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Retrograde resonances in compact multi-planetary systems: a feasible stabilizing mechanism

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Abstract. Multi-planet systems detected until now are in most cases characterized by hot-Jupiters close to their central star as well as high eccentricities. As a consequence, from a dynamical point of view, compact multi-planetary systems form a variety of the general N-body problem (with $N \geqslant 3$), whose solutions are not necessarily known. Extrasolar planets are up to now found in prograde (i.e. direct) orbital motions about their host star and often in mean-motion resonances (MMR). In the present paper, we investigate a theoretical alternative suitable for the stability of compact multi-planetary systems. When the outer planet moves on a retrograde orbit in MMR with respect to the inner planet, we find that the so-called retrograde resonances present fine and characteristic structures particularly relevant for dynamical stability. We show that retrograde resonances and their resources open a family of stabilizing mechanisms involving specific behaviors of apsidal precessions. We also point up that for particular orbital data, retrograde MMRs may provide more robust stability compared to the corresponding prograde MMRs.

Keywords. celestial mechanics, planetary systems, methods: numerical, statistical

1. Introduction

To identify the dynamical state of multi-planetary systems, we use the MEGNO technique (the acronym of Mean Exponential Growth factor of Nearby Orbits; Cincotta & Simò 2000). This method provides relevant information about the global dynamics and the fine structure of the phase space, and yields simultaneously a good estimate of the Lyapunov Characteristic Numbers with a comparatively small computational effort. From the MEGNO technique, we have built the MIPS package (acronym of Megno Indicator for Planetary Systems) specially devoted to the study of planetary systems in their multi-dimensional space as well as their conditions of dynamical stability.

Particular planetary systems presented in this paper are only used as initial condition sources for theoretical studies of 3-body problems. By convention, the reference system is given by the orbital plane of the inner planet at $t = 0$. Thus, we suppose the orbital inclinations and the longitudes of node of the inner (noted 1) and the outer (noted 2) planets (which are non-determined parameters from observations) as follows : $i_1 = 0°$ and $\Omega_1 = 0°$ in such a way that the relative inclination and the relative longitude of nodes are defined at $t = 0$ as follows : $i_r = i_2 - i_1 = i_2$ and $\Omega_r = \Omega_2 - \Omega_1 = \Omega_2$. The MIPS maps presented in this paper have been confirmed by a second global analysis technique (Marzari *et al.* 2006) based on the Frequency Map Analysis (FMA; Laskar 1993).

2. Fine structure of retrograde resonance

Studying conditions of dynamical stability in the neighborhood of the HD 73526 two-planet system (period ratio: 2/1, see initial conditions in Table 1), we only find one stable and robust island (noted (2)) for a relative inclination of about 180° (see Fig. 2a). Such a relative inclination (where in fact $i_1 = 0°$ and $i_2 = 180°$) may be considered to a coplanar system where the planet 2 has a retrograde motion with respect to the planet 1. From a kinematic point of view, it amounts to consider a scale change of 180° in relative inclinations. Taking into account initial conditions inside the island (2) of Fig. 1a, we show that the presence of a strong mean-motion resonance (MMR) induces clear stability zones with a nice V-shape structure, as shown in Fig. 1b plotted in the $[a_1, e_1]$ parameter space. Let us note the narrowness of this V-shape, namely only about 0.006 AU wide for the inner planet (it is 5 times larger in the Jupiter-Saturn case). A similar V-shape structure is obtained in $[a_2, e_2]$ with about 0.015 AU wide. Due to the retrograde motion of the outer planet 2, this MMR is a 2:1 retrograde resonance, also noted 2:-1 MMR.

