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A NOTE ON (r, λ)-SYSTEMS

BY
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An (r, λ)-system is an arrangement of v objects (or varieties) into subsets (or blocks) such that each variety appears in exactly r blocks and each pair of distinct varieties appears in exactly λ blocks. To avoid trivial designs, we assume that $1 \leq \lambda < r$.

An (r, λ)-system which contains either a complete block or a complete set of singletons is called reducible. Otherwise, it is called irreducible. If $\lambda(v-1) > r(r-1)$, the corresponding system is called hyperbolic.

Stanton and Mullin [2] made the following conjecture and proved it for $\lambda = 1$.

Conjecture 2. For $\lambda \leq 2$ (and perhaps all λ), all hyperbolic systems are reducible.

Vranch [3] claims that his results support this conjecture for arbitrary values of λ.

In [1], it is shown that this conjecture is true for all λ in the cases $r = \lambda + 1$ and $r = \lambda + 2$.

For $r = \lambda + 3$, we exhibit counterexamples when $\lambda \geq 3$.

The following irreducible hyperbolic systems provide counterexamples to Conjecture 2 for $\lambda = 3$ and $\lambda = 4$.

Irreducible hyperbolic $(6, 3)$-system on thirteen varieties.

1	2	3	4	5	6	7	8	10	12
1	2	3	4	5	6	7	9	11	13
1	2	3	4	8	9	10	11	12	13
1	7	8	9						
1	6	10	11						
1	5	12	13						
2	5	6	7	8	9	10	11	12	13
2									
2									
3	5	8	11						
3	6	9	12						
3	7	10	13						
4	5	9	10						
4	6	8	13						
4	7	11	12.						

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Irreducible hyperbolic (7, 4)-system on twelve varieties.

1	2	3	4	6	8	9	10
1	2	3	4	7	10	11	12
1	2	3	5	8	9	11	12
1	2	5	6	7	8	10	12
1	3	5	6	7	9	10	11
1	4	6	7	8	9	11	12
1	4	5					
2	3	4	5	6	7	8	11
2	4	5	6	9	10	11	12
2	7	9					
3	4	5	7	8	9	10	12
3	6	12					
8	10	11.					

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