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THE KINETIC FRICTION OF SNOW

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"At 36 below zero, the sledges (with steel runners) dragged hard over young ice covered with an inch of granular snow. Sand could hardly have been worse." D.B. Macmillan (1925)

ABSTRACT. Three components of the kinetic friction of snow are described but only the lubricated component of friction is treated in detail. This component depends upon the thickness of water films which support a slider on snow grains over a small fraction of its area. The thickness of the film decreases with ambient temperature in a manner which is sensitive to the thermal conductivity of the slider. The minimum value of friction at any temperature is reached at an intermediate value of speed because friction decreases as the slider first begins to move and the films form but then increases at higher speeds because of the shear resistance. At sub-freezing temperatures a small area in the front part of the slider is dry and the friction is high. Once the water film is formed it increases in thickness towards an equilibrium value which can be very sensitive to slider properties, speed, and temperature. It appears that the mechanisms may be very different for hydrophobic and hydrophilic sliders. From the equations derived here it is clear why friction decreases with repeated passes over the same snow.

LIST OF SYMBOLS

c	Ratio of area to weight
c_p	Specific heat of slider
f	Coefficient of total friction
f_D	Coefficient of dry friction
f_s	Coefficient of suction friction due to excess water
f_w	Coefficient of lubricated friction
F	Force on an ice grain
h	Water-film thickness
h_∞	Water-film thickness for 0 °C at infinite distance along slider
H	Thickness of slider
k_i	Thermal conductivity of ice
k_s	Thermal conductivity of slider
l	Length of slider
l_D	Length of dry area
L	Latent heat of fusion
$\dot{m}$	Mass rate of water production
N	Number of load-bearing grains
q	Heat flow at slider—ice contact
q_s	Heat flow into slider
Q	Total displacement rate from one contact
r	Radius of water film
S	Supplemental water source per contact
t	Time
T	Temperature
T_0	Initial surface temperature
T_s	Surface temperature of slider—ice contact
$\overline{T'}$	Average temperature gradient at lower surface of slider
u	Slider speed
u_c	Critical slider speed for onset of lubricated friction
v	Water-film extrusion speed
w	Width of slider
W	Slider weight
x	Coordinate along slider
y	Coordinate below water—ice contact
α	Ratio of supplemental water source to water supplied by melt
β	Coefficient in f_D
β_j	Represents four heat-flow cases described in Table I
γ	Coefficient in f_s
ε	Coefficient in f_D
κ	Thermal diffusivity of ice
κ_s	Thermal diffusivity of slider
μ	Water viscosity at 0 °C
ρ	Water density at 0 °C
ρ_s	Density of slider
τ	Minimum time for slider bottom to reach 0 °C
τ_1	Average time a point on slider makes contact with water film
ϕ	Fractional area of contact

INTRODUCTION

The kinetic friction of snow has been studied most intensively because of the widespread interest in recreational skiing (e.g. Bowden and Tabor, 1964). There are many other interests such as slip of snow tires but the challenge of designing a faster ski has received the most attention. Evans and others (1976) explained the basic mechanism for ice skating, but most knowledge of snow friction is based on skier experiences or experimentation. The theory of sliding on snow given here considers a variety of conditions, but most of the attention is given to the intermediate values of water-film thickness where friction is minimized.

It is clear that sliding over snow involves a water layer, as does sliding over ice. This layer has been observed directly (Kuroiwa, 1977) and measured by its capacitance (Ambach and Mayr, 1981); its existence is further shown by the heat-dissipation calculations made here and field observations of the icing of snow surfaces by skis.

Sliding over snow is necessarily more complicated than sliding over ice because of the large variety of possible snow conditions. Snowfall often consists of angular dendrites or needles which are "aggressive" and increase the solid-to-solid component of the friction. Snow surfaces sometimes consist of vapor-deposited crystals like the "surface-hoar" crystal shown in Figure 1a. More often, however, we are interested in snow which has been aged enough to eliminate the sharp edges of faceted crystals. The cluster of grains from wet snow shown in Figure 1b is more typical of the type of snow we consider here.

Snow crystals are highly variable (Colbeck, in press) so that the three components of snow friction suggested by Klein (1947) — solid-to-solid, lubricated, and capillary suction — are somewhat dependent on the prevailing crystal type as well as the common parameters of snow — temperature and liquid-water content. These three components of

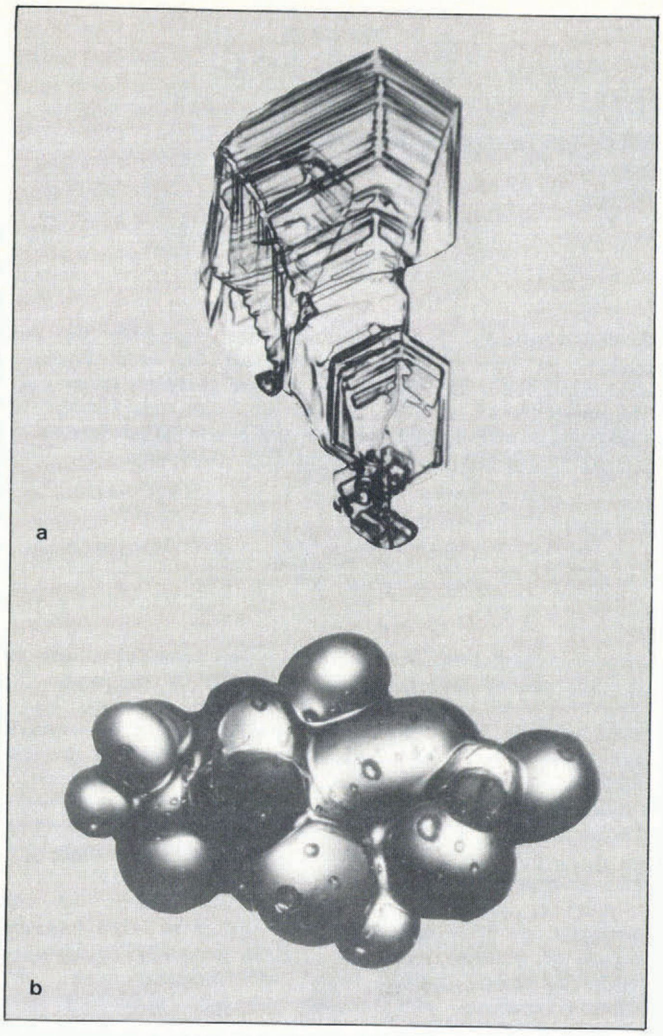

Fig. 1. a. A vapor-grown crystal on the snow surface called "surface hoar". These are commonly 3 mm to 3 cm in size. b. A cluster of ice crystals with liquid water held along the grain-boundary grooves and in interior veins. In wet snow the single crystals are typically 1 mm in size.