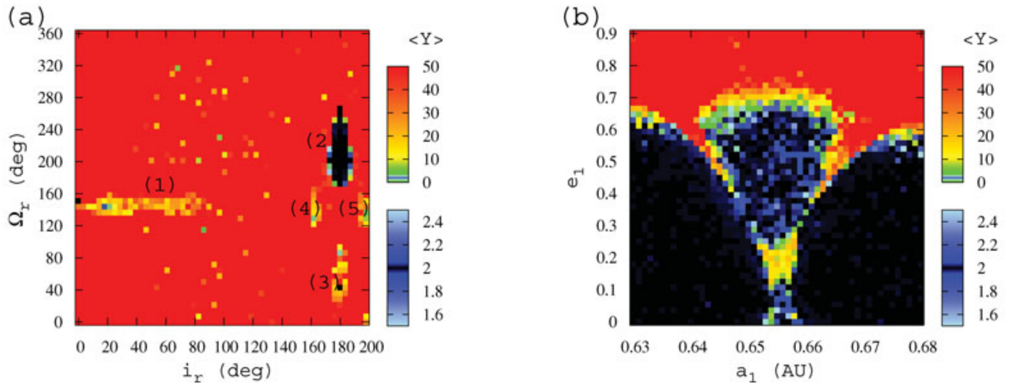

Figure 1. Panel (a): Stability map in the $[i_r, \Omega_r]$ non-determined parameter space of the HD 73526 planetary system (see Table 1). Panel(b): Stability map in the $[a_1, e_1]$ parameter space for initial conditions taken in the stable zone (2) of panel (a). Note that masses remain untouched whatever the mutual inclinations may be; they are equal to their minimal observational values. Black and dark-blue colors indicate stable orbits ($< Y >= 2 \pm 3\%$ and $< Y >= 2 \pm 5\%$ respectively with $< Y >$, the MEGNO indicator value) while warm colors indicate highly unstable orbits.

3. Efficiency of retrograde resonances

Fig. 2 exhibits stability maps in the $[i_r, \Omega_r]$ parameter space considering a scale reduction of the HD 82943 planetary system (see Table 1) according to a factor 7.5 on semi-major axes (masses remaining untouched). The dynamical behavior of the reduced system (Fig. 2b) with respect to the initial one (Fig. 2a) points up the clear robustness of retrograde configurations contrary to prograde ones. The "prograde" stable islands completely disappear while only the "retrograde" stable island resists, persists and even extends more or less. Even for very small semi-major axes and large planetary masses, which should a priori easily make a system unstable or chaotic, stability is possible with counter-revolving orbits.

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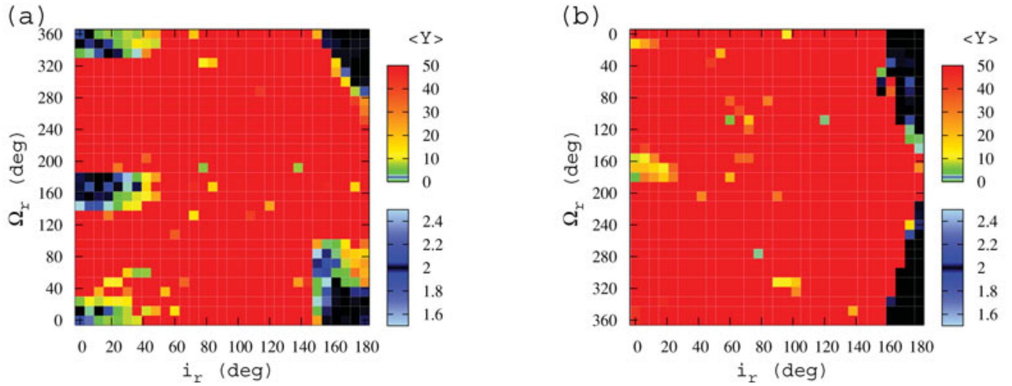

Figure 2. Stability maps in the $[i_r, \Omega_r]$ parameter space. Panel (a): initial HD 82943 planetary system (see Table 1). Panel (b): scale reduction of the HD 82943 planetary system according to a factor 7.5 on semi- major axes. Masses in Panel (a) and Panel (b) are identical. Color scale is the same as in Fig. 1.

In the case of the 2:1 retrograde resonance, although close approaches happen more often (3 for the 2:-1 MMR) compared to the 2:1 prograde resonance, the 2:-1 MMR remains very efficient for stability because of faster close approaches between the planets. A more detailed numerical study of retrograde resonances can be found in Gayon & Bois (2008).

