

ANGULAR DIAMETER-REDSHIFT AND FLUX DENSITY TESTS

# RADIO SOURCE ANGULAR SIZES AND COSMOLOGY

R.D. Ekers  
 Kapteyn Laboratory, Groningen  
 G.K. Miley  
 Huygens Laboratory, Leiden

## 1. INTRODUCTION

The main cosmological tests using radio sources as probes are summarized below.

Test	Advantages	Difficulties
Source counts <sup>1</sup> N(s)	Only requires information from a radio source survey.	i) Survey must be complete and unbiased with s. ii) Interpretation depends on the radio luminosity function (RLF).
Hubble relation <sup>2</sup> for radio sources s(z)	Independent of the RLF.	i) Knowledge of z requires complete optical identifications to avoid bias on z with s. ii) RLF is broad and very nearly critical. <sup>3</sup>
Angular size - redshift <sup>4,5,6</sup> $\theta(z)$	i) Complete sample is not critical unless its incompleteness is a function of $\theta$ . ii) Independent of the RLF.	Requires a large sample of objects with both $\theta$ and z measured.
Angular size - flux density <sup>7,8</sup> $\theta(s)$	i) Complete sample in s is not critical. ii) Only requires a radio catalogue of angular sizes.	i) Interpretation depends on both the RLF and the linear size distribution function.
Angular size <sup>8</sup> distribution N( $\theta, s$ )	i) Includes N(s) relation but $\theta$ gives additional constraints.	Will be affected by any correlation between radio power and linear size.

Footnotes to table:

1. Jauncey (1975); 2. Bolton (1966); 3. von Hoerner (1973); 4. Hoyle (1959); 5. Wardle and Miley (1974); 6. Hewish et al. (1974); 7. Swarup (1975); 8. Kapahi (1975a).

There are three tests involving angular sizes of radio sources which appear to be at least as good as the traditional  $N(s)$  test. These have not been used extensively until now, probably because of a lack of high resolution angular size measurements over a wide range of radio source flux densities. This situation is changing rapidly especially with the increasing resolution and sensitivity of aperture synthesis telescopes. In this review these methods and some of the recent results are discussed.

## 2. MEASURES OF THE ANGULAR SIZE OF RADIO SOURCES

When using photographic observations of galaxies the simplest measure is an isophotal diameter - in contrast to this the radio astronomer can usually measure directly the more fundamental metric diameter. However, there is still considerable choice regarding which characteristic of the angular distribution of emission to use. General considerations to keep in mind are that the measure of diameter

- i) should not depend on relative resolution,
- ii) should not depend on relative sensitivity,
- iii) should be independent of frequency and
- iv) can be used on the largest possible fraction of the data.

The most common measure is the separation of the components of a double source and since the majority of strong extragalactic radio sources are double this is a simple and effective definition. In order to satisfy points i) and iv) it is necessary to extend it to partially resolved and unresolved sources by modelling them on the basis of the characteristics of the more resolved sources. To satisfy point ii) a cut off must be set on the acceptable ratio of intensities for the two components. These conditions will not introduce bias providing the brightness distribution or the fractions with different morphology are not themselves a function of distance. Other possibilities are to use some characteristic in the visibility function, e.g. the spacing for which the visibility first falls to 0.5, or the first moment of the source distribution which can be derived either from the visibility function or the brightness distribution (Burn and Conway, 1976). These have the advantage that they can easily be applied to partially resolved sources. Another estimate sometimes used is the distance between outermost contours. This is a bad measure since it depends on receiver sensitivity and resolution, and is not a purely metric diameter.

There is a general problem with complex sources. Is the correct measure for a tail source like NGC 1265 (e.g. Wellington et al., 1973) the separation of the two components of the tail or the length of the tail? A general answer is that it does not matter so long as it is done consistently. A less physical choice will increase the dispersion in the linear size distribution and hence increase the scatter in the angular sizes but this is not as bad as introducing a bias in diameter with flux

density or distance. For this example the more physical measure is the separation of the two components of the tail but this will introduce a bias unless the total sample is restricted to those objects of sufficiently large angular size that this distinction could, in principle, be made for all of them.

In the following discussion we have used the separation of components of double sources, or an equivalent measure.

3. LINEAR SIZE DISTRIBUTION FUNCTION

The distribution of linear sizes for the 3CR radio sources with  $z < 0.25$  and  $b > 10^0$  are shown in Figure 1.

