STUDIES THROUGH RADIAL VELOCITY MEASUREMENTS OF THE PECULIAR MOTIONS OF STARS IN GALACTIC GLOBULAR CLUSTERS

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Abstract. In a brief introduction, we first compare, in the framework of globular cluster dynamics, the relative importance of the amounts of information acquired, during the past decades, through proper motion and radial velocity measurements. Next, we review the most recent and important studies based on measurements of radial velocities obtained with single-object and multi-object spectrometers and interpreted with the use of King-Michie and Fokker-Planck models or with non-parametric methods. Then, we present the results obtained from integrated-light spectra in the cores of a few collapsed globular clusters. We conclude with a summary of the most important scientific outputs secured from the radial velocity measurements of stars in globular clusters.

1. Introduction

As clearly stated by Oort & van Herk (1959) in their paper on M3, which was one of the first modern dynamical studies of a globular cluster (GC), "*the purpose of the investigation (...) is to see in how far the observed distribution of the stars corresponds to what we should expect theoretically, and to investigate what the comparison between observation and theory can teach us concerning the unobserved faint stars*". Nowadays, the aim of their work is still completely valid.

Oort & van Herk (1959) were already mentioning the potential observa-

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tional constraint to be obtained from the random motions of stars. Actually, at that time, so little was known concerning random motions in star clusters that this was not yet a sensitive test.

Random, or peculiar (u,v,w), motions were known only for stars in the solar neighbourhood. The velocity distributions of stars were known to be different along each axis and essentially gaussian (Blaauw & Schmidt 1965). This led to a distribution function in (u,v,w) space which has now acquired the name of "Schwarzschild velocity ellipsoid", as a reference to the elegant theoretical description of this problem (Schwarzschild 1907, 1908).

Proper motions and radial velocities of stars in GCs are significantly more difficult to measure than for field stars, since the typical projected velocity dispersion of a galactic GC is of the order of $\sigma_p = 5 - 10\ \mathrm{km\,s^{-1}}$.

1.1. PROPER MOTIONS: ERRORS $\sim \sigma_p$(GCS)

For a relatively nearby GC located at a distance of 5 kpc and with projected velocity dispersion $\sigma_p = 5\ \mathrm{km\,s^{-1}}$, the corresponding proper motion equals 20 marcsec/century, i.e., 1.5 μm in 80 years on a Yerkes plate. This has made proper motions difficult to measure with the required precision. K.M. Cudworth has been the pioneer of this field (see Meylan & Pryor 1993 for some references). But even in the best studied GCs, the errors in the proper motions have been comparable in size to the motions themselves. Reijns et al. (1993) report on the largest proper motion work, concerning a few thousand stars in ω Centauri (see Section 4.3 below). Proper motions represent a gold mine of information which has still to be exploited.

1.2. RADIAL VELOCITIES: ERRORS $< \sigma_p$(GCS)

For the same nearby GC (with d = 5 kpc and $\sigma_p = 5\ \mathrm{km\,s^{-1}}$), cross-correlation techniques have provided, for nearly two decades, high-quality radial velocities V_r with errors typically $\leq 1\ \mathrm{km\,s^{-1}}$. This has opened the way to essential observational constraints allowing numerous dynamical studies.

2. V_r from cross-correlation techniques

There is only one GC — ω Centauri — which was studied, before the advent of cross-correlation techniques, by use of V_r of individual stars. Harding (1965) measured the radial velocities of 40 individual stars, deriving the mean radial velocity of the cluster $\langle V_r \rangle = 238\ \mathrm{km\,s^{-1}}$, and its mean projected velocity dispersion $\sigma_p = 9.9 \pm 6.6\ \mathrm{km\,s^{-1}}$. Using the best 13 V_r measurements, Harding (1965) detected the presence of rotation in this cluster, although in a more qualitative than quantitative way.

The basic idea of cross-correlation techniques — matching superimposed spectra — is not new: Evershed (1913) used the method for measuring solar spectra, but its application to stellar spectra has been suggested by Felget (1953) and Babcock (1955). Griffin (1967) was the pioneer of the field in being the first to build a modern instrument capable of using cross-correlation techniques in a high-performance mode. In this first study, Griffin quotes a standard deviation of one photoelectric observation equal to 1.06 ± 0.07 $\mathrm{km\,s^{-1}}$. This started the beginning of a new era for dynamics of GCs.