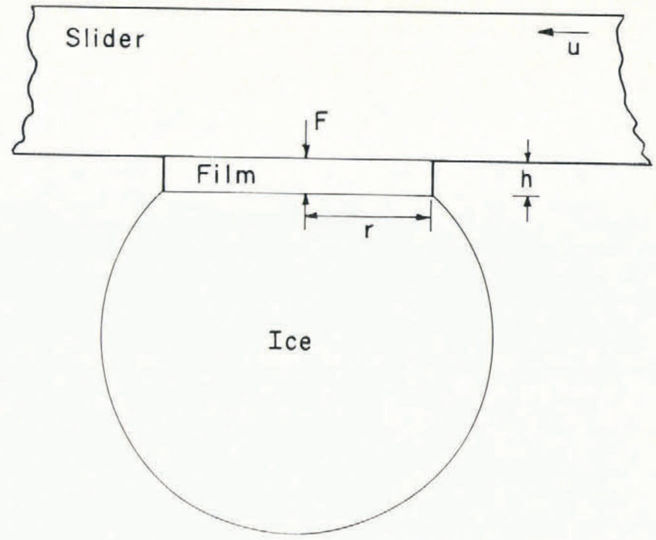

Fig. 3. A slider moving over a load-bearing ice grain with an intervening water film. The film is sheared by the movement of the slider.

snow friction are relatively important, depending on the thickness of the water film over which the slider moves. As suggested in Figure 2, the friction is high when the thickness of the water film is insufficient to prevent ploughing by solid-to-solid contacts. As the water film thickens and solid-to-solid interactions become less frequent, the slider should only have to overcome the viscous resistance of the water film between the supporting snow grains and the slider (Fig. 3). It is important to recognize that not all of the snow grains carry the load of the slider but, as shown later, relatively few of the grains develop a pressurized water film that supports the weight of the slider and resists its movement viscously. As the water production increases, the extrusion and shearing of water out of the pressurized water films increases along with the films' thicknesses. Both the increased expulsion and the greater quantity of melt water generally present in snow at higher temperatures must account for the often observed increase in friction when the snow is very wet. The apparent explanation for the frictional increase at large film thicknesses shown in Figure 2 is capillary attraction between ice grains and the slider as simulated in Figure 4. This bridging would occur to ice grains which are not carrying any of the load. These liquid bridges are under "tension" at all separations for small contact angles as well as for separations greater than about 10% of the radius of the particle at large contact angles (Hwang and others, 1987). The liquid bridge shown in Figure 4 spans a very large separation between hydrophilic and hydrophobic surfaces, and it is very likely that the bridge is under a state of capillary attraction and that the energy required to separate the surfaces and break the connecting bridge is large. However, even if the water were not in the usual state of capillary tension, the water held between the slider and the non-supporting grains would still exert a drag on the slider.

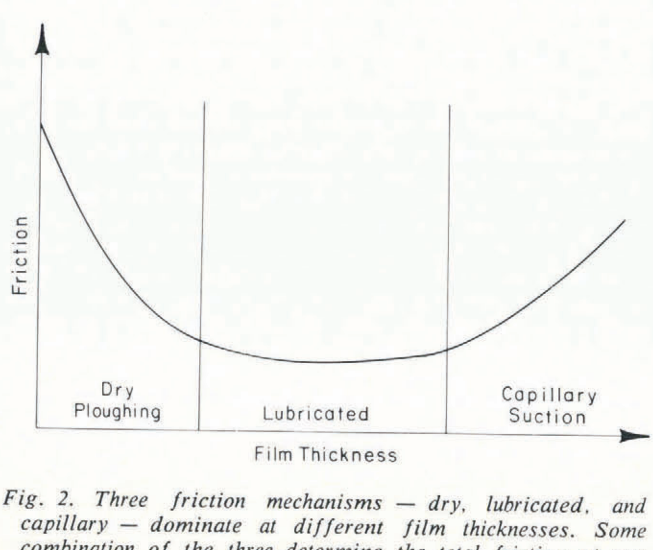

Fig. 2. Three friction mechanisms — dry, lubricated, and capillary — dominate at different film thicknesses. Some combination of the three determine the total friction at any film thickness.

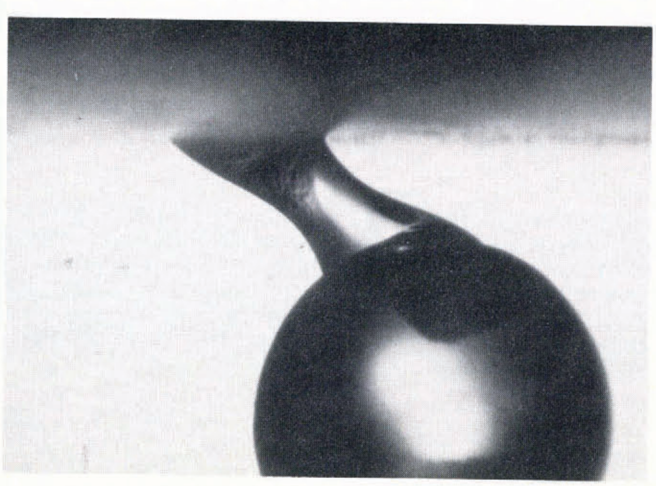

Fig. 4. Water held between a hydrophilic bead and a teflon slider while the slider is moving to the left. This water bridge would exert a drag on the slider without supporting any weight.

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The mechanisms of dry friction are not described here but are important at very low speeds or high roughnesses. The principles developed elsewhere (e.g. Lim and Ashby, 1987) could be applied to ice, but the interesting thing about ice is that it generates its own lubricating layer. Only the component of the coefficient of friction due to the lubricating layer f_W is described in detail. We return to the dry and capillary frictions later.

FRICTION AT THE MELTING TEMPERATURE

The water film shown in Figure 3 is being displaced by shearing as the slider moves over it, and the film is extruded by the weight of the slider it supports. The extrusion speed v is (Moore, 1965)

$$v = \frac{Fh^2}{3\pi\mu r^3} \tag{1}$$

where F is the force on the particle, h is the thickness of the water film, μ is the viscosity of water at 0 °C, and r is the radius of the film. The average pressure in the film is $F/\pi r^2$. The removal of water by the shearing action of the slider is taken to be one-half of the volume of the film for a displacement equal to the root-mean-square value of the contact diameter. Therefore, the total displacement rate Q from one contact is

$$Q = 2\pi rhv + rhu \tag{2}$$

where u is the speed of the slider.

