

# THE DYNAMICS OF WOLF-RAYET WINDS

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**Abstract.** We review the dynamics of Wolf-Rayet stellar winds, with emphasis on how multi-line scattering can lead to mass loss rates for which the wind's radial momentum flux  $\dot{M}v_\infty$  greatly exceeds the limit for single-scattering of the star's radiative momentum  $L_*/c$ . The geometrical considerations that allow this to occur are illuminated through connections to multiple scattering in an effectively gray medium. The so-called "WR wind momentum problem" would be better characterized as an "opacity problem" of simply identifying a sufficiently dense ensemble of optically thick lines.

**Key words:** stars: Wolf-Rayet – winds – model atmosphere

## 1. Introduction

The radial momentum of a Wolf-Rayet (WR) wind is commonly compared to that associated with the star's radiative luminosity  $L_*$  through the dimensionless ratio  $P_{wind} \equiv \dot{M}v_\infty / (L_*/c)$ , whereby it is often noted that best estimates for WR winds imply values well in excess of unity, perhaps as high as  $P_{wind} \approx 50$  (Schmutz *et al.* 1989; Willis 1991; Hamann *et al.* 1993). Over the last decade, there have been several papers (Panagia & Machetto 1982; Friend & Castor 1983, hereafter FC; Abbott & Lucy 1986; Kato & Iben 1992; Lucy & Abbott 1993; Springmann 1994) showing that multiple scattering of radiation can in principle yield  $P_{wind} > 1$ . Nonetheless, among many researchers in hot-star winds, there apparently remains considerable uncertainty about exactly how, and by how much, this single-scattering limit can be exceeded. A primary objective of this review is to clarify these issues through pedagogical discussion of multiple scattering grounded in the ideas of gray radiative transfer. An important point is that the cumulative effect of scattering in strongly overlapping spectral lines can be heuristically treated as "effectively gray". The key requirement for achieving the inferred momentum factors for WR winds is simply to have a sufficiently dense packing of optically thick lines, without large gaps in their spectral distribution.

## 2. Multiple momentum deposition

### 2.1 EXAMPLE OF A STATIC GRAY ENVELOPE

Consider a static, spherical envelope of gray opacity  $\kappa$  (defined here as a cross section per unit mass of material) surrounding a central luminosity source  $L_*$ . Each differential shell of geometric thickness  $dr$ , radius  $r$ , and mass density  $\rho(r)$  has a column mass  $dm = \rho dr$ , and so will intercept a fraction  $d\tau = \kappa dm$  of the local radiative flux. For a static gray medium, radiative equilibrium requires that the flux density at each radius is simply

$F = L_*/4\pi r^2$ , implying then that the radiative momentum imparted to each shell is given by  $d\dot{p}_{rad} = 4\pi r^2 F d\tau/c = L_* d\tau/c$ . Integration over all such shells therefore means that the total radial momentum imparted to the envelope is just

$$\dot{p}_{rad} = \frac{L_*}{c}\tau, \quad (1)$$

where  $\tau$  is the total radial optical depth of the envelope. This shows that the radial momentum deposited by radiation can become *arbitrarily large* in a sufficiently opaque medium. Indeed, one extreme example of this is stellar interiors, for which the optical depths are truly enormous, typically of order  $10^{10}$ ! However, in this case the implied huge source of radiative momentum is simply lost to the equally huge momentum “sink” of gravity, and thus merely helps to maintain (together with the gas pressure) a hydrostatic equilibrium. Hence, although the radiation provides a large momentum, it does no net work.

In a stellar wind, the radiative momentum deposition must become strong enough to exceed this gravitational loss, and thereby drive a flow. In this case, the radiation must supply *energy* as well as momentum, by doing work to accelerate material and lift it out of the star’s gravitational well. As radiation is scattered in this expanding wind, it is thereby red-shifted, which is sometimes termed “photon tiring.” Energy conservation then imposes the more fundamental mass-loss limit that  $\dot{M}(v_{esc}^2 + v_\infty^2)/2 < L_*$ , where now  $L_*$  and  $v_{esc}$  are the star’s radiative luminosity and escape speed at the subsonic wind base. However, since characteristically  $v_{esc} \lesssim v_\infty \ll c$ , this still allows quite a large wind momentum, in principle as high as  $P_{wind} \approx 2c/v_\infty$ , which is of order a few hundred for a typical WR star. For most WR stars with inferred  $P_{wind} \approx 10 - 20$ , photon tiring is thus only about a 5-10% effect, and so to this level may be ignored in discussing mechanisms for satisfying the wind momentum requirements.

