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Compact Subsets of the Glimm Space of a C^*-algebra

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Abstract. If A is a σ-unital C^*-algebra and a is a strictly positive element of A, then for every compact subset K of the complete regularization $\mathrm{Glimm}(A)$ of $\mathrm{Prim}(A)$ there exists $\alpha > 0$ such that $K \subset \{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq \alpha\}$. This extends a result of J. Dauns to all σ-unital C^*-algebras. However, there exist a C^*-algebra A and a compact subset of $\mathrm{Glimm}(A)$ that is not contained in any set of the form $\{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq \alpha\}$, $a \in A$ and $\alpha > 0$.

1 Introduction

The lack of good separation properties on the primitive ideal space of a C^*-algebra is a serious obstacle in obtaining useful non-commutative versions of the Gelfand–Naimark theorem for the commutative algebras. One way to circumvent this impediment is to pass to the complete regularization of the primitive ideal space. A method to obtain the complete regularization of $\mathrm{Prim}(A)$ for a C^*-algebra A was given in [4, III, §3], and we shall briefly review it here. Two primitive ideals P_1 and P_2 are equivalent, $P_1 \approx P_2$, if $f(P_1) = f(P_2)$ for every bounded continuous real function f on $\mathrm{Prim}(A)$. Each equivalence class is a hull-kernel closed subset of $\mathrm{Prim}(A)$ and one associates with it its kernel. The family of ideals thus obtained was denoted in [2, p. 351] by $\mathrm{Glimm}(A)$, and the quotient space $\mathrm{Prim}(A)/\approx$ was naturally identified with it. The quotient map q_A takes a primitive ideal P to the kernel of its equivalence class. Two topologies that can be put on $\mathrm{Glimm}(A)$ are of interest to us. One is the quotient topology τ_q; the other is the weakest topology for which the functions on $\mathrm{Glimm}(A)$, defined by dropping to this space the bounded real continuous functions on $\mathrm{Prim}(A)$, are continuous. The latter topology is completely regular and is denoted τ_{cr}. Obviously $\tau_q \geq \tau_{cr}$ and cases when equality does or does not occur were discussed in [2, 7]. Ways to represent A as an algebra of continuous fields with base space $(\mathrm{Glimm}(A), \tau_{cr})$ were discussed in [2, 4]. Other uses of $\mathrm{Glimm}(A)$ can be found in [6].

The continuous fields that appear in [4] have to vanish at infinity. To topologize the disjoint union $\mathrm{Glimm}(A) \cup \{\infty\}$ one has to use the complements in this set of a family of τ_{cr}-compact sets that is closed under finite unions and whose union is $\mathrm{Glimm}(A)$. The family considered in [4] is that of the τ_{cr}-compact sets

$$\{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq \alpha\}, \quad a \in A \quad \text{and} \quad \alpha > 0.$$

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The complements of these sets are the basic neighbourhoods of the point at infinity. This is the topology that appears in the statement of [4, Corollary 8.13]. A more natural choice would be the family of all the τ_{cr}-compact subsets of Glimm(A), which yields the finest one point compactification. This is the standard one point compactification of Glimm(A). Wanting to work with this topology in the context of [4] motivated [3] where it was shown that if A is quasicentral, that is, no primitive ideal of A contains the center of A, then each τ_{cr}-compact subset of Glimm(A) is included in a set of the form $\{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq \alpha\}$ for some $a \in A$ and $\alpha > 0$, meaning that the two compactifications are the same; see [3, Theorem 2.5]. In [2, p. 351] this result was improved by showing that a can be taken in the center of A. Dauns pointed out in [3, p. 43] that the general case, that is, without an additional assumption on the algebra A, remained open. Here one should point out the fact that despite the claim in [3, p. 42], the standard one point compactification of Glimm(A) is Hausdorff only when the latter is locally compact, and this is not always the case.