Da Costa et al. (1977) measured, with a mean accuracy of 1.5 $\mathrm{km\,s^{-1}}$, 11 stars in the cluster NGC 6397 and determined $\sigma_p = 3.1 \pm 0.7$ $\mathrm{km\,s^{-1}}$. Such a low velocity dispersion would not have been measured without cross-correlation techniques. A milestone was reached, from both theoretical and observational points of view, with the work by Gunn & Griffin (1979) on M3. They measured, with a mean accuracy of 1.0 $\mathrm{km\,s^{-1}}$, radial velocities for 111 member stars of the cluster. Density and velocity dispersion profiles were simultaneously fitted to multi-mass anisotropic dynamical models based on the King-Michie form of the phase-space distribution function $f_i(E, J) \propto [\exp(-A_i E) - 1] \exp(-\beta J^2)$. Their results were consistent with the absence of spectroscopic binaries in the cluster, a result which misled the astronomical community until such binaries were discovered about a decade later by Pryor et al. Gunn & Griffin (1979) also found two high-velocity stars with constant radial velocities at 3.5 and 4.5 times the mean cluster velocity dispersion. These two interlopers are difficult to explain from both a dynamical and a statistical point of view.

3. Single-object spectrometers and King-Michie models

The late 70's and early 80's saw the development of cross-correlation techniques in different astronomical institutes. All techniques were based on the acquisition of single-star V_r in two different ways: (i) with spectrometers making online cross-correlation (CORAVEL type) and (ii) with spectrograph-CCD combinations recording the stellar spectra and allowing cross-correlation after the observations. Dynamical studies of galactic GCs, implying samples of V_r for 68 to 469 stars, were published following the study of Gunn Griffin (1979) on M3: for 47 Tuc by Mayor et al. (1984) and Meylan (1988, 1989); for M92 by Lupton et al. (1985); for M2 by Pryor et al. (1986); for M13 by Lupton et al. (1987); for ω Cen by Meylan (1987a) and Meylan et al. (1995); for M15 by Peterson et al. (1989) and Grabhorn et al. (1992); for NGC 6397 by Meylan & Mayor (1991); for NGC 362 by Fischer et al. (1993) and for NGC 3201 by Da Costa et al. (1993).

Kinematical studies of a few other GCs were done with fewer (∼ 20) stars, e.g., by Peterson & Latham (1986) and by Pryor et al. (1989, 1991).

3.1. ω Centauri: THE MOST MASSIVE GALACTIC GLOBULAR CLUSTER

The most recent dynamical study of this type, containing also the largest number of such V_r measurements, concerns ω Cen (Meylan et al. 1995). The mean radial velocities obtained with CORAVEL for 469 giant stars in this galactic GC, with a mean accuracy of 0.6 $\mathrm{km\,s^{-1}}$, are used to derive the velocity dispersion profile. It increases significantly from the outer parts inwards: the 16 outermost stars, located between 19.2′ and 22.4′ from the center, have a velocity dispersion $\sigma_p = 5.1 \pm 1.6$ $\mathrm{km\,s^{-1}}$, while the 16 innermost stars, located within 1′ from the center, have a velocity dispersion $\sigma_p = 21.9 \pm 3.9$ $\mathrm{km\,s^{-1}}$. This inner value of about $\sigma_p(0) = 22$ $\mathrm{km\,s^{-1}}$ is the largest velocity dispersion value obtained in the core of any galactic GC.

TABLE 1. ω Cen: the four best (lowest χ^2) K-M models

m_{hr}	x	M_{hr} %	M_{wd} %	r_a/r_c	M_{tot} $[10^6 m_\odot]$	M/L_V
1.4	1.50	2	15	2.0	5.23	4.32
1.4	1.25	4	21	3.0	4.55	3.74
2.0	1.75	1	10	2.0	5.63	4.21
2.0	1.50	3	15	3.0	4.88	3.90

The results of the four best (lowest χ^2) simultaneous fits of these radial velocities and of the surface brightness profile to multi-mass King-Michie dynamical models are displayed in Table 1 (Meylan et al. 1995). Column (1) gives the stellar mass m_{hr} of the heavy remnants, column (2) the mass function exponent x, columns (3) and (4) the fractions of the total mass in heavy remnants and in white dwarfs, column (5) the anisotropy radius r_a, column (6) the cluster total mass M_{tot}, and column (7) the cluster mass-luminosity ratio M/L_V.

