaciers and ice shelves and allowing the propagation of water-filled crevasses from the surface to the bottom of ice shelves and ice sheets (Benn and others, 2007). Historically, fracture propagation in ice has been treated using linear elastic fracture mechanics (LEFM) (Lawn, 1993; Van der Veen, 2007) or the Nye zero-stress model (Nye, 1957; Jezek, 1984; Nick and others, 2010). In our two recent papers (Duddu and Waisman, 2012, 2013) we proposed a nonlocal continuum damage mechanics (CDM) approach as an alternative to the commonly used LEFM- and Nye-model-based approaches. In the first paper (Duddu and Waisman, 2012), we presented a viscoelastic constitutive damage model for polycrystalline ice aimed at capturing its time-dependent creep behavior at low stresses leading to failure. An interesting finding of our study is that a power-law-based creep damage model is sufficient to phenomenologically capture the tertiary creep behavior of ice. In a follow-up paper (Duddu and Waisman, 2013), we presented a nonlocal damage mechanics formulation of the constitutive model within a finite-element framework that alleviates the pathological mesh dependence of damage computations. In Duddu and Waisman (2013), tensile creep fracture was studied and crack propagation was simulated under uniaxial and biaxial tension.

In their comment, Gagliardini and others (2013) claim that we did not accurately consider the specificities of ice rheology under compression and that, as a consequence, the damage mechanics model is inappropriate for studying crevasse propagation. However, it is important to note that crevasses are tensile cracks, so Galiardini and others' main criticism regarding the damage description under compression in Duddu and Waisman (2012) is not relevant to crevasse propagation, which is our main interest. The appropriateness of our model for investigating tensile-stress-induced surface crevasse propagation in glaciers and ice sheets is demonstrated in our recent publication (Duddu and others, 2013). In the next section, we briefly respond to the two criticisms by Gagliardini and others (2013) and clarify the important aspects of the damage model under compression. We conclude with a discussion on the calibration and validation of damage models for studying crevasse propagation and iceberg calving.

2. CRITICISMS BY GAGLIARDINI AND OTHERS (2013)

Criticism 1: *The model in Duddu and Waisman (2012), based on the formalism of Murakami (1983), is not appropriate for ice as it only applies to ductile failure associated with diffusion-controlled cavity growth. As a consequence of this wrong hypothesis, damage is initiated using a strain threshold as opposed to a stress or a strain-rate threshold. The strain threshold will cause damage to accumulate even for very low stresses when no damage is observed to occur, so it is unphysical. A strain threshold is not adapted for many places in ice sheets and glaciers where tensile stress remains too small to initiate any damage, whatever the deformation level.*

Gagliardini and others (2013) associate our CDM-based approach with a diffusion-controlled cavity growth mechanism (CGM-)based approach. For a more detailed account of the differences between the CDM- and CGM-based approaches we refer the reader to a review paper by Yao and others (2007). The formalism of Murakami (1983), originally developed for polycrystalline metals, improved the Kachanov (1958) and Rabotnov (1963) model by considering the effect of damage-induced anisotropy. However, Murakami (1983) does not associate the model with any particular failure behavior (i.e. ductile or brittle) because the formulation incorporates the Hayhurst (1972) criterion to account for different types of failure. Moreover, Murakami (1983) does not even mention 'diffusion-controlled cavity growth' because damage models are developed phenomenologically with no strict specification of the underlying physical mechanisms associated with material degradation. Thus, the compromise is that one loses the ability to resolve, much less decipher, microstructure deterioration. Since our goal is to represent ice behavior at the macroscale (i.e. at the scale of the ice sheet or ice shelf), it is not practical to resolve all the microscale features (i.e. crystal grains, dislocations, microcracks, etc.) and mechanisms (i.e. grain boundary sliding, dislocation pile-up formation, etc.). Therefore, we have proposed a CDM approach that is simpler and more amenable to computational analysis but can still reveal important macroscale fracture processes in glacial ice.

The strain threshold defined in Duddu and Waisman (2012) is essentially a phenomenological strain criterion for damage initiation and is not based on the hypothesis of diffusion-controlled cavity growth or ductile damage. Numerical calculations using our damage model indicated that a stress threshold or a strain-rate threshold cannot capture (1) the occurrence of a minimum strain rate at a constant strain or (2) the time required to initiate softening for different applied stresses, as observed in experiments (Mellor and Cole, 1982; Jacka, 1984; Glen and Ives, 1988). This is because the stress threshold is not a constant material parameter and depends on the applied strain rate; thus more model parameters are needed to account for its strain-rate dependency and more experimental data are required for calibration. In contrast, the results obtained using a strain criterion (threshold) are in better agreement with the experimental data, confirming that it does not lead to unphysical results. For example, when ice at $T = -5^{\circ}$C is subjected to an octahedral shear stress of 50 kPa (0.5 bar), with a strain threshold of 0.8%, our model does not predict any damage even after 10 000 hours, which is in agreement with the experimental data of Jacka (1984). Moreover, under low compressive stresses, damage quickly saturates at low levels and our model does not predict failure by rupture (characterized by a steep increase in strain rate) but it rather predicts a gradual failure process (characterized by a steady-state strain rate). Note that the internal damage variable in our model lumps together the softening effects of all the operative microscale mechanisms.