Figure 1 provides a geometric illustration of how multiple momentum deposition occurs in an optically thick envelope. Figure 1a shows the case of a hollow shell with optical thickness  $\tau = 5$ , wherein a photon is backscattered within the hollow sphere roughly  $\tau$  times before escaping, having thus cumulatively imparted  $\tau$  times the single photon momentum, as given by eq. (1). This idea of hemispheric crossing has been the usual physical interpretation of multiple momentum deposition, as first discussed by Panagio & Machetto (1982) in the context of reflection between line-resonance layers. However, as we discuss further below, line resonances really only backscatter at most half the photons, and so such hemispheric crossing between lines can never account for more than about a factor of two momentum enhancement. Large momentum factors can, however, be achieved by a dense packing of lines, wherein photons locally *diffuse* between line resonances.

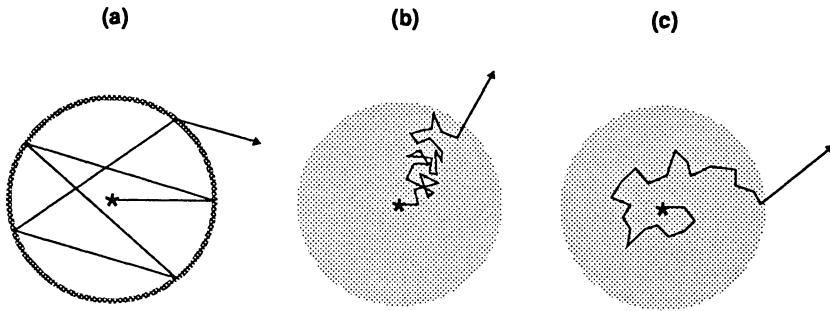


Fig. 1. Multiscattered photon trajectories in (a) hollow and (b) filled gray spheres with the same central  $\tau = 5$ . The latter is shown ‘untangled’ in part (c) to illustrate that the multiple momentum deposition associated with hemispheric crossing in (a) is analogous to an effective ‘winding angle’ of the photon diffusion in (b).

For the case of gray scattering, Figure 1b shows such a diffusive photon trajectory for a solid sphere with the same central optical depth as figure 1a, wherein the same factor 5 momentum deposition now occurs *without* unphysical backscattering or hemispheric crossing. Figure 1c illustrates how these diffusive vs. direct-flight pictures of multiple momentum deposition can be reconciled by thinking in terms of an effective “winding angle” around the envelope. For each scattering within a spherical envelope the radial momentum deposition is unchanged by arbitrary rotations about a radius through the scattering point. For each such scattering, let us imagine a rotation that brings the scattered trajectory into a single plane, with the azimuthal component always of the same sense, say clockwise viewed from above the plane. In this artificial construction, scattering leads to a *systematic* (vs. random walk) drift of the photon along one azimuthal direction, implying a cumulative *winding* of the photon trajectory in such a way as to impart the same radial momentum as the original convoluted trajectory. For the above example of a homogeneous sphere, each mean-free-path segment  $R/\tau$  yields a winding-angle increment of roughly  $\Delta W \sim 1/\tau$ . During the  $\tau^2$  scatterings needed to escape, photons viewed in this way thus accumulate a total effective winding angle of order  $W \sim \tau$ . This winding angle is the filled-sphere analog of the crossing number for a hollow sphere, and it again implies a radial momentum deposition that scales as  $\tau L_*/c$ , in accord with eq. (1).

Relations similar to eq. (1) have been discussed previously by Netzer & Elitzur (1993) for dust driven winds from cool stars, and by Kato & Iben (1992) for a continuum-driven model of WR winds. In WR winds, a dominant source of continuum opacity is electron scattering, which is indeed gray; but even though such winds can have moderately large electron scat-

tering optical depths  $\tau_e \sim 10$ , the associated radiative momentum cannot *by itself* drive a wind. The reason is that the electron scattering opacity has a nearly constant mass absorption coefficient  $\kappa_{es}$ , implying that the associated radiative acceleration is nearly a constant fraction  $\Gamma \equiv \kappa_{es} L_*/4\pi GM_*c$  (the Eddington factor) of the gravitational acceleration. However, for the underlying star to remain gravitationally bound requires that  $\Gamma < 1$ . This means that electron scattering can help reduce the effective stellar gravity, but other opacity sources are required to actually overcome it and accelerate a wind outflow. Kato & Iben (1992) arbitrarily assumed an enhancement by factors 2-5 in the continuum opacity in the outer stellar envelope, thus attaining an effective  $\Gamma > 1$ , and so an outward wind acceleration. To justify this, they cite expected opacity enhancements due to the large number of iron lines only now being incorporated into opacity projects, but, as we shall now argue, such line transitions should not be so naively treated as contributing to an effective continuum opacity without taking into account of the effects of flow acceleration.