Our aim is to show that one can cover the τ_{cr}-compact subsets of the Glimm space by sets determined by norm inequalities as in [3, Theorem 2.5] in two other situations: when the C^*-algebra has a countable approximate identity or when the quotient map q_A onto the Glimm space is open. We hope this result will help to invigorate the interest in representing C^*-algebras as continuous sections of certain bundles. We also show that there are situations when the τ_{cr}-compact sets cannot be covered in this manner.

We shall use the well-known fact that a C^*-algebra has a countable approximate identity (often also called a σ-unital C^*-algebra) if and only if it has a strictly positive element a that is, an element such that aAa is dense in A; see [8, 3.10.5]. We shall freely make use of the following two equalities for x an element of the C^*-algebra A and $\alpha > 0$:

$$q_A\big(\{P \in \mathrm{Prim}(A) \mid \|x + P\| > \alpha\}\big) = \{G \in \mathrm{Glimm}(A) \mid \|x + G\| > \alpha\},$$
$$q_A\big(\{P \in \mathrm{Prim}(A) \mid \|x + P\| \geq \alpha\}\big) = \{G \in \mathrm{Glimm}(A) \mid \|x + G\| \geq \alpha\}.$$

They follow immediately from these facts:

(i) $q_A(P) \subset P$ for every $P \in \mathrm{Prim}(A)$;
(ii) for each $G \in \mathrm{Glimm}(A)$ and $x \in A$ there is $P \in \mathrm{hull}(G)$ such that $\|x + P\| = \|x + G\|$.

The last claim is a consequence of [5, 3.3.6].

2 Results

Theorem 2.1 *Let A be a C^*-algebra with a countable approximate identity and $a \in A$ a strictly positive element. Then for every τ_{cr}-compact subset K of* Glimm(A) *there exists* $\alpha > 0$ *such that* $K \subset \{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq \alpha\}$.

Proof The existence of a countable approximate identity implies $\tau_q = \tau_{cr}$ by [7, Theorem 2.6] so we shall work with τ_q in what follows.

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For every natural number n let

$$O_n := \{P \in \mathrm{Prim}(A) \mid \|a + P\| > 1/n\},$$
$$C_n := \{P \in \mathrm{Prim}(A) \mid \|a + P\| \geq 1/n\},$$
$$K_n := q_A(C_n),$$

where $q_A\colon \mathrm{Prim}(A) \to \mathrm{Glimm}(A)$ is the quotient map. Then O_n is open in $\mathrm{Prim}(A)$, and C_n is compact by [5, Propositions 3.3.2 and 3.3.7]. Thus K_n is a compact, hence closed, subset of the Hausdorff space $\mathrm{Glimm}(A)$. We have $\mathrm{Prim}(A) = \cup_{n=1}^{\infty} O_n = \cup_{n=1}^{\infty} C_n$, since a is strictly positive, $\mathrm{Glimm}(A) = \cup_{n=1}^{\infty} K_n$, and $K_n \subset K_{n+1}$ for every n.

A subset F of $\mathrm{Glimm}(A)$ is closed if and only if each $F \cap K_n$ is closed. It is clear that if $F \subset \mathrm{Glimm}(A)$ is closed, then $F \cap K_n$ is closed. Suppose now that $F \cap K_n$ is closed for every n. We want to show that $q_A^{-1}(F)$ is closed. To this end, let $P_0 \in \overline{q_A{}^{-1}(F)}$. Then $P_0 \in O_m$ and $q_A(P_0) \in K_m$ for some m. Let U be an arbitrary open neighbourhood of $q_A(P_0)$ in $\mathrm{Glimm}(A)$. Thus $q_A^{-1}(U) \cap O_m$ is an open neighbourhood of P_0, hence there exists $P \in q_A^{-1}(F) \cap q_A^{-1}(U) \cap O_m$. We get $q_A(P) \in F \cap U \cap K_m$. We showed that $U \cap F \cap K_m \neq \varnothing$ and we can conclude that $q_A(P_0) \in \overline{F \cap K_m} = F \cap K_m$. But then $P_0 \in q_A^{-1}(F \cap K_m) \subset q_A^{-1}(F)$. It follows that $\overline{q_A{}^{-1}(F)} = q_A^{-1}(F)$.