Indeed, under very low stresses, large strains can accumulate in the ice sheet over several years or even decades without any damage, and this deformation is primarily due to dislocation creep (Budd and Jacka, 1989). However, our damage model, proposed as an alternative to the LEFM models, is intended for regions of larger tensile stress (0.5–1 MPa) where fractures do develop, in which case purely viscous-creep-based flow models are not appropriate.

Also, our damage model is developed for short timescales on which fracture occurs (i.e. days to weeks) and is not intended for longer timescales (i.e. years to decades). Model parameters under tension are calibrated using the creep test data of Mahrenholtz and Wu (1992) at three applied uniaxial tensile stresses of 0.64, 0.82, 0.93 MPa. Therefore, our model is best suited to evolve damage accumulation in ice in the tensile stress range 0.5–1 MPa that is appropriate for simulating the propagation of surface crevasses in the vicinity of pre-existing defects. Also, for this range of tensile stresses, the strain or stress threshold does not seem to affect the results much.

Criticism 2: *The model in Duddu and Waisman (2012) is calibrated using experimental creep tests by Jacka (1984) and Mellor and Cole (1982), for which damage might only explain a small part of the total deformation (Mellor and Cole, 1982) or not even occur during the tests (Jacka, 1984). The underlying hypothesis is that all the tertiary creep deformation is due to damage, neglecting other softening processes such as dynamic recrystallization. The delayed elastic strain is not enhanced by damage, contrary to the postulate in Duddu and Waisman (2012).*

In section 3.2 of Duddu and Waisman (2012) it was clearly stated that dynamic recrystallization is an important mechanism at play during the tertiary creep stage under compression, and our model captures its effects with a single internal state (damage) variable. Given the sparse experimental data available for ice it may be reasonable to use a single scalar damage variable. However, we do agree that a single variable is not ideal for modeling the complex behavior of ice under compression that is governed by several physical mechanisms (e.g. recrystallization, microcracking, etc.). A more rigorous model can be developed (Brown and others, 1989) but will require sophisticated experimental data for model calibration and validation, which is currently not available for ice. An important point is that dynamic recrystallization, observed under low compressive stresses, is not relevant for studying crevasse propagation driven by tensile stresses over a timescale of days to weeks.

Previously, Sjölind (1987) and Karr and Choi (1989) postulated the delayed elastic strain to be a function of the effective stress, which is a function of the damage variable. In Duddu and Waisman (2012), we followed their description. However, Gagliardini and others (2013) point to the work presented in two conference papers (Meyssonnier and Duval, 1989; Weiss, 1999) that showed otherwise. But, as noted in Duddu and Waisman (2012), at low stresses or deformation rates the delayed elastic strain is negligibly small compared to the viscous strain during secondary and tertiary creep. From numerical calculations, we find that the delayed elastic behavior plays a significant role only in the primary creep stage wherein damage is negligible. Therefore the model results presented in Duddu and Waisman (2012) are still valid as they are unaffected by the lack of any damage enhancement of delayed elastic strain.

3. CONCLUSION

Our damage model is a phenomenological, single internal variable constitutive model, that describes the softening behavior of ice at macroscale by lumping the effects of all the microscale mechanisms. Under tension, Mahrenholtz and Wu (1992) report microcracking as the main damage mechanism, and in this case our damage representation is exactly the same as that of Kachanov (1958). Our model is constructed and calibrated based on (small-scale) laboratory experimental data (Mahrenholtz and Wu, 1992) and is used for studying (large-scale) surface crevasse propagation in glaciers due to the tensile stresses generated by gravity-induced creep flow (Duddu and others, 2013). This approach of model development from laboratory experiments is a common practice in geomechanics for characterizing other natural materials (e.g. soils and clays). Alternatively, as suggested by Gagliardini and others (2013), the construction and calibration of the model may be performed at the large scale using field data; however, such a model will not be capable of identifying the interplay between macroscale processes (e.g. tensile creep fracture, hydrofracture due to meltwater) and more so the microscopic processes. In contrast, the field data from remote-sensing or satellite imagery can be used to validate our model predictions and to discover the role of various macroscale fracture processes associated with full-depth crevasse propagation that can eventually lead to iceberg calving.

Our nonlocal continuum approach is an improvement over the traditional LEFM-based approaches because our approach does not neglect the viscous effects such as stress relaxation at crack tips and time-dependent creep deformation that drives deeper crack propagation. The simulation results indicate that crevasses propagate to a depth where the tensile stress vanishes, in accordance with the Nye zero-stress model, which illustrates the viability of our approach. The advantage of a CDM-based model is that it allows for complex interaction between crevasses and boundary conditions, which is not possible using analytic formulations. Definitely, more work is required to improve the damage model and its numerical implementation, which is a direction of our future research, and our nonlocal CDM approach (Duddu and Waisman, 2012, 2013) is an important first step in this direction. We believe our approach will lead to a more reliable modeling framework than the existing approaches based on LEFM and Nye models; therefore, it can be used for predicting the propagation of crevasses leading to iceberg calving due to time-dependent tensile fracture.

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