The mean estimate of the total mass equals $M_{tot} = 5.1\ 10^6 m_\odot$, with a corresponding mean mass-to-light ratio $M/L_V = 4.1$. There is evidence for 0.6 - 1.3 $10^6 m_\odot$ of dark remnants in the form of white dwarfs and neutron stars. These results emphasize the fact that ω Cen is not only the brightest but also, by far, the most massive galactic GC.

3.2. PRIMORDIAL BINARIES IN ω Cen

The monitoring, over more than 10 years, of the radial velocities of 310 of the giant stars observed in ω Cen allows a search for primordial spectroscopic binaries (Mayor et al. 1996). The present period range for such binaries is limited, for short periods, by the onset of Roche-lobe overflow and, for long periods, by dynamical friction. Most of the primordial binaries among the red giants of ω Cen should have periods between 200 and 4,000 days. The two main important results of this study are as follows:

Duplicity among the chemically peculiar giants of ω Cen: The binary frequency observed among the 32 chemically peculiar giants in ω Cen is very low when compared to the binary frequency of similar stars in the field. This suggests either that enrichment mechanisms in ω Cen are quite different from those in the field, or that these stars may be the extreme tail of the abnormal abundances in ω Cen (Mayor et al. 1996).

Global percentage of binaries in ω Cen: Within the period range of 200 and 4,000 days, the binary frequency in ω Cen is about five times lower than for the field G dwarfs, and about 10 times lower than in the giants of the open cluster NGC 2477. Since about 20% of the nearby G dwarfs belong to binary systems with periods below 10^4 days, the binary rate estimated in ω Cen is only a fifth of that rate, i.e., about 4% (Mayor et al. 1996).

3.3. ROTATION IN GLOBULAR CLUSTERS

Rotation, suspected for a long time to be at least partly responsible for the small flattening of GC, has now been actually observed. Non-cylindrical differential rotation has been measured in ω Cen, with $v_{rot}^{max} = 8.0$ km s^{-1}, between 3-4 r_c, and in 47 Tuc, with $v_{rot}^{max} = 6.5$ km s^{-1}, between 11-12 r_c (Meylan & Mayor 1986). For $i = 90°$ and $60°$, the ratio of ordered to random motions $v_\circ/\sigma_\circ = 0.35$ and 0.39 in ω Cen and $v_\circ/\sigma_\circ = 0.40$ and 0.46 in 47 Tuc. Even with $i = 45°$, the importance of rotation remains weak compared to random motions: the ratio of rotational to random kinetic energies is $\simeq 0.1$, confirming the fact that GCs are hot stellar systems. Given their small ellipticities ($0.00 \lesssim \langle\varepsilon\rangle \lesssim 0.12$), GCs are located in the lower-left corner of the $v_\circ/\sigma_\circ$ vs. $\langle\varepsilon\rangle$ diagram, an area characterized by isotropy or mild anisotropy of the velocity-dispersion tensor (Pryor et al. 1986). A very clear relation between $\langle\varepsilon\rangle$ and v_{rot}^{max} is presented in Fig. 5 in Meylan (1987b), pointing towards flattening by rotation.

Recently a non-parametric estimate of the mean line-of-sight velocity field on the plane of the sky has been obtained by Merritt et al. (in preparation) using the 469 stars from Meylan et al. (1995). Isorotation curves increase from 1 to 5 km s^{-1} and confirm, both qualitatively and quantitatively, Fig. 3b in Meylan & Mayor (1986).

Models based on three integrals of the motion (E,J_z,I_3) have been developed for galaxies (see, e.g., Dehnen & Gerhard 1993) and applied with a few subpopulations to GCs (Lupton & Gunn 1987, Lupton, Gunn & Griffin 1987). The third integral being still unknown, these two studies use guessed approximations for I_3. The advantage of the presence of rotation does not counterbalance totally, in the case of GCs, the difficulties of fitting processes with an increased number of free parameters.

4. Multi-object spectrometers and non-parametric studies

Acquiring even a few hundred stellar radial velocities one at a time is a slow and tedious job, even on 4-m class telescopes. But the number of stellar velocities in GCs has recently grown explosively because of new technologies made available. Fiber-fed, multi-object spectrographs like ARGUS at Cerro Tololo, HYDRA at Kitt Peak, and AUTOFIB at the Anglo Australian Observatory can obtain velocities for about 25 stars simultaneously. Similar gains result from using the Rutgers Fabry-Perot interferometer. The number of stars observed per GC may typically reach a few thousands in a few seasons instead of a few hundreds in more than a decade. Such large samples call for more subtle dynamical interpretation.

