If O be the middle point of the base, P, Q, R, S coincide at O, and the six distances between the four points are

$$OF - OE = \frac{2(a^2 - c^2)}{3\sqrt{4a^2 - c^2}} \qquad OG - OE = \frac{c(a-c)}{\sqrt{4a^2 - c^2}}$$

$$OH - OE = \frac{a^2 - c^2}{\sqrt{4a^2 - c^2}} \qquad OF - OG = \frac{(a-c)(2a-c)}{3\sqrt{4a^2 - c^2}}$$

$$OH - OF = \frac{a^2 - c^2}{3\sqrt{4a^2 - c^2}} \qquad OH - OG = \frac{a(a-c)}{\sqrt{4a^2 - c^2}}.$$

Here two cases are to be considered, when $a > c$ and when $a < c$; for if $a = c$, the triangle is equilateral, and the four points coalesce in one.

If $a > c$, the points will be situated as in fig 46 where $HF = \frac{1}{3}EH$, or $EF = \frac{2}{3}EH$, and $EG < \frac{1}{2}EH$.

In this case O falls in HE produced beyond E so that $OE = \frac{c^2}{2\sqrt{4a^2 - c^2}}$.

If $a < c$, the points will be situated as in fig. 47 where $HF = \frac{1}{3}EH$ or $EF = \frac{2}{3}EH$, but $EG > \frac{1}{2}EH$.

In this case O falls in EH produced beyond H so that $HO = \frac{2a^2 - c^2}{2\sqrt{4a^2 - c^2}}$; whence if $2a^2 < c^2$, the point O falls between H and E.

Sixth Meeting, April 8th, 1886.

Dr Ferguson, F.R.S.E., President, in the Chair.

On the Divisibility of certain Numbers.

By J. S. Mackay, M.A.

Of the following properties the first two are obvious; the third was communicated to the Royal Society of London by the Rev. Dr James Booth* in 1854; the others, obtained many years ago, have not as far as I know been remarked. They seem to be more curious than useful.

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* See *Proceedings of the Royal Society of London*, (London, 1856) vol. vii. pp. 42-43.

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Any number consisting of

1. One figure repeated is divisible by 11.						
2. Two figures	,,	,,	,,	,,	101.
3. Three	,,	,,	,,	,,	,,	7, 11, 13.
4. Four	,,	,,	,,	,,	,,	73, 137.
5. Five	,,	,,	,,	,,	,,	11, 9091.
6. Six	,,	,,	,,	,,	,,	101, 9901.
7. Seven	,,	,,	,,	,,	,,	11, 909091.
8. Eight	,,	,,	,,	,,	,,	17, 5882353.
9. Nine	,,	,,	,,	,,	,,	7, 11, 13, 19, 52579.

The proofs of these properties are very simple. Take for example the third.

The smallest number consisting of three figures repeated is 001001, and all the others are multiples of it.

But $1001 = 7 \times 11 \times 13$.

Dr Booth's proof is:

A number N of six places may be thus written:

$N = 100000a + 10000b + 1000c + 100d + 10e + f$,

which, when divided by 7, will give a quotient q, and a remainder $5a + 4b + 6c + 2d + 3e + f$.

Now if $d = a$, $e = b$, $f = c$, this remainder may be written $7(a + b + c)$, which is divisible by 7, whatever be the values of a, b, c.

Similarly for the divisors 11 and 13.

On consulting Dr Booth's paper the other day I find that he states 17 to be a divisor of any number consisting of eight figures repeated. He does not appear to have observed that the other divisor 5882353 is a prime.

Projective Geometry of the Sphere.

By R. E. Allardice, M.A.

[*Abstract.*]

If A and B be two fixed points on a great circular arc and P a variable point on the arc, there are two and only two possible positions of the point P corresponding to a given value of the ratio sinAP/sinBP, provided arcs measured in one direction from A or B be considered positive, and in the opposite direction negative; and these two points are antipodal.

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