The shearing of the film generates heat at the rate of $\mu(\mathrm{d}v/\mathrm{d}y)^2$ (Bird and others, 1960) where $\mathrm{d}v/\mathrm{d}y$ is u/h. Accordingly, the mass rate of water production at one contact $\dot{m}$ is

$$\dot{m} = \frac{\mu\pi}{L}\frac{u^2r^2}{h} \tag{3}$$

where L is the latent heat of fusion. When the melt and displacement rates balance so that the film thickness is steady,

$$\frac{\pi\mu}{\rho L}u^2r^4 = \frac{2F}{3u}h^4 + r^3h^2u. \tag{4}$$

The load carried by a contact F is equal to W/N, where W is the total weight of the slider and N is the number of load-bearing grains.

Snow friction is reported to be independent of weight (Perla and Glenne, 1981) which is a consequence of the true contact area being proportional to the load (Bowden and Tabor, 1956). Accordingly, it follows that

$$\pi r^2N = cW \tag{5}$$

where c is a constant. From the measurements of Bowden and Hughes (1939), c is about $7.25 \times 10^{-7} r^{1/2}$ Pa^{-1} where r is in mm. The lubricated component of the coefficient of friction f_W is the shear force divided by F, where the shear stress is the shear force times the area πr^2. The shear stress is $\mu u/h$ so the shear force is $\pi r^2\mu u/h$. Using NF equal to W, it follows that

$$f_W = c\mu u/h \tag{6}$$

which is a variation on plane Coutte flow.

This equation describes lubricated friction in terms of the slider speed and film thickness, which are related themselves through Equation (4). From Equations (4) and (5)

$$\frac{\pi\mu}{L\rho} = \frac{2\pi}{3\mu c}\left[\frac{h^2}{ur}\right]^2 + \frac{h^2}{ur} \tag{7}$$

which can be resolved to show

$$\frac{h^2}{ur} = \frac{3\mu c}{4\pi}\left[\left(1 + \frac{8}{3}\frac{\pi^2}{c\rho L}\right)^{1/2} - 1\right]. \tag{8}$$

From Equation (7) the ratio of the expulsion rate by squeeze to that by shear is $2\pi h^2/3\mu cur$ or, using Equation (8), about 0.02. This shows that we are justified in ignoring the contribution of the squeeze flow in determining the shear-strain energy, and we will use a simplified version of Equations (7) and (8) to find the film thickness,

$$h^2 = \frac{\pi\mu}{\rho L}ur. \tag{9}$$

This indicates that, even for fairly icy conditions and in the absence of heat flow into the slider or the snow, the film thickness never exceeds about 2 µm in snow skiing. This value is about one-fifth of that calculated by Ambach and Mayr (1981) from capacitance measurements; a possible explanation for this difference in terms of hydrophilic versus hydrophobic surfaces is given later. Evans and others (1976) reported a film thickness of 0.3 µm for ice skating, so it is clear that friction can be low even at these low film thicknesses.

It is assumed here that Nr^2 is constant, but the evidence strongly suggests that r^2 increases with repeated passes of a slider over snow, even in hard, boot-packed snow on race courses. Also, with repeated passes, the friction decreases even though the total supporting area $N\pi r^2$ remains constant. From Equations (6) and (9) it is clear that the lubricated friction decreases as $(r)^{\frac{1}{2}}$ increases. Thus, with repeated passes, f_W and N decrease while the track becomes icier.

The lubricated friction could be reduced by the addition of a water source S which operates at each contact and is represented by

$$S = \alpha\frac{\mu u^2\pi r^2}{Lh}. \tag{10}$$

Using Equations (3) and (9), the film thickness increases as $(\alpha + 1)^{1/2}$ as shown in Figure 5, and according to Equation (6), the lubricated component of friction should drop as $(\alpha + 1)^{-1/2}$. The effect of adding a water source or heat to accomplish more melting is moderated by the fact that, at greater film thicknesses, the heat generated by shearing is reduced and the removal of water by shearing is increased. Thus, the lubricated component of friction is reduced by the introduction of water but not greatly. In addition, excessive amounts of water are known to increase the total friction, presumably by the coupling shown in Figure 4.

The amount of water extruded per contact per pass of a slider is rlh, where l is the length of the slider. In high-speed skiing this is about enough water to bond 30 non-supporting grains to the ski. If the tension in the attached water were 1300 N/m^2, a value which has been measured in snow (Wankiewicz, 1979), and the axes of the water bridges were bent to 45°, the total force on a ski would be about 30 N if all of the melt water bonded to the ski after expulsion from the load-bearing films. This is equal to about the down-slope component of gravity on a typical ski on a 10° slope, so the adhesion of the slider to unloaded snow grains by capillarity is a very powerful deterent to forward motion. The common practices of roughening the bottom of a ski "to shed water" or streaking the ski to provide grooves for water have evolved to reduce this problem on warm days. From this calculation it would appear that this could be a factor even on cooler days.

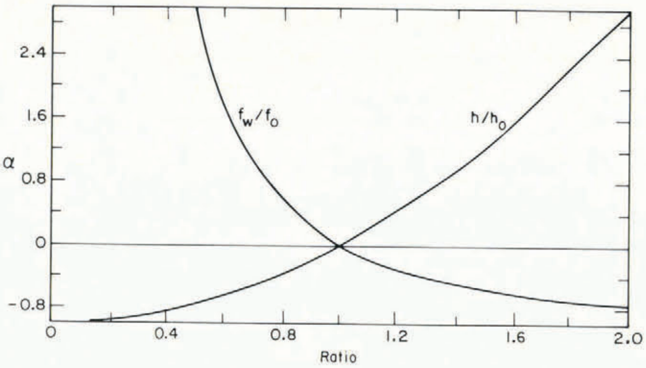

Fig. 5. The variation of the film thickness and lubricated friction versus α, representing a water source (α = 1 means that the water supplied equals the amount from melt). h_0 is the film thickness and f_0 the lubricated friction when there is no source or sink.