4.1. PARAMETRIC AND NON-PARAMETRIC METHODS

In order to build King-Michie type models, two essential steps have to be made: (i) the choice of the integrals of the motion, and (ii) the choice of the dependence of the phase-space distribution function on these integrals, i.e., the functional form of $f(E)$, $f(E, J)$, or $f(E, J_z, I_3)$. The results can be strongly biased by the above two assumptions: e.g., when going from a distribution function $f(E, J)$ to the gravitational potential $\Phi(r)$, the solution may not be unique, the presence or not of anisotropy and different potentials can mimic identical surface-brightness profiles.

For a few years, D. Merritt has been the pioneer of non-parametric methods in the framework of dynamical models (see Merritt & Tremblay 1994, and references therein). The general aim is to infer the gravitational potential $\Phi(r)$ and the phase-space distribution function $f(E)$, given the observations of the surface density and velocity dispersion profiles of a "tracer" population. Briefly, in the case of a GC, (i) the projected density $I(R)$ provides the space density $\nu(r)$, (ii) the projected velocity dispersion $\sigma^2(R)$ provides the space velocity dispersion $v^2(r)$, (iii) the Jeans equation provides the gravitational potential $\Phi(r)$, and (iv) the Eddington equation provides the phase-space distribution function $f(E)$. Nevertheless, a disadvantage of such techniques arises from the delicate process of deprojection using Abel integrals.

4.2. GCS STUDIED WITH NON-PARAMETRIC METHODS

Already for four GCs non-parametric studies have been published using samples from a few hundred up to a few thousand stars: 47 Tuc (Gebhardt & Fischer 1995), NGC 362 (Gebhardt & Fischer 1995), NGC 3201 (Gebhardt & Fischer 1995, Côté et al. 1995), and M15 (Gebhardt et al. 1994, Gebhardt & Fischer 1995). Non-parametric mass density and M/L_V profiles are compared with theoretical slopes for core-collapse clusters. The two non-collapsed GCs, viz., NGC 362 and NGC 3201, seem to exhibit significant differences from the two possibly collapsed GCs, 47 Tuc and M15. The derived phase-space distribution functions are not consistent with King models: NGC 362 and NGC 3201 have significantly more tightly-bound stars than King models, and systematic differences appear between 47 Tuc and M15 and either the King models or the two less concentrated GCs. Côté et al. (1995), using King, King-Michie, and non-parametric models, present, for NGC 3201, an interesting comparison between the different results, a way to disentangle the consequences of the assumptions and disadvantages of each approach.

4.3. A MAJOR STUDY OF ω Centauri

The large collection of data announced by Reijns et al. (1993), yields radial velocities and proper motions for thousands of stars in ω Cen and illustrates what has become possible.

The proper motions measurements come from 50 early-epoch plates which are the best out of a collection of 443 plates taken of this cluster during the 1930's with the Yale–Columbia refractor while in South Africa. An equal number of 2^{nd}-epoch plates have been obtained during the last decade at Mt. Stromlo. Nearly final proper motions with an internal 1-σ accuracy of $\sim 7\ \mathrm{km\,s^{-1}}$ (0.3 marcsec/yr) have been determined for around 9800 stars brighter than V=16.5, of which an estimated 7500-8500 are members of ω Cen. Accuracies are well sufficient to investigate internal motions (Seitzer, private communication).

For the last four observing seasons, a major effort has been undertaken with ARGUS at Cerro Tololo to get accurate radial velocities ($\sim 1\ \mathrm{km\,s^{-1}}$ accuracy) for as many of these stars as possible. Some 4,500 velocities of over 3,500 stars have been obtained over the entire spatial extent of the cluster, which will permit investigation of the 3-D space velocity distribution. The same spectra are also being used to determine metallicity, to investigate the correlation between metallicity, radius, and kinematics (Seitzer, private communication).

5. Integrated-light spectra in the cores of GCs

In very high-concentration (collapsed) GCs, the measurement of the V_r of individual stars becomes very difficult in the core because of crowding problems. A way to alleviate the problem consists of either obtaining integrated light spectra or of using a Fabry-Perot interferometer (e.g., Gebhardt et al. 1994). The former technique has been used by, e.g., Illingworth (1976), Peterson et al. (1989), Dubath et al. (1990), Zaggia et al. (1992), and Dubath et al. (1994a,b).