Elements	HD 73526	HD 82943	HD 128311	HD 160691	HD 202206
M_{star} $(M_\odot)$	1.08 ± 0.05	1.15	0.84	1.08 ± 0.05	1.15
m sin i_l (M_J)	2.9 ± 0.2 2.5 ± 0.3	1.85 1.84	1.56 ± 0.16 3.08 ± 0.11	1.67 ± 0.11 3.10 ± 0.71	17.4 2.44
a (AU)	0.66 ± 0.01 1.05 ± 0.02	0.75 1.18	1.109 ± 0.008 1.735 ± 0.014	1.50 ± 0.02 4.17 ± 0.07	0.83 2.55
e	0.19 ± 0.05 0.14 ± 0.09	0.38 ± 0.01 0.18 ± 0.04	0.38 ± 0.08 0.21 ± 0.21	0.20 ± 0.03 0.57 ± 0.1	0.435 ± 0.001 0.267 ± 0.021
ω (deg)	203 ± 9 13 ± 76	124.0 ± 3 237.0 ± 13	80.1 ± 16 21.6 ± 61	294 ± 9 161 ± 8	161.18 ± 0.30 78.99 ± 6.65
M (deg)	86 ± 13 82 ± 27	0 75.21 ± 1.96	257.6 ± 2.7 166 ± 2	0 12.6 ± 11.2	105.05 ± 0.48 311.6 ± 9.5

Table 1. Orbital parameters of the HD 73526, HD 82943, HD 128311, HD 160691 and HD 202206 planetary systems. Data sources come from Tinney *et al.* (2006), Mayor *et al.* (2004), Vogt *et al.* (2005), McCarthy *et al.* (2004) and Correia *et al.* (2005) respectively. For each system and each orbital element, the first line corresponds to the inner planet and the second one to the outer planet.

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Data sources	Period ratio	Prograde MMR	Retrograde MMR
HD 73526	2/1	17	500
HD 82943	2/1	755	1000
HD 128311	2/1	249	137
HD 160691	5/1	ε	320
HD 202206	5/1	ε	631

Table 2. Statistical results. For each type of MMR (prograde or retrograde), 1000 random systems have been integrated in the error bars of each data source. The proportion of stable systems over 1000 is indicated in each case. ε designates a very small value that depends on the size of the random system size. Data sources come from Tinney *et al.* (2006), Mayor *et al.* (2004), Vogt *et al.* (2005), McCarthy et al. (2004) and Correia *et al.* (2005) respectively (see Table 1).

4. Occurrence of stable counter-revolving configurations

The occurence of stable two-planet systems including counter-revolving orbits appears in the neighborhood of a few systems observed in 2:1 or 5:1 MMR. New observations frequently induce new determinations of orbital elements. It is the case for the HD 160691 planetary system given with 2 planets in McCarthy *et al.* (2004) then with 4 planets in Pepe *et al.* (2007). Hence, systems related to initial conditions used here (see Table 1) have to be considered as *academic* systems. Statistical results for stability of these academic systems are presented in Table 2, both in the prograde case ($i_r = 0°$) and in the retrograde case ($i_r = 180°$). For each data source, 1000 random systems taken inside observational error bars have been integrated. Among these random systems, the proportion of stable systems either with prograde orbits or with counter-revolving orbits is given in Table 2. In all cases, a significant number of stable systems is found in retrograde MMR. Moreover, in most data sources, retrograde possibilities predominate.

5. Resources of retrograde resonances

The 2:1 (prograde) MMRs preserved by synchronous precessions of the apsidal lines (ASPs) are from now on well understood (see for instance Lee & Peale 2002, Bois *et al.* 2003, Ji *et al.* 2003, Ferraz-Mello *et al.* 2005). The MMR-ASP combination is often very effective; however, ASPs may also exist alone for stability of planetary systems. Related to subtle relations between the eccentricity of the inner orbit (e_1) and the relative apsidal longitude $\Delta\tilde{\omega}$ (i.e. $\tilde{\omega}_1 - \tilde{\omega}_2$), Fig. 3 permits to observe how the 2:1 retrograde MMR brings out its resources in the $[\Delta\tilde{\omega}, e_1]$ parameter space :

• In the island (1) (i.e. inside the $[a, e]$ V-shape of Fig. 1b), the 2:-1 MMR is combined with a uniformly prograde ASP (both planets precess on average at the *same rate* and in the *same prograde direction*).

• In the island (2) (i.e. outside but close to the $[a, e]$ V-shape of Fig. 1b), the 2:-1 *near*-MMR is combined with a particular apsidal behavior that we have called a *rocking* ASP (see Gayon & Bois 2008): both planets precess at the *same rate* but in *opposite directions*.

• The $[\Delta\tilde{\omega}, e_1]$ map also exposes a third island (3) that proves to be a wholly chaotic zone on long term integrations.