TABLE I. FOUR APPROACHES TO HEAT FLOW

Case	*Assumption*	*Value of* β_j
1	Slider insulated; heat flow in ice only	$\pi^{\frac{1}{2}} k_i T_0 r/(\kappa x u)^{\frac{1}{2}}$
2	Heat flow at contacts only; point on slider at T_0 through entire thickness when contact is made	$\beta_1 + 2T_0(2rk_s\rho_s c_p/u)^{\frac{1}{2}}$
3	Entire lower surface at 0°C	$\beta_1 + \pi k_s T_0 wlr/cWHu$
4	Average temperature gradient calculated from periodic solution to on, off boundary temperature.	$\beta_1 + \pi k_s T_0 r/Hu$

FRICTION BELOW THE MELTING TEMPERATURE

The problem of heat conduction into the ice can be solved as shown by Evans and others (1976). Heat is conducted into the weight-bearing ice grains as the transients of temperature propagate downward. The slider also conducts away heat but in a steady fashion once the slider itself reaches steady state.

Treating each ice grain as a semi-infinite solid whose surface is held at 0°C for a time l/u, the temperature varies as (Carslaw and Jaeger, 1959)

$$T = T_0 \operatorname{erf}(y/2(\kappa t)^{\frac{1}{2}}) \tag{11}$$

where T_0 is the initial surface temperature in °C, y is the distance below the water film, κ is the thermal diffusivity of ice, t is the time since the slider made contact, and l is the length of the slider. The heat flow q at the contact is

$$-k_i \partial T/\partial y|_0$$

or

$$q = -\frac{k_i T_0}{(\pi \kappa t)^{\frac{1}{2}}} \tag{12}$$

where k_i is the thermal conductivity of ice. This reduces the melt-water production rate by q/L, where t is x/u when x is between 0 and l. Equation (9) can be re-derived to find the thickness of the water film including heat flow into the ice:

$$h = \left[\beta_j + \left[\beta_j^2 + 4\pi(\alpha + 1)\rho L \mu u r\right]^{\frac{1}{2}}\right] \Big/ 2\rho L \tag{13}$$

where for an insulated slider,

$$\beta_1 = \frac{\pi^{\frac{1}{2}} k_i T_0 r}{(\kappa x u)^{\frac{1}{2}}} \tag{14}$$

and j represents cases 1 through 4 which are described in Table I. The effect of temperature on the film thickness after 1 m of water production is shown as case 1 in Figure 6 for an insulated slider, where it can be seen that the film thickness decreases almost linearly with temperature. This decrease is not in itself sufficient to explain the great decrease in friction which is commonly observed. A linear decrease in film thickness was measured by Ambach and Mayr (1981) but with a stronger temperature dependence, depending somewhat on the conditions. As shown next, the thermal conductivity of the slider further increases the film's temperature dependence, and all sliders maintain a dry surface in the front which can increase their friction on cold snow.

In the presence of a water source and heat conduction into both snow and slider, the water-production rate per contact $\dot{m}$ for a slider which is dry everywhere except at load-bearing contacts is derived in the Appendix as case 2 and is given by

$$\dot{m} = (\alpha + 1)\frac{\mu\pi}{L}\frac{u^2 r^2}{h} + \frac{\pi r^2 k_i T_0}{L(\pi\kappa x/u)^{\frac{1}{2}}} + \frac{2T_0 r}{L}(2urk_s\rho_s c_p)^{\frac{1}{2}} \tag{15}$$

where k_s, ρ_s, and c_p are the thermal conductivity, density, and specific heat of the slider. Equation (10) was used to represent the source and Equation (12) the heat flow into the snow. Again assuming that all of the water is displaced by shear, Equation (13) still describes the thickness of the water film but β_j assumes an additional term to account for heat flow into the slider, or

$$\beta_2 = \beta_1 + 2T_0(2rk_s\rho_s c_p/u)^{\frac{1}{2}}. \tag{16}$$

In Figure 6 the thickness of the water film beneath the slider is shown to vary drastically with temperature for a highly conductive aluminum slider whose base is dry everywhere except at the load-bearing contacts. This

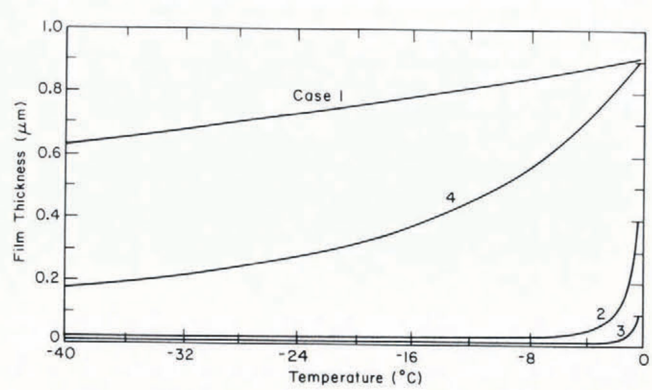

Fig. 6. The thickness of the water film after 1 m of water-film production in perfectly insulated and aluminum sliders versus temperature (u = 10 m/s, H = 10 mm, and r = 5 mm). Cases 1 through 4 are for different heat-flow assumptions as described in Table I.

suggests that the temperature at which the roughness elements begin to interact with the slider is very sensitive to the thermal conductivity of the slider. This result also shows the very great advantage achieved by packing runways for ski-equipped aircraft, because packing or icing greatly increases r and therefore reduces the effect of the heat-flow terms in these equations. This is especially important for such applications as aluminum aircraft skis at low temperatures. While qualitatively correct, the result shown in Figure 6 for an aluminum slider whose base is dry everywhere except at the load-bearing contacts overestimates the effect of heat flow through the slider, at least at high speeds. Bowden (1953) found that aluminum skis have a much lower friction than suggested by the curve for case 2 in Figure 6 for a dry base. However, at low speeds the assumptions behind the derivation of Equation (16) appear to be reasonable, and the result shown in Figure 6 is consistent with Bowden's measurements. Nevertheless, at speeds greater than 5 m/s the friction is so low that the heat flow into the slider must be less than that derived in the Appendix. The apparent problem is that we assumed that the temperature was T_0 throughout the entire thickness when a point on the slider came in contact with a water film.

We examine the heat flow at high speeds in another way by assuming that the temperature of the upper surface is at T_0 and the temperature of the entire lower surface is $0\,^\circ$C. Then, the total heat flow through the slider is $-k_s T_0 wl/H$ and Equation (13) still applies where

$$\beta_3 = \beta_1 + \pi k_s T_0 wlr/cWHu. \tag{17}$$

As shown in Figure 6 for case 3, the slider is even more temperature sensitive if its base is everywhere at the melting point because of the larger heat flow.