Now we claim that if $K \subset \mathrm{Glimm}(A)$ is τ_q-compact, then there exists m such that $K \subset K_m = \{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq 1/m\}$. If not, then there exists $G_n \in K \setminus K_n$ for every n. We shall show that the infinite set $F := \{G_n \mid n \in \mathbb{N}\}$ is both closed and discrete. Being a subset of the compact set K, this a contradiction, and by this our claim will be established. Let F_1 be any subset of F. Since the sequence $\{K_n\}$ is non-decreasing, $F_1 \cap K_n$ is finite for every n. By the previous paragraph, F_1 is closed, and we are done. ■

For a quasicentral C^*-algebra with a countable approximate identity one can improve a little the strengthened version of [3, Theorem 2.5] outlined in [2, p. 351].

Corollary 2.2 *Let A be a quasicentral C^*-algebra with a countable approximate identity. Then its center, $Z(A)$, contains a strictly positive element z of A so for every τ_{cr}-compact subset K of* $\mathrm{Glimm}(A)$ *there exists $\alpha > 0$ such that*

$$K \subset \{G \in \mathrm{Glimm}(A) \mid \|z + G\| \geq \alpha\}.$$

Proof As observed in [2, p. 351], $\mathrm{Glimm}(A)$ is homeomorphic to the maximal ideal space of $Z(A)$. Thus the latter space is σ-compact, hence $Z(A)$ contains a strictly positive element z for $Z(A)$. But no positive linear functional of A can be trivial on $Z(A)$, since A is quasicentral. Thus z is strictly positive for A, and the conclusion follows from Theorem 2.1. ■

It is possible to describe precisely when a τ_{cr}-compact subset of the Glimm space of a C^*-algebra can be covered by a compact set determined by a norm inequality: it must be the image by the quotient map of a compact subset of the primitive ideal space.

Proposition 2.3 *Let A be a C^*-algebra and K a τ_{cr}-compact subset of* $\mathrm{Glimm}(A)$. *There are $a \in A$ and $\alpha > 0$ such that $K \subset \{G \in \mathrm{Glimm}(A) \mid \|a+G\| \geq \alpha\}$ if and only if there exists a compact subset $C \subset \mathrm{Prim}(A)$ such that $q_A(C) = K$. A τ_{cr}-compact subset that satisfies the above conditions is also τ_q-compact.*

Proof Suppose the compact subset C of $\mathrm{Prim}(A)$ satisfies $q_A(C) = K$. For every $P_0 \in C$ let x_0 be a positive element in $A \setminus P_0$. Then

$$P_0 \in \{P \in \mathrm{Prim}(A) \mid \|x_0 + P\| > \|x_0 + P_0\| /2\},$$

and in this way we get an open cover of C. By compactness we get positive elements $\{x_i\}_{i=1}^n$ of A and positive scalars $\{\alpha_i\}_{i=1}^n$ such that

$$C \subset \cup_{i=1}^n \{P \in \mathrm{Prim}(A) \mid \|x_i + P\| > \alpha_i\}.$$

With $x := \sum_{i=1}^n x_i$ and $\alpha := \min\{\alpha_i \mid 1 \leq i \leq n\}$, we have

$$C \subset \cup_{i=1}^n \{P \in \mathrm{Prim}(A) \mid \|x_i + P\| > \alpha_i\} \subset \{P \in \mathrm{Prim}(A) \mid \|x + P\| > \alpha\}.$$

Then

$$\begin{aligned} K = q_A(C) &\subset q_A\big(\{P \in \mathrm{Prim}(A) \mid \|x+P\| > \alpha\}\big) \\ &= \{G \in \mathrm{Glimm}(A) \mid \|x+G\| > \alpha\}. \end{aligned}$$