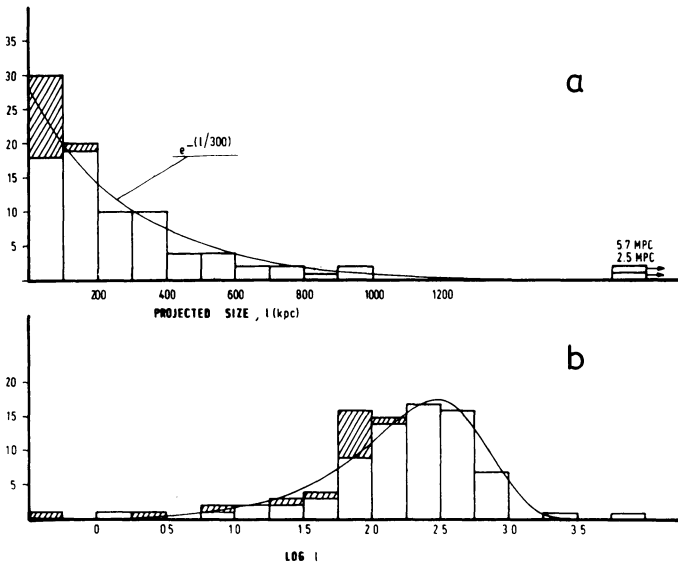


Figure 1. Distribution of projected linear sizes of 3CR radio galaxies from Kapahi (private communication), assuming  $H = 50$ . a) linear scale with equal interval bins, b) logarithmic scale with logarithmic binning. In each case the same exponential of form  $k \exp(-l/300)$  is shown.

Jacobs (this symposium) has pointed out that the use of logarithmic binning and logarithmic scales as in (b) has led to the erroneous notion that the number of radio sources peak at a characteristic separation of about 300 kpc. This has not only confused the development of radio source theories but has led to an overdependence on the largest angular size (LAS) statistics. Figure 1(a) shows the same data in a linear plot with equal bins and it can be seen that an exponential distribution of projected sizes is a better fit. It is clear that for samples drawn from such a distribution the mean or the median of the sample will have much

smaller sampling uncertainty than the largest member of the sample. The effects of random projection on this distribution are relatively unimportant.

#### 4. ANGULAR SIZE - REDSHIFT RELATION

Miley (1968) first showed that there was an angular size-redshift relation for quasars and further work on this has been reported by Legg (1970), Miley (1971) and Wardle and Miley (1974). These observations showed that the upper envelope of both the angular separation distribution and component sizes of quasars decreased approximately as  $z^{-1}$  up to  $z \sim 2.5$ . This is expected in a Euclidian universe but is a faster decrease than is expected for most cosmological models unless there is a linear size-redshift relation of the form,  $\ell \propto \ell_0(1+z)^n$  with  $n \sim -1$ , which just cancels the geometrical effects (Kellermann, 1972). Such a linear size-redshift relation is not however unexpected (e.g. van der Kruit, 1973). Figure 2 summarizes the angular size-redshift data as given in Wardle and Miley (1974) by estimating the median of the angular size distribution for four ranges of  $z$ .

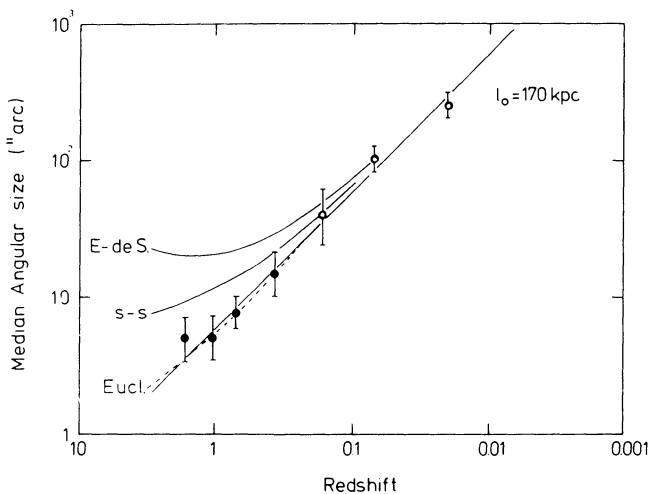


Figure 2. Median angular sizes of quasars, ●, Wardle and Miley (1974), and 3CR radio galaxies, ○, plotted against redshift. N.B.: Selection effects discussed in text. The result for three world models are indicated by the continuous line. The broken line is for an Einstein-de Sitter model with  $\ell \propto \ell_0(1+z)^{-2}$ .

