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# CORRECTION

NEAL, P. (2006). Stochastic and deterministic analysis of SIS household epidemics. *Adv. Appl. Prob.* **38,** 943–968.

Lemma 4.1 in the above paper is incorrect. Lemma 4.1 is only required for Corollary 4.1, which can be proved using susceptibility processes introduced in Neal (2008, Section 3). This requires a different, but equivalent, description of the epidemic process outlined below. For further details, see Neal (2008).

For $s \geq 0$, let $\mu_m(s)$ denote the mean number of infectives, in stationarity, within a household of size $m$, given that the household is subjected to a constant global infectious pressure, $s$. That is, global infectious contacts are made with members of the household at the points of a homogeneous Poisson point process of rate $\lambda_G s$. Now $\mu_m(s) = m\varpi_m(s)$, where $\varpi_m(s)$ is the probability that an individual in a household of size $m$ subjected to a constant global infectious pressure, $s$, is infectious. We proceed by showing that, for all $m \geq 1$, $\varpi_m(s)$ is a concave function of $s$, which in turn implies that $\mu_m(s) = m\varpi_m(s)$ is a concave function of $s$ (see Equation (4.2) of the above paper).

Fix $m \geq 1$ and the global infectious pressure $s \geq 0$. Label the individuals in the household $1, 2, \ldots, m$ and focus upon individual 1. For $i = 1, 2, \ldots, m$, we follow Neal (2008, Section 2) in assigning local infectious processes $\{\eta_i^L, V_i^L, \chi_i\}$ to individual $i$ as follows. Let $\eta_i^L$ be a homogeneous Poisson point process of rate $\lambda_L$ at which individual $i$ makes infectious contacts, if infectious, and let $V_i^L$ determine the individual contacted by a particular local infectious contact. (The individual contacted is chosen uniformly, at random, from the remaining individuals in the household.) Let $\chi$ be a homogeneous Poisson point process of rate $\gamma$ at which an individual recovers from the disease, if infectious. For $i, j = 1, 2, \ldots, m$ and $u \leq t$, we say that $i_u \rightsquigarrow j_t$ if there exists a path of local infection from individual $i$ at time $u$ to individual $j$ at time $t$. That is, if individual $i$ is infectious at time $u$ then individual $j$ will be infectious at time $t$ regardless of the global infectious process. Then the susceptibility process of individual 1 at time $t$ is given by

$$\mathcal{S}_1^t = \{S_1^{t,u};\, u \leq t\},$$

where $S_1^{t,u} = |\{j = 1, 2, \ldots, m;\, j_u \rightsquigarrow 1_t\}|$. The susceptibility time of individual 1 at time $t$ is defined to be

$$T_1^t = \int_{-\infty}^{t} S_1^{t,u}\, \mathrm{d}u,$$

the total amount of time that individuals in the household need to avoid global infectious contacts for individual 1 to be susceptible at time $t$. Note that the epidemic behaviour within a household is time homogeneous, so $T_1^t \stackrel{\mathrm{D}}{=} T_1^0$. Given $\mathcal{S}_1^0$, the probability that individual $i$ is susceptible at time 0 is

$$\exp\biggl(-\int_{-\infty}^{0} \lambda_G s S_1^{0,u}\, \mathrm{d}u\biggr) = \exp(-\lambda_G s T_1^0).$$

Hence,

$$\varpi_m(s) = 1 - \mathrm{E}[\exp(-\lambda_G s T_1^0)],$$

which is concave, as required, since

$$\frac{\mathrm{d}}{\mathrm{d}s}\varpi_m(s) = \mathrm{E}[\lambda_G T_1^0 \exp(-\lambda_G s T_1^0)] > 0$$

and

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\varpi_m(s) = \mathrm{E}[-\lambda_G^2 (T_1^0)^2 \exp(-\lambda_G s T_1^0)] < 0.$$

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I would like to thank the anonymous referee of a subsequent paper submission for pointing out the error.

## Reference

Neal, P. (2008). The SIS great circle epidemic model. *J. Appl. Prob.* **45,** 513–530.