Thus $K \subset \{G \in \mathrm{Glimm}(A) \mid \|x+G\| \geq \alpha\}$

Suppose now that $K \subset \{G \in \mathrm{Glimm}(A) \mid \|x+G\| \geq \alpha\}$ for some $x \in A$ and $\alpha > o$. Then $C := q_A^{-1}(K) \cap \{P \in \mathrm{Prim}(A) \mid \|x+P\| \geq \alpha\}$ is compact, being a relatively closed subset of the compact set $\{P \in \mathrm{Prim}(A) \mid \|x+P\| \geq \alpha\}$, and $q_A(C) = K$, since

$$q_a\big(\{P \in \mathrm{Prim}(A) \mid \|x+P\| \geq \alpha\}\big) = \{G \in \mathrm{Glimm}(A) \mid \|x+G\| \geq \alpha\}.$$

Obviously, $K = q_A(C)$ is τ_q-compact. ■

The first part of the previous proof shows that the question for the primitive ideal space, analogous to that which we treat here for the Glimm space, always has a positive solution. Namely, for every C^*-algebra A and every compact set $C \subset \mathrm{Prim}(A)$, there exist $a \in A$ and $\alpha > 0$ such that $C \subset \{P \in \mathrm{Prim}(A) \mid \|a+P\| \geq \alpha\}$.

To obtain another situation in which the conclusion of [3, Theorem 2.5] holds, we need the following lemma.

Lemma 2.4 *Let the Hausdorff space Y be a quotient of the locally compact space X. If the quotient map, q, is open, then for every compact subset K of Y there exists a compact subset C of X such that $q(C) = K$.*

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Proof For each $x \in q^{-1}(K)$ let E_x be a compact subset of X such that $x \in \operatorname{int}(E_x)$. Then

$$K \subset \bigcup_{x \in q^{-1}(K)} q\big(\operatorname{int}(E_x)\big).$$

By passing to a finite subcover of this open cover of K we get $\{x_i\}_{i=1}^n \subset q^{-1}(K)$ such that $K \subset \bigcup_{i=1}^n q(\operatorname{int}(E_{x_i})) \subset \bigcup_{i=1}^n q(E_{x_i})$. Now $D := \bigcup_{i=1}^n E_{x_i}$ is a compact subset of X and $C := q^{-1}(K) \cap D$ is a relatively closed subset of it hence compact too. Clearly $q(C) = K$. ■

If q_A is τ_q-open, then $\tau_q = \tau_{cr}$ by [2, p. 351]; if q_A is τ_{cr}-open then obviously it is also τ_q-open. The following proposition follows immediately from Proposition 2.3 and Lemma 2.4.

Proposition 2.5 *Let A be a C^*-algebra for which the quotient map $q_A\colon \operatorname{Prim}(A) \to \operatorname{Glimm}(A)$ is open. Then for every τ_{cr}-compact $K \subset \operatorname{Glimm}(A)$ there exist $a \in A$ and $\alpha > 0$ such that $K \subset \{G \in \operatorname{Glimm}(A) \mid \|a + G\| \geq \alpha\}$.*

Observe that by [2, Theorem 3.3] Proposition 2.5 applies to the class of quasi-standard C^*-algebras.

In view of the above results it is natural to ask if the conclusion of [3, Theorem 2.5] always holds when A is a C^*-algebra for which $\tau_{cr} = \tau_q$ on $\operatorname{Glimm}(A)$.

3 Examples

We know now three classes of C^*-algebras for which the τ_{cr}-compact subsets of their Glimm spaces satisfy the conclusion of [3, Theorem 2.5]: the quasicentral C^*-algebras, the C^*-algebras that have a countable approximate identity, and those for which the quotient map of the primitive ideal space onto the Glimm space is open. In this section we shall show that none of these classes contains any other.

In the following ω will denote the first infinite ordinal, and Ω will stand for the first uncountable ordinal. All the spaces of ordinals will be considered with their order topology.