A similar statistic is given for 3C radio galaxies. The median is a convenient estimate to use in this case since sources which only have upper limits can also be included. This diagram clearly shows the relation and also shows a continuity between radio galaxies and quasars if

the quasar redshifts are assumed to be at cosmological distances. The models shown are for the linear size distribution function for radio galaxies given in Figure 1. A good fit can be obtained for a Euclidian model and also for other cosmologies provided  $\ell \propto \ell_0(1+z)^n$  with  $n$  between  $-1$  and  $-2$ .

At this stage a strong word of warning is still required. The data available in the literature, especially for the quasars, is still very heterogeneous and contain unknown frequency dependent effects so a more detailed analysis of these results should not be attempted until a more systematic analysis of diameters subject to the points raised in Sections 2 and 3 has been made.

One such investigation is now in progress as a follow up to the work which led to the discovery of the  $(\theta, z)$  correlation. Hartsuijker and Miley have mapped 117 quasars with the Westerbork telescope at 5 GHz with a resolution of  $6'' \times 6'' \cos \delta$ . The sample includes all those quasars whose redshifts were published up to 1972, with  $LAS > 7''$  or with unknown structure. Together with the smaller diameter sources this gives structural information on 211 quasars with known redshift. 30% of these are 3C and 35% are 4C sources so several complete sub samples can be isolated. These measurements give both the intensity and polarization distribution. Some preliminary results are:

- i) Central components are detected in 41 out of the 43 cases with sufficient resolution to make this separation. For 24 of these the central components comprise more than 10% of the total flux density of the source. Comparison with the results of Fanti (this symposium) shows that this fraction is much larger than is the case for the radio galaxies.
- ii) For 75 sources that are sufficiently resolved, 85% are symmetrical doubles (D1) or triples and only 15% are asymmetric doubles (D2) like 3C273.
- iii) The number of sources on the  $\theta(z)$  diagram has been almost doubled from Miley (1972) but its form remains substantially unaltered. There is no apparent difference in the behaviour of 3C, 4C, Parkes and Bologna sources. More detailed analysis is still in progress.

An interesting variation on this test is to use the interplanetary scintillations as a measure of the angular sizes of compact components in extragalactic sources (Hewish et al., 1974). Some further results from this technique are described by Hewish (this symposium).

## 5. ANGULAR SIZE FLUX DENSITY RELATION

Early attempts (Longair and Pooley, 1969) and (Fanaroff and Longair, 1972) to use this test were not very promising mainly because of the small samples of extended sources available at low flux densities. Now that large amounts of information on the angular sizes of radio sources can be routinely obtained it is possible to exploit this test more fully. The hope is that the very large data samples which are available will offset the double smoothing of the  $\theta(s)$  relation by the RLF and the linear size distribution function.

The most complete published analysis of  $\theta(s)$  data comes from the lunar occultation sample with the Ooty telescope combined with the 3C data (Swarup 1975, Kapahi 1975b). A sample of data at lower flux densities has been obtained by analysing the angular size information on background sources found during routine mapping observations with the Westerbork Synthesis Radio Telescope (Ekers, Hummel and Jacobs - in preparation). These two sets of data are summarized in Figure 3.

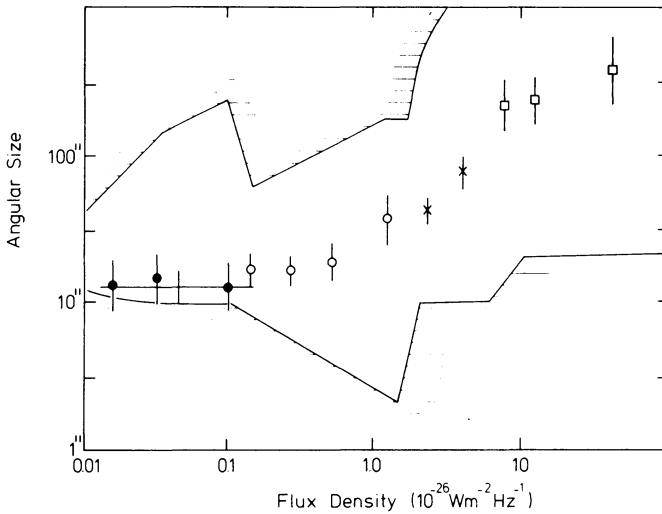


Figure 3. 30 percentile points in the angular size distribution of radio sources plotted against flux density. Data is from Westerbork (Ekers et al., in preparation), ●, and Katgert (1976), +, Ooty (Kapahi, 1975b), o, BDFL (Bridle et al., 1972), x, and all sky (Robertson, 1973), □. The hatched area is excluded by selection effects.