In the final attempt to characterize the heat flow through the slider, we again assume that the bottom of the slider is at the melting temperature at the contacts and at T_0 everywhere else. The average temperature gradient over the entire slider is $-cWT_0/wlH$ as calculated in the Appendix from the temperatures at the boundary. The film thickness is then given by Equation (13) where

$$\beta_4 = \beta_1 + \pi k_s T_0 r/Hu. \tag{18}$$

As shown in Figure 6 for case 4, the film thickness decreases with temperature but not as much as for cases 2 and 3. This approach gives about the correct temperature dependence and is physically reasonable, so we use it for the remainder of this work. Nevertheless, it is somewhat arbitrary since we have assumed an instantaneous decay in the surface temperature which probably leads to an underestimation of the amount of heat flow into the slider.

Bowden's (1953) results suggest a sudden transition between two conditions, as if there is an easy glide and a hard glide with separate mechanisms. He also reported that an aluminum ski could suddenly stick to the snow at $-10\,^\circ$C when the speed fell below a critical value. This critical speed u_c probably occurs when the total heat generated by sliding is less than the heat flow through the slider plus the heat flow into the ice. For an insulated slider this critical speed is given by

$$u_c = \frac{4c^2 k_i^2 T_0^2}{\pi \kappa l f^2} \tag{19}$$

where f is the coefficient of total friction. For plastic skis this critical speed is about 0.10 m/s at $-10\,^\circ$C, below which some of the melt water probably begins to freeze on to the ski after being displaced from the load-bearing films. As shown next, the friction increases sharply at lower speeds for another reason too.

Equation (6) shows how the lubricated component of the coefficient of friction varies with speed and the thickness of the water film. Equation (13) with β_4 constrains the thickness at any speed and temperature. Using these equations, we calculate the coefficient of the lubricated friction for a plastic slider as shown in Figure 7a and for an aluminum slider as shown in Figure 7b. At sub-freezing temperatures this coefficient passes through a minimum value at a speed which increases as the temperature decreases. At low speeds the friction is high because heat flow away from the contacts begins to dominate the heat balance, and at high speeds friction increases as $(u)^{\frac{1}{2}}$ because of the shear force in the water films. At the lowest speeds the effect of heat flow up through the slider is probably greater as shown in Figure 6 for case 2 but, as discussed later, at lower speeds the processes of dry friction begin to operate so the total friction is limited to the dry value as the water film vanishes.

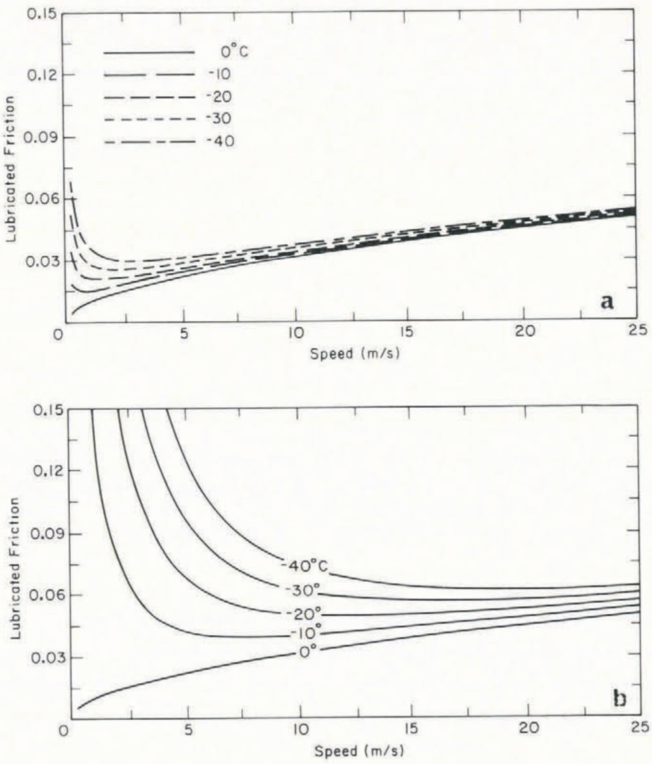

Fig. 7. The lubricated friction versus speed at various temperatures (H = 10 mm, r = 1 mm, x = 1 m, wl = 0.14 m^2, W = 380 N). Part (a) is for plastic and part (b) is for aluminum.

FRICTION ALONG THE SLIDER

So far we have assumed that the slider is lubricated along its entire length, but actually the slider is dry over some part of its front. The dry friction is larger than the lubricated friction, at least at the intermediate values of film thickness shown in Figure 2. The rate of heat production is correspondingly higher. When the moving slider first contacts a snow grain, the slider is in an overall state of thermal equilibrium but the surface temperature T_s of the snow grain increases as (Carslaw and Jaeger, 1959)

$$T_s = T_0 + \frac{2q}{k_i}\left(\frac{\kappa t}{\pi}\right)^{\frac{1}{2}} \tag{20}$$

where q is the heat flux into the ice grains. We assume that the heat is conducted into the slider only at the contacts and that the heat flow is uniform up through the slider. The heat is conducted into the ice grains over an area of $\phi w l_D$, where l_D is the length of the dry area and ϕ is the fractional area of contact.

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Over the dry area the heat produced by dry friction ($f_D Wul_D/l$) just balances the heat conducted up through the slider ($-0.5T_0 k_s wl_D/H$) and the heat conducted down into the snow ($q\phi wl_D$). Using Equation (20) and the average heat balance over the dry area, the slider is dry over the length given by

$$l_D = \frac{\pi u k_i^2}{\kappa}\left[-\frac{2f_D u}{cT_0} - \frac{k_s}{\phi H}\right]^{-2}. \qquad (21)$$

Once the interface reaches the melting temperature and the water film begins to form, the friction decreases from the dry value to one which is given by some combination of the three components of friction. Given the assumption that has been used for heat flow, the length of this dry area is found to be small except for metals at low temperature. According to Equation (21), a plastic ski is dry over only a few centimeters at −40°C. For an aluminum aircraft ski the dry part on the front of the ski could be considerable in certain areas of aircraft operation. For down-hill skiing, however, it appears to be small.

Once the water film begins to form, it thickens at a rate

$$\frac{dh}{dt} = \frac{\dot{m}}{\rho\pi r^2} - \frac{uh}{\pi r} \qquad (22)$$

which balances generation with drainage. Using u as dx/dt and the energy balance for case 4,

$$\frac{dh}{dx} = \frac{(\alpha + 1)\mu u}{\rho L h} + \frac{k_i T_0}{\rho L(\pi\kappa u x)^{\frac{1}{2}}} + \frac{k_s T_0}{\rho L H u} - \frac{h}{\pi r} \qquad (23)$$

describes the increase in film thickness along the length of the slider. The film appears to build up much faster along the slider at higher speeds since the last three terms are negative. Also, this equation shows that the film thickens more rapidly at higher temperatures and for non-conductive sliders.