For the first two examples, at the referee's suggestion, we shall use the C^*-algebra A of [1, Example 4.12]. This is the algebra of all the sequences $x = \{x_n\}$ of 2×2 scalar matrices such that $\{x_{3n}\}$ converges to the diagonal matrix $\operatorname{diag}(\lambda_1(x), \lambda_2(x))$, $\{x_{3n+1}\}$ converges to $\operatorname{diag}(\lambda_2(x), \lambda_3(x))$, and $\{x_{3n+2}\}$ converges to $\operatorname{diag}(\lambda_3(x), \lambda_1(x))$. A is unital hence $\tau_q = \tau_{cr}$ by [2, p. 351]. Every primitive ideal of A is of the form $P_n = \{x \in A \mid x_n = 0\}$, $n \geq 1$, or $Q_m = \{x \in A \mid \lambda_m(x) = 0\}$, $m = 1, 2, 3$. Clearly A is quasicentral. The sequence $\{P_{3n}\}$ of $\operatorname{Prim}(A)$ converges to Q_1 and Q_2, and so on. The Glimm space of A consists of the sequence $\{P_n\}$ together with its limit $Q_1 \cap Q_2 \cap Q_3$, and it is obvious that q_A is not τ_q-open.

Example 3.1 Let A_1 be the direct sum of the above C^*-algebra A with an abelian non σ-unital C^*-algebra, for instance $C_0([0, \Omega))$. Then A_1 is quasicentral and does not have a countable approximate identity, and q_{A_1} is not τ_q-open.

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Example 3.2 Now let H be a separable infinite dimensional Hilbert space, $\mathcal{K}(H)$ the algebra of compact operators on H, and $A_2 := A \oplus \mathcal{K}(H)$. Then A_2 is a separable C^*-algebra, hence σ-unital, for which q_{A_2} is not τ_q-open. A_2 is not quasicentral, since its direct summand $\mathcal{K}(H)$ is not quasicentral, having a trivial center.

Example 3.3 Now we are going to discuss an example of a non σ-unital non quasicentral C^*-algebra A for which q_A is open. As above let $\mathcal{K}(H)$ be the C^*-algebra of all the compact operators on an infinite dimensional Hilbert space. Then $A := \mathcal{C}_0([0,\Omega), \mathcal{K}(H))$ is not σ-unital, since $\mathrm{Prim}(A)$ is homeomorphic to $[0,\Omega)$, and this space is not σ-compact. The center of A is trivial, thus A cannot be quasicentral. Finally, q_A is just the identity map on $\mathrm{Prim}(A)$.

Lastly we shall show that there exist a C^*-algebra A and a τ_{cr}-compact subset K of $\mathrm{Glimm}(A)$ that is not τ_q-compact. By Proposition 2.3, for such K, there do not exist $a \in A$ and $\alpha > 0$ such that $K \subset \{G \in \mathrm{Glimm}(A) \mid \|a + G\| \geq \alpha\}$. Moreover, for this C^*-algebra, τ_{cr} is locally compact, so the local compactness of this topology on the Glimm space does not guarantee the possibility of covering the τ_{cr}-compact sets by sets defined by a norm inequality as above. The C^*-algebra that we treat is a particular case of a class of C^*-algebras constructed by D. W. B. Somerset in the Appendix of [7] for which $\tau_{cr} \neq \tau_q$.

Example 3.4 Let

$$Y := [0,\Omega] \times [0,\omega] \setminus \{(\Omega,\omega)\}, \quad S := \{\Omega\} \times [0,\omega), \quad \text{and} \quad T := [0,\Omega) \times \{\omega\}.$$

Let y be an element not in Y and consider the space $X := Y \cup \{y\}$ with the topology determined by the requirement that Y is embedded homeomorphically into X and $\{y\}$ is an open subset whose closure is $S \cup \{y\}$. Then X is a locally compact T_0-space, and, following [7, p. 155], we shall produce a C^*-algebra A whose primitive ideal space is homeomorphic to X.