The 30 percentile points of the angular size distribution (i.e. 30% of the sources are larger than the size given) for a number of flux density bins are plotted against flux density. Since the Westerbork data is all obtained at 1400 MHz, the BDFL catalogue (Bridle et al., 1972) has been used instead of the 3C sample. No other data is available in the intermediate flux density range so the Ooty sample has been shifted using a mean spectral index of  $-0.75$  between 327 and 1400 MHz. Similarly, the all sky sample (Robertson, 1973) has been corrected from 408 to 1400 MHz. A further measurement of the angular sizes in the  $0.01 - 0.1 \text{ Jy}^*$  range at 1400 MHz can be obtained from the Westerbork observations of the 5C2 regions (Katgert, 1976). The 30 percentile

\*  $1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$

points derived from this data are also shown in Figure 3. Error bars indicate statistical uncertainty only.

The hatched area in Figure 3 is a rough indication of the area excluded by selection effects. The upper limit results from the use of peak rather than integrated flux densities in radio source surveys and the lower limits result from the finite resolution and sensitivity of the radio telescope used to measure the angular sizes. The separation between these regions becomes uncomfortably small for some ranges of flux density and indicates that further attention to these effects is warranted. The fact that the lower angular size limit for the Westerbork data is very close to the measurements is just a consequence of using the 30 percentile points since this percentile was specifically chosen to avoid this selection effect.

A very suggestive comparison of the  $(\theta, s)$  and  $(N, s)$  results are illustrated in Figure 4 in which we show the data normalized by the Euclidian values. There is a striking similarity between these two plots.

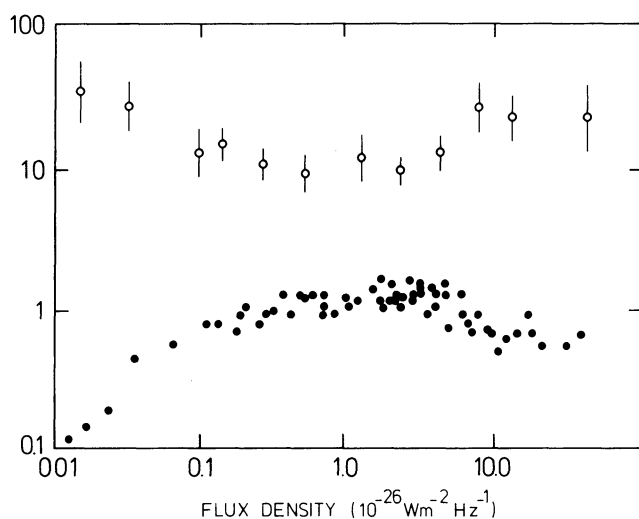


Figure 4. The normalized  $(\theta, s)$  relation,  $\circ$ , and normalized differential  $(N, s)$  relation,  $\bullet$ , plotted against flux density.

Both relations are close to the Euclidian result over the same flux density range. In both cases there is a much steeper slope in the 3-10 flux unit range and at low flux densities both relations become less steep than Euclidian. A few remarks on this result which indicate the potential of the  $(\theta, s)$  test can be made.

i) The  $(\theta, s)$  results are independent of the  $(N, s)$  results since the measurement of  $\theta$  for the sources is independent of the density of these sources in a catalogue. Hence an explanation of the steep  $N(s)$  relation in the 3-10 flux unit range as a statistical fluctuation in the number of strong sources would not explain the  $(\theta, s)$  result.



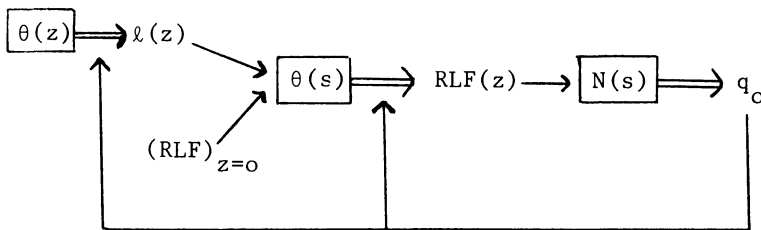
ii) If the deficiency of strong sources were due to selection against strong extended sources such as DA240 (Willis et al., 1974) this would have given smaller rather than larger sizes in the  $(\theta, s)$  data in this density range.

iii) A more likely explanation of this similarity is that the relation between average flux density and distance, which depends on the RLF and its evolution, will be the same for both relations.