At the melting temperature, Equation (2) has the solution

$$h = h_\infty\left[1 - e^{2(l_D - x)/r}\right] \qquad (24)$$

where h_∞, the value at infinite distance at 0°C, is given by

$$h_\infty^2 = \frac{(\alpha + 1)\pi r \mu u}{\rho L}. \qquad (25)$$

As shown in Figure 8, the film thickness increases most rapidly at the melting temperatures for any slider. Even for well-insulated sliders there is an effect of decreasing the ambient temperature because of the heat conducted into the snow. At −20°C this effect is not great for a plastic slider except that the dry area at the front of the slider is extended and the transition zone where the film thickens is longer. For a 10 mm thick, aluminum slider travelling at 10 m/s, the heat lost through the slider is about six times greater than that lost to the ice, so the thickness of the water film is greatly reduced. These effects were known to the early polar explorers who noted the difference between nickel-plated and all-wood sledge runners (Bowden and Tabor, 1950), and at least qualitatively, these results are correct.

To calculate the friction along the slider, it is necessary to quantify more than just the variation of film thickness with position. Three components of friction have been discussed, and all three vary with the film thickness. The friction due to plowing f_D decreases as the film thickens because there are fewer solid-to-solid contacts when the surfaces are separated by a lubricating film. We assume this takes the form

$$f_D = \varepsilon e^{-\beta h}. \qquad (26)$$

Taking the wet and dry frictions as parallel processes, the total friction f is

$$f = f_s + \frac{f_D f_w}{f_D + f_w} \qquad (27)$$

where the friction due to surface forces f_s is assumed to be an additional, rather than a parallel, process. We further assume that the capillary attraction increases as the rate of water expulsion, so from Equations (1) and (4),

$$f_s = \gamma h^3. \qquad (28)$$

We construct an example of how these terms interact by arbitrarily taking ε as 0.12, β as 1.0 μm^{-1}, γ as 1.5 × 10^{-2} μm^{-3} and u as 10 m s^{-1}. The result is shown in Figure 9, where the lowest value of friction occurs at a film thickness of 0.6 μm, which corresponds roughly to the film thickness calculated earlier for 0°C. The values of ε, β, and γ were chosen so that the results would agree with the dry friction for P.T.F.E. reported by Bowden and Tabor (1956) as well as their observation that ski friction is a minimum at 0°C. Below 0°C, dry friction and lubricated friction both increase with dry friction being the limiting process as the water film disappears entirely. Above 0°C, where much snow melting is taking place, the capillary friction increases and assumes control as the other frictions decrease.

The variation of friction along an aluminum slider is shown in Figure 10. The leading edge of the slider is dry when the temperature is below the melting temperature. This effect is very small for an aluminum slider at −10°C. Once melting begins, the water film increases in thickness as shown in Figure 8 and the friction decreases as shown

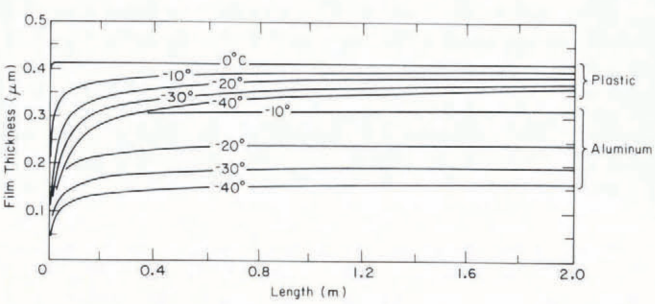

Fig. 8. Water-film thickness versus distance along the lubricated area for plastic and aluminum sliders. The result for 0°C would apply to any material (H = 10 mm, u = 10/ms, and r = 1 mm).

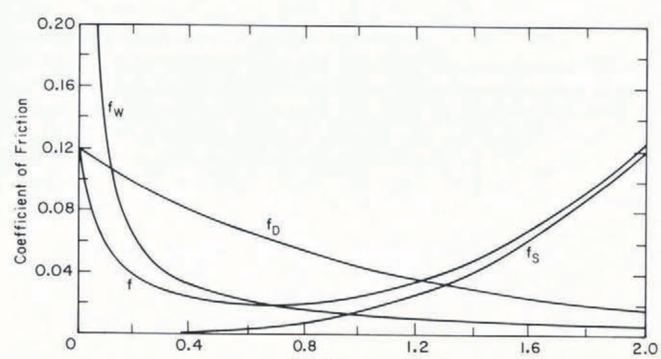

Fig. 9. Total friction f, dry friction f_D, lubricated friction f_w, and capillary friction f_s versus water-film thickness (H = 10 mm, r = 1 mm, and u = 10 m/s).

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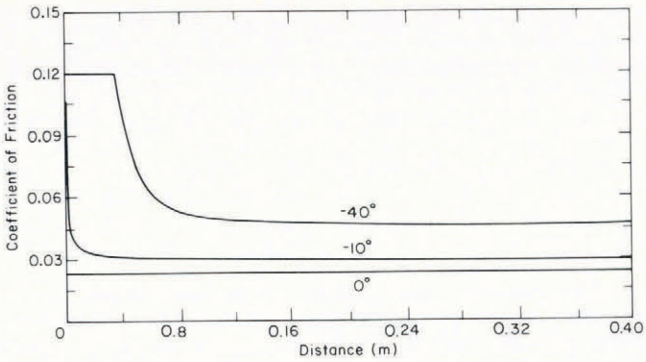

Fig. 10. Total friction versus length along an aluminum slider at various temperatures (r = 1 mm, u = 10 m/s, and H = 10 mm).

in Figure 9. It is apparent from Figure 10 that there are several differences between the slider's friction at 0 °C and at −40 °C. Because friction increases with water-film thickness below 0.4 μm, the friction over the trailing part of the slider is quite temperature-dependent. The total friction over the length of this slider, or the area under the curve, is about 30% greater at −10 °C and 150% greater at −40 °C. These increases are due to both the dry area at the front of the slider and the decreased film thickness due to thermal effects.

SLIDER TRANSIENT TEMPERATURES

At rest the entire slider–snow interface is dry. The interface warms by dry friction once the slider begins to move and the generated heat is conducted into both the snow and the slider. A lower limit can be placed on the time required to warm the slider to an equilibrium temperature profile by assuming that all of the heat produced at the interface is absorbed by the slider. The minimum time τ for the slider's lower surface to reach 0 °C at a constant speed from Equation (20) is

$$\tau = \frac{\pi}{\kappa_s}\left[\frac{-k_s w l T_0}{2 f_D W u}\right]^2 \tag{29}$$

where $f_D Wu/wl$ was taken as the heat flow. At a constant speed of 10 m/s, a typical plastic ski would have to travel more than 10 m to reach equilibrium. This long response distance suggests that many sliders are always adjusting thermally since in skiing, for example, the speed is highly variable.