Let us note that the division between islands (1) and (2) is related to the degree of closeness to the 2:-1 MMR.

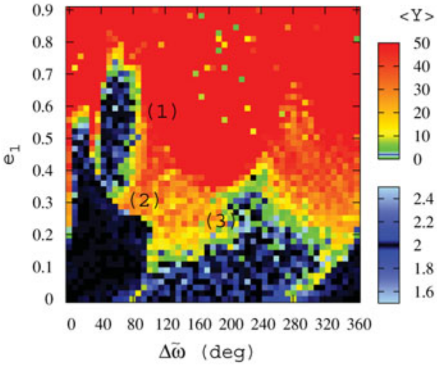

Figure 3. Stability map in the $[\Delta\tilde{\omega}, e_1]$ parameter space. A similar distribution of stable islands is obtained in $[\Delta\tilde{\omega}, e_2]$. Color scale and initial conditions are the same as in Fig. 1 with in addition the i_r and Ω_r values chosen in the island (2) of Fig. 1a.

6. Conclusion

We have found that retrograde resonances present fine and characteristic structures particularly relevant for dynamical stability. We have also shown that in cases of very compact systems obtained by scale reduction, only the "retrograde" stable islands survive. From our statistical approach and the scale reduction experiment, we have expressed the efficiency for stability of retrograde resonances. Such an efficiency can be understood by very fast close approaches between the planets although they are in greater number.

We plan to present an Hamiltonian approach of retrograde MMRs in a forthcoming paper (Gayon, Bois, & Scholl, 2008). Besides, in Gayon & Bois (2008), we propose two mechanisms of formation for systems harboring counter-revolving orbits. Free-floating planets or the Slingshot model might indeed explain the origin of such planetary systems.

In the end, we may conclude that retrograde resonances prove to be a feasible stabilizing mechanism.

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References

Bois, E., Kiseleva-Eggleton, L., Rambaux, N., & Pilat-Lohinger, E. 2003, *ApJ*, 598, 1312

Cincotta, P. & Simó, C. 2000, *A&AS*, 147, 205

Correia, A. C. M., Udry, S., Mayor, M., Laskar, J., Naef, D., Pepe, F., Queloz, D., Santos, N. C. 2006, *A&A*, 440, 751

Ferraz-Mello, S., Michtchenko, T. A., Beaugé, C., & Callegari, N. 2005, *Lecture Notes in Physics*, 683, 219

Gayon, J. & Bois, E. 2008, *A&A*, accepted, [arXiv:0801.1089v2]

Gayon, J., Bois, E., & Scholl, H. 2008, *Celestial Mechanics and Dynamical Astronomy*, Special Issue : "Theory and Applications of Dynamical Systems", to be submitted

Ji, J., Kinoshita, H., Liu, L., Li, G., & Nakai, H. 2003, *Celestial Mechanics and Dynamical Astronomy*, 87, 113

Laskar, J. 1993, *Physica D*, 67, 257

Lee, M. H. & Peale, S. J. 2002, *ApJ*, 567, 596

Mayor, M., Udry, S., Naef, D., Pepe, F., Queloz, D., Santos, N. C., Burnet, M. 2004, *A&A*, 415, 391

Marzari, F., Scholl, H., & Tricarico, P. 2006, *A&A*, 453, 341

McCarthy, C., Butler, R. P., Tinney, C. G., Jones, H. R. A., Marcy, G. W. Carter, B., Penny, A. J., & Fischer, D. A. 2004, *ApJ*, 617, 575

Pepe, F., Correia, A. C. M., Mayor, M., Tamuz, O., Couetdic, J., Benz, W., Bertaux, J.-L., Bouchy, F., Laskar, J., Lovis, C., Naef, D., Queloz, D., Santos, N. C., Sivan, J.-P., Sosnowska, D., & Udry, S. 2007, *A&A*, 462, 769

Tinney, C. G., Butler, R. P., Marcy, G. W., Jones, H. R. A., Laughlin, G., Carter, B. D, Bailey, J. A., & O'Toole, S. 2006, *ApJ*, 647, 594

Vogt, S. S., Butler, R. P., Marcy, G. W., Fischer, D. A., Henry, G. W., Laughlin G., Wright, J. T., Johnson, J. A. 2005, *ApJ*, 632, 638