Model calculations including the RLF and the linear size distribution and their evolution are discussed in the following contribution by Kapahi.

## 6. SUMMARY

Good information on the angular sizes of radio sources over a large range of flux densities and redshifts is now available. This information can be used to place additional constraints on world models and to give more information on the evolution of radio source sizes and intensities. A schematic illustration of the way in which angular size information could in principle be used to determine results of cosmological significance is illustrated below.



The  $\theta(z)$  relation depends only on the cosmology and the size evolution function  $\lambda(z)$  so for a given cosmological model, quantified by  $q_0$  in this simple outline, we can deduce the  $\lambda(z)$  relation. This plus the local RLF can be used to determine the evolution of the RLF from the  $\theta(s)$  relation. Once we have determined  $RLF(z)$  independently of the  $N(s)$  relation we can put it in the  $N(s)$  relation to determine the cosmology. If this differs from the original assumption we could iterate to search for a consistent solution.

## REFERENCES

- Bolton, J.G. 1966, *Nature* 211, 917.  
 Bridle, A.H., Davis, M.M., Fomalont, E.B. and Lequeux, J. 1972, *Astron. J.* 77, 405.  
 Burn, B.J. and Conway, R.G. 1976, *Mon.Not.R.Astr.Soc.* 175, 461.  
 Fanoroff, B.L. and Longair, M.S. 1972, *Mon.Not.R.Astr.Soc.* 146, 361.  
 Hewish, A., Readhead, A.C.S. and Duffett-Smith, P.J. 1974, *Nature* 252, 657.  
 von Hoerner, S. 1973, *Astrophys. J.* 186, 741.  
 Hoyle, F. 1959, *Paris Symposium on radio astronomy*, p.529, ed. R.N. Bracewell, Stanford University Press.

- Jauncey, D.L. 1975, Ann.Rev.Astron.and Astrophys. 13, 23.  
Kapahi, V.K. 1975a, Mon.Not.R.Astr.Soc. 172, 513.  
Kapahi, V.K. 1975b, Ph.D.Thesis, Tata Institute of Fundamental Research, Bombay.  
Katgert, P. 1976, Astron. and Astrophys. 49, 221.  
Kellermann, K.I. 1972, Astron. J. 77, 531.  
van der Kruit, P.C. 1973, Astrophys. Lett. 15, 27.  
Legg, T.H. 1970, Nature 226, 65.  
Longair, M.S. and Pooley, G.G. 1969, Mon.Not.R.Astr.Soc. 145, 121.  
Miley, G.K. 1968, Nature 218, 933.  
Miley, G.K. 1971, Mon.Not.R.Astr.Soc. 152, 477.  
Swarup, G. 1975, Mon.Not.R.Astr.Soc. 172, 501.  
Wardle, J.F.C. and Miley, G.K. 1974, Astron. and Astrophys. 30, 305.  
Wellington, K.J., Miley, G.K. and van der Laan, H. 1973, Nature 244, 502.  
Willis, A.G., Strom, R.G. and Wilson, A.S. 1974, Nature 250, 625.

## DISCUSSION

*Baldwin:* Is it possible to improve the error bars on the angular diameters in the  $\theta - s$  diagram at high flux densities?

*Ekers:* Yes. We could obtain a complete sample of angular sizes for the Southern hemisphere sources.

*Wampler:* How does one correct for the fact that the sample contains objects such as 3C 273, with one component of the double source on the QSO and the other off the QSO? Does one double the angular separation?

*Ekers:* One possibility would be to give any difference and just say that this class is one of the contributions to the linear size distribution function. Providing the relative number of these objects does not change with redshift this will not introduce any bias. If one has sufficiently high spatial resolution to recognize these objects for the whole sample then the scatter in the  $(\theta, z)$  relation may be slightly decreased by including this correction.

*McCrea:* There is one effect that would account qualitatively for the effects in numbers and sizes at larger redshifts, but I do not know how well it would do so quantitatively. If two sources are confused and counted as one, the result is to get the size too large and the number too few.