DISCUSSION OF HYDROPHILIC VERSUS HYDROPHOBIC SURFACES

Equation (9) gives the film thickness when no heat is conducted away from the slider and the thickness of the film is determined by a dynamic balance between heat generated in and shear displacement of the films. According to this balance, the film thicknesses are less than those determined from capacitance measurements by Ambach and Mayr (1981). If there had been no shear displacement of the water in or heat flow out of their films, the thickness could have been increased to a value of 330 μm because of the frictional energy that was available to generate the melt layer. Thus it seems possible that their values are correct.

One possible explanation for the difference between their values and those calculated here is the observed difference in the behavior of thin water films between two glass plates and between one glass plate and a waxed ski. By placing a known quantity of water between two smooth microscope slides, I observed the friction between these smooth hydrophilic surfaces with water films varying in thickness between 2.5 μm and one-quarter of the wavelength of light, or about 0.15 μm. These glass slides always exhibited easy glide, indicating a low value of friction even at these thicknessses as suggested by Evans and others (1976). The behavior of water films between a glass slide and a waxed ski was very different, however, since in that case the water film showed a strong tendency to push the objects apart to a spacing of about 10 μm and to occupy a correspondingly smaller area between the ski and the glass. Another difference between the two hydrophilic glass surfaces and one hydrophobic and one hydrophilic surface was that, upon sliding, the water film would be sheared out from between the two glass plates in the manner described above but the water film tended to slide along the hydrophobic surface. Thus, it seems possible that our calculated film thicknesses are correct, but only for hydrophilic surfaces such as ice and metals, and that Ambach and Mayr's (1981) values are correct for waxed skis. If the water film is not sheared by a hydrophobic surface, then the theory given here could underestimate the thickness of the lubricating film for those surfaces. More observations are necessary to determine the nature of the films when one surface is hydrophobic. Both the mechanism for removing water by shear and for generating the heat by shear may be different than what is assumed here.

One way to analyze friction, if the surface of the slider is hydrophobic, is to assume that all the water is removed by squeezing, since shearing appears to be ineffective. In that case, combining Equations (1), (3), and (5), we find that

$$h^4 = \frac{3c\mu^2 u^2 r^2}{2\rho L} \tag{30}$$

which gives a somewhat thicker water film but the thickness is still limited by the amount of shear energy available for melting. It is difficult to see how enough heat could be generated to achieve film thicknesses of 8–10 μm unless the process is nearly 100% efficient and little of the water is displaced by either shear or squeeze. It is possible that we have underestimated the value of c. If c were larger, our results would agree more closely with the film thicknesses determined by Ambach and Mayr (1981) and the area determination of Kuroiwa (1977) but would not agree with the measured values or other statements of Bowden (1953).

IMPLICATIONS OF THE THEORY

Many aspects of the kinetic friction of snow that have been learned by experience can be explained and understood in a more quantitative way by this theory. For example, from these equations the lubricated component of friction decreases as the size of the load-bearing surfaces increases, i.e. as the snow becomes icy, the thickness of the water film increases. Also, the coefficient of friction decreases as temperature increases, which is perhaps the best-known result from all of snow- and ice-friction observations.

Bowden and Tabor (1950) showed that a highly conductive ski has more friction than a well-insulated one and that the difference between them increases as temperature decreases. Equation (23) shows that the rate of increase in thickness of the lubricating film along the length of a slider decreases with temperature, and Equation (21) shows that the dry area at the front of a slider increases somewhat with temperature drop. Both of these friction-enhancing effects are potentiated by the high thermal conductivity of metal sliders. The effect of thermal conductivity and temperature is shown in Figure 11 for an infinitely long slider. The equilibrium water-film thickness obtained from Equation (23) is very sensitive to the product $k_s T_0$ over the range of values from 10^3 to 10^5 W/m. For a good insulator like plastic, temperature has rather little effect on the equilibrium film thickness and most of the temperature effect on friction is the extension of the transition area shown in Figure 8. However, for an aluminum slider there is a large effect of temperature, since the dry area is extended and the water-film thickness decreases rapidly with decreasing temperature. This result

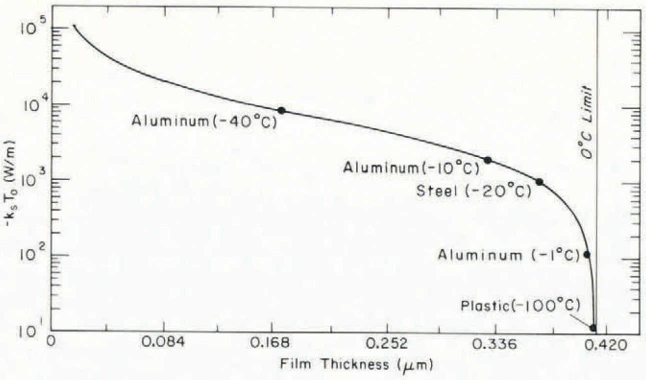

Fig. 11. Water-film thickness versus $-k_sT_0$ for an infinitely long slider (H = 10 mm, u = 10 m/s, and r = 1 mm) from Equation (23) minus the heat flow into the ice. Various temperatures and materials are indicated.

suggests that a long slider does not have a proportionately higher friction than a short slider because, once the water-film thickness approaches its equilibrium value, friction is minimized.

Bowden and Tabor (1964) reported that at low speeds the value of friction is close to the static value. On Figure 7 is it shown that lubricated friction drops rapidly as speed increases from zero, especially at higher temperatures. However, the friction passes through a minimum at an intermediate value of speed because of two counteracting effects. As an object begins to move and the water layer is first generated, the friction decreases just because of the existence of the lubricating layer. When everything is at the melting temperature, however, this occurs at any finite speed, so friction at that temperature increases as $(u)^{\frac{1}{2}}$ as shown by Equations (6) and (9). At lower temperatures there is a combination of these two effects with lubricated friction decreasing initially but then increasing with $(u)^{\frac{1}{2}}$ as the slider reaches higher speeds. Kuroiwa's (1977) data suggest that the total friction increases much more rapidly over this range of velocities.