In the case of the extension of the CORAVEL technique to the integrated-light spectra, the cross-correlation technique produces a cross-correlation function (CCF) — relative light intensity as a function of radial velocity — which is nearly a perfect Gaussian whose σ does not depend on the metallicity. The fit of such a function provides three physical quantities: (1) the abscissa of its minimum, equal to the radial velocity V_r, (2) its depth D, related to the metallicity, and (3) its standard deviation $\sigma_{\rm CCF}$, related to line broadening mechanisms. Comparison of the cross-correlation function ($\sigma_{\rm CCF}$) of a GC spectrum with the cross-correlation functions ($\sigma_{\rm ref}$) of standard star spectra unveils the broadening of the cluster cross-correlation function (σ_p) produced by the Doppler line broadening present in the integrated-light spectra because of the random spatial motions of the stars along the line of sight. A precise estimate of the projected stellar velocity dispersion σ_p in the integration area is then given by the following quadratic difference, $\sigma_p{}^2 = \sigma_{\rm CCF}{}^2(\text{cluster}) - \sigma_{\rm ref}{}^2$.

Depending on the relative numbers of bright and faint stars, such techniques present some potential sampling problems. When the light in the sampling area is dominated by one star, $\sigma_{\rm CCF}$ is narrower and the derived velocity dispersion is too small; when the light is dominated by two stars with an unusually large radial velocity difference, $\sigma_{\rm CCF}$ is wider and the derived velocity dispersion is too large.

5.1. THE CORE VELOCITY DISPERSION OF M15

M15 $\equiv$ NGC 7078 has been, for about two decades, the prototypical collapsed GC (see Peterson et al. 1989). Using the ESO New Technology Telescope with a $1'' \times 8''$ slit, Dubath et al. (1994) obtained five high-resolution integrated-light echelle spectra over the core of M15, covering a total central area of $5'' \times 8''$. By taking advantage of the spatial resolution along the slit, they extracted spectra at 120 different locations over apertures $\sim 1''$ square. The Doppler velocity broadening of the CCFs of these integrated-light spectra is always ≤ 17 km s^{-1}, at all locations in the $5'' \times 8''$ area and with a mean velocity dispersion $\sigma_p = 11.7 \pm 2.6$ km s^{-1}. The individual radial velocities of the 14 best-resolved (spatially or spectroscopically) bright stars are also determined; they give $\sigma_p = 14.2 \pm 2.7$ km s^{-1}, a value consistent with the above determination. Two of the brightest central stars, separated by 2.5″, have radial-velocity values differing by 45.2 km s^{-1}. This study agrees with the recent work by Gebhardt et al. (1994), who measured the radial velocities of 216 stars located within 1.5′ of the cluster centre. From 0.1′ to 0.4′, their data suggest a constant velocity dispersion of about $\sigma_p = 11$ km s^{-1}. These measurements therefore provide no evidence for the velocity dispersion cusp observed by Peterson et al. (1989). Nevertheless, given the general shapes of its surface brightness and velocity dispersion profile, M15 remains the best GC candidate for being close to a state of deep core collapse.

5.2. CORE VELOCITY DISPERSION SURVEY

In order to study the globular cluster masses and mass-to-light ratios as a function of galaxy types and environments, Dubath et al. (1993 and in preparation) have, in the framework of their survey, obtained integrated-light spectra of the cores of about 60 Galactic, Magellanic, and Fornax globular clusters. Zaggia et al. (1992, 1993) have developed a similar technique which they applied to seven galactic globular clusters. The integrated absolute magnitudes, velocity dispersions, and core radii of the survey clusters are used to investigate the fundamental plane correlations for Galactic and Magellanic clusters (Dubath et al. in preparation). These correlations, which are analogous to the fundamental plane correlations for elliptical galaxies, have already been discussed (e.g., Djorgovski & Meylan 1994) and are consistent with the scaling law expected from the Virial Theorem. This suggests that globular clusters are virialized systems with a universal and constant M/L ratio to within the measurement errors.

6. Main scientific outputs from the radial velocities

The main scientific outputs from V_r studies are as follows: (i) rotation is present in GCs (especially in ω Cen), but GCs remain hot dynamical systems; (ii) typical GC mass $\sim 10^5 M_\odot$ with $10^4 M_\odot \lesssim$ mass $\lesssim 10^6 M_\odot$ and $M/L_V \sim 2.5 \pm 1.0$; (iii) spectroscopic binaries do exist in GCs; (iv) velocity dispersion profiles imply no very massive haloes around GCs; ¡(v) study of global properties of GC systems, e.g., fundamental plane.

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