For $B := \mathcal{C}_0(Y)$ and $D := \mathcal{C}_0(S)$ let $\pi_1 \colon B \to D$ be the restriction map. Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of infinite dimensional mutually orthogonal projections on the infinite dimensional separable Hilbert space H. We define an embedding $\rho \colon D \to \mathcal{L}(H)$ by $\rho(f) := \sum_{n=1}^{\infty} f(\Omega, n) p_n$. Remark that $\rho(D) \cap \mathcal{K}(H) = \{0\}$. Set $E := \rho(D) + \mathcal{K}(H)$ and let $\pi_2 \colon E \to D$ be the natural quotient map. We denote by A the pullback of π_1 and π_2; that is, $A := \{(b,e) \in B \oplus E \mid \pi_1(b) = \pi_2(e)\}$. It is easily seen that $\mathrm{Prim}(A)$ is homeomorphic to X.

The quotient map q_A maps every point in $Y \setminus S$ to itself, and $S \cup \{y\}$ is a $\approx$-class that we shall denote by z. Thus the Glimm space of A can be identified with $\mathcal{G} := (Y \setminus S) \cup \{z\}$. Moreover, the restriction of q_A to the open subset $Y \setminus S$ of X is a homeomorphism onto the τ_q-open subset $Y \setminus S$ of $(\mathcal{G}, \tau_q)$. We claim that $K := T \cup \{z\}$ is a τ_{cr}-compact subset of $\mathcal{G}$ that it is not τ_q-compact. The τ_{cr}-compactness of K follows immediately once we prove that every τ_{cr}-neighbourhood of z contains a set of the form $((\alpha,\Omega) \times \{\omega\}) \cup \{z\}$ for some ordinal α, since $[0,\alpha] \times \{\omega\}$ is clearly compact. A basic τ_{cr}-neighbourhood $\mathcal{U}$ of z is given by a bounded real valued continuous function g on $\mathcal{G}$ such that $g(z) = 1$: $\mathcal{U} = \{t \in \mathcal{G} \mid |g(t) - 1| < 1\}$. The continuity of g on $([0,\Omega) \times \{n\}) \cup \{z\}$ for $0 \leq n < \omega$ entails the existence of

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an ordinal α_n, $0 \le \alpha_n < \Omega$, such that $g(\beta, n) = 1$ whenever $\alpha_n < \beta < \Omega$. Set $\alpha := \sup_n \alpha_n$; then $0 \le \alpha < \Omega$ and $g(\beta, n) = 1$ for $\alpha < \beta < \Omega$ and $0 \le n < \omega$. It follows that $g(\beta, \omega) = 1$ if $\alpha < \beta < \Omega$, thus $((\alpha, \Omega) \times \{\omega\}) \subset \mathcal{U}$ as needed. The subset $T = q_A(T)$ of K is τ_q-closed, since $q_A^{-1}(T) = T$ and T is closed in X. Therefore K cannot be τ_q-compact, since its τ_q-closed subset T, that is homeomorphic to the subset T of X, is not τ_q-compact.

We show now that τ_{cr} on $\mathcal{G}$ is locally compact (and Hausdorff, of course). The points of $Y \setminus S$ clearly have a neighbourhood base of compact sets. We claim that $\mathcal{U}' := \{t \in \mathcal{G} \mid |g(t) - 1| \le 1/2\}$ is a τ_{cr}-compact neighbourhood of z contained in $\mathcal{U}$. To prove the compactness of this set let $\{t_\iota\}$ be a net contained in $\mathcal{U}' \setminus \{z\}$ with $t_\iota := (\omega_\iota, n_\iota)$. We may suppose that $\{\omega_\iota\}$ converges to Ω. Given a bounded real valued continuous function h on $\mathcal{G}$ one can find, as above, an ordinal α_h such that $h(\beta, n) = h(z)$ for $\alpha_h < \beta < \Omega$ and $0 \le n < \omega$. We conclude that $\{h(t_\iota)\}$ converges to $h(z)$, and we are done.

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