Equations (6) and (9) also show that the lubricated component of friction decreases as $(r)^{\frac{1}{2}}$ increases, the size of the load-bearing areas. Ice always appears to have a lower friction than snow, and Perla and Glenne (1981) reported that friction does increase with decreasing grain-size. This effect arises from the increasing water-film thickness with grain-size and the corresponding decrease in lubricated friction. This phenomenon probably explains the decreased friction after ski racers have "set" a track; the icing in such tracks is clearly visible. In these tracks the load-bearing area, $\pi r^2 N$, should remain constant by decreasing the number of loaded surfaces as the average size increases. This could be accomplished in part by melting the loaded areas, although the average crystal would not melt away until after tens of passes. Observations of this phenomenon are needed.

CONCLUSION

The idea that the friction of snow and ice is reduced by a layer of melt water is still not completely accepted (Perla and Glenne, 1981). Melting is considered in other materials as well (Lim and Ashby, 1987) but is probably easiest to demonstrate for ice. This has been done by direct observations, indirect measurements, and energy-balance calculations. The thickness of the melt-water layer is highly dependent on size of the contacts, speed, temperature, and thermal characteristics of the slider. It does not depend on the load on the slider.

The calculations made here suggest that the frictional force on the slider varies rather little over an intermediate range of film thicknesses from about 0.2 to 1.2 μm. This insensitivity to film thickness accounts for the slight change in friction over a wide range of speeds and temperatures. This behavior arises in part from the interaction between heat generation by shearing and water removal by shearing. Even if a separate source of water is added, the thickness of the water film does not increase dramatically because of the accelerated removal at larger thicknesses. The lubricated component of friction decreases with the addition of a water source but not by much. In fact, a water source might even be counter-productive except in cases like that of a thin aluminum slider. These can have very thin water films at low temperatures because of the large amount of heat conducted through them.

The quantitative results presented here provide some of the answers to problems of snow friction but there are too many assumptions. The idea that load-bearing area is proportional to load is supported by the theoretical and experimental conclusions that friction is independent of load. However, with multiple passes of a slider, the snow surface changes and these changes should be observed and then included in the theory. Only one component of friction was examined in detail here; the others must be treated more extensively. It is especially important to examine the nature of the capillary attraction between the slider and non-supporting snow grains since this component of friction may be unique to snow. The calculation of heat flow up through the slider is only approximate and, as shown in Figure 6, the final results are quite sensitive to the results of that calculation. In spite of these limitations, this theory provides a quantitative way of thinking about snow friction and should help guide future experiments to resolve some of the outstanding issues. As with friction on ice (Evans and others, 1976), there are several processes which allow sliding, the thin water film being the most important feature.

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APPENDIX

TWO CASES WITH HEAT FLOW INTO SLIDER WHEN DRY BETWEEN CONTACTS

Case 2. Any point on the slider passes through a contact traversing its root-mean-square diameter, or $\pi r/2$, in a time $\pi r/2u$. Since the average distance between contacts is $lw/2rN$, the point on the slider passes between contacts in a time $lw/2urN$. The fraction of the time spent in contact with a water film is $N\pi r^2/wl$, the fraction of the slider which is in contact with the load-supporting films. Using Equation (5), this fraction is cW/wl or, for a typical ski, about 0.002. Because of the relatively large amount of time that a point on the slider spends between contacts, we assume that the slider temperature at that point returns to the ambient temperature, T_0 over the entire slider thickness, before the next contact is reached. Then the heat flow into the slider at each contact is given by Equation (12) for the slider,

$$q_s = -\frac{k_s T_0}{(\pi \kappa_s t)^{\frac{1}{2}}} \qquad \text{(A1)}$$

where the subscript s represents the slider and $0 < t < \pi r/2u$. The average heat flow at the point on the slider $\overline{q}_s$ during the traverse of the contact is given by

$$\overline{q}_s = -\frac{2uk_s T_0}{\pi^{3/2}\kappa_s^{1/2} r}\int_0^{\pi r/2u}\frac{dt}{t^{1/2}} \qquad \text{(A2)}$$

or

$$\overline{q}_s = -\frac{2}{\pi}T_0(2uk_s\rho_s c_p/r)^{1/2} \qquad \text{(A3)}$$

where ρ_s and c_p are the density and heat capacity of the slider. The average heat flow into the slider from one contact at any time is then

$$-2T_0\, r(2urk_s\rho_s c_p)^{\frac{1}{2}}.$$

Accordingly, the melt rate at a contact when the bottom of the slider is dry between the contacts is

$$\dot{m} = (\alpha + 1)\frac{\mu\pi}{L}\frac{u^2r^2}{h} + \frac{k_i T_0\,\pi r^2}{L(\pi\kappa x/u)^{\frac{1}{2}}} + \frac{2T_0 r}{L}(2urk_s\rho_s c_p)^{\frac{1}{2}} \qquad \text{(A4)}$$

which is used in the text as case 2.

Case 4. Another approach to heat flow into the slider is also based on the assumption that the temperature at its lower surface is either 0°C or T_0, depending on whether or not that point on the slider is in contact with a water film. Carslaw and Jaeger (1959, p. 107) gave the temperature distribution on the slider for this periodic boundary condition. From this temperature distribution we calculate the temperature gradients at the lower surface for periods of 0°C and periods of T_0. By taking the average for each period of 0°C and each period of T_0,

$$\overline{T'}(T_0) = \frac{2T_0}{H(\tau_1 - \tau)}\sum_{n=1}^{\infty}\frac{1 - e^{\alpha_n(\tau-\tau_1)} - e^{\alpha_n\tau_1} + e^{\alpha_n\tau}}{(1 - e^{\alpha_n\tau})\alpha_n} \qquad \text{(A5)}$$

and

$$\overline{T'}(0\,^\circ\mathrm{C}) = -\frac{T_0}{H} + \frac{\tau_1 - \tau}{\tau_1}\overline{T'}(T_0) \qquad \text{(A6)}$$

where τ_1 is the average time a point on the slider is in contact with a water film, τ is the time period of the contact–no contact–contact cycle, and

$$\alpha_n = \kappa_s n^2\pi^2/H^2. \qquad \text{(A7)}$$

The average temperature over the entire cycle is

$$\overline{T'} = \frac{\tau_1}{\tau}\,\overline{T'}(0\,^\circ\mathrm{C}) + \frac{\tau - \tau_1}{\tau}\,\overline{T'}(T_0) \qquad \text{(A8)}$$

which reduces to

$$\overline{T'} = \frac{T_0\tau_1}{H\tau}. \qquad \text{(A9)}$$

Since the fraction of time that a point on the slider is in contact with a water film is the fraction of the surface in contact with water films, $N\pi r^2/wl$,

$$\overline{T'} = -cWT_0/wlH. \qquad \text{(A10)}$$

This average temperature gradient over the entire slider is used in the text as case 4.

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