## 2.2 MULTI-LINE TRANSFER IN AN EXPANDING WIND

The radiative acceleration associated with *line* opacity is, in fact, ideally suited to driving a stellar wind, because it has a natural coupling to the flow velocity. As required, it tends to be small in the nearly hydrostatic regions at the wind base, where the line flux saturates, but then naturally increases with the wind acceleration as the local line frequency is Doppler-shifted out of the shadow of underlying material.

On the other hand, the nature of line opacity makes it rather less effective at keeping photons trapped within the wind envelope. Unlike in a gray medium, photon scattering by single, isolated lines in an expanding supersonic wind is confined to relatively narrow resonance layers. For a very optically thick line, photons scatter repeatedly within this layer, but unlike in a gray shell, there is not an effective backscattering, since photons eventually escape with roughly equal probabilities in the fore or aft directions. Furthermore, since the upward and downward recoils within the narrow layer nearly cancel, the net radial momentum imparted is nearly the same as for a single scattering, no matter how optically thick the line. This again is in strong contrast to the gray scattering case described in eq. (1) above.

A single, thick line with a frequency near the peak of a star's flux spectrum can scatter about a fraction  $v_\infty/c$  of the total stellar luminosity within the wind, and so the radial momentum from each such isolated line is roughly  $\dot{p}_{rad} = v_\infty L_*/c^2$ . For  $N_{thick}$  non-overlapping lines, the total radiative momentum factor is thus  $P_{rad} \equiv N_{thick} v_\infty/c$ . Hence, to within an order unity factor to account for wind gravitational support, the mass loss that can be

driven is roughly

$$\dot{M} \approx N_{thick} \frac{L_*}{c^2}. \quad (2)$$

Furthermore, without line overlap within the wind, there can be at most about  $N_{thick} \approx c/v_\infty$  thick lines spread throughout the spectrum, and so the wind momentum in this case is limited to  $P_{wind} \lesssim 1$ . To produce the mass loss rates of WR winds with  $P_{wind} \gg 1$ , a line-driven wind model must therefore necessarily incorporate line-overlap effects. An essential complication of this multi-line scattering is that the radiation field impinging on the resonance layer of any given line is no longer just that from the stellar core, but includes photons scattered in another line somewhere else in the wind.

An approach used by Lucy & Abbott (1993) has therefore been to forgo detailed solution of the local momentum balance, and instead derive the mass loss rate from a Monte Carlo calculation of the global momentum deposition from a detailed line list. Although this model assumed a fixed, parameterized velocity law, and so did not attempt to satisfy local dynamical balance, it did nonetheless show that such multi-line scattering can globally deposit sufficient momentum to support mass loss rates as high as  $\dot{M} v_\infty \approx 10L_*/c$ . Springmann (1994) has recently presented more heuristic Monte Carlo calculations aimed at further clarifying the physical requirements of such multiple momentum deposition.

An alternative approach introduced by FC was to seek direct solutions to the multi-line radiative transfer by assuming that the frequency distribution of the line ensemble is approximately Poissonian. The implied *statistical independence* of the lines then again allows a local expression for the cumulative line acceleration, making possible a direct extension the usual CAK formalism for OB winds to incorporate multi-line scattering effects. Of particular interest in the context of our discussion of gray transfer is that FC were able to represent the *collective* effects of this statistical line ensemble in terms of an *effectively gray* opacity, even though the opacity of each individual line is the antithesis of gray. In this case, however, this effectively gray ensemble opacity no longer characterizes a physical absorption cross section per unit mass of wind material, but instead is now equal to the inverse of the expected mass column an “inter-line” photon must propagate before it is red-shifted into resonance with an optically thick line by the wind expansion. If  $\Delta v$  is the mean wavelength separation (in velocity units) between thick lines, then along a ray coordinate  $l$  with direction cosine  $\mu$  relative to the radial direction, the effectively gray opacity of the line ensemble is

$$\kappa_{lines}(\mu) = \frac{1}{\rho \Delta v} \frac{dv_l}{dl}, \quad (3)$$

where  $dv_l/dl$  is the gradient of the projected velocity along the ray. Since this velocity gradient typically differs along different directions, the effective

line opacity is *non-isotropic*. Along the radial direction ( $\mu = 1$ ), the effective optical depth of the line ensemble over the wind is given simply by

$$\tau_r = \int_{R_*}^{\infty} \kappa_{lines}(1)\rho(r)dr = \frac{v_{\infty}}{\Delta v}, \quad (4)$$

where now  $\Delta v$  is appropriately averaged over the wind.

Despite its somewhat different physical origin, this effectively gray opacity leads to a radiative transfer that, apart from the anisotropy, is quite analogous to that in the simple gray envelope cited above. This means that, within order unity angular corrections, eq. (1) can still be used to estimate the momentum deposition in the expanding envelope of a stellar wind,

$$P_{rad} \approx \tau_r = \frac{v_{\infty}}{\Delta v}. \quad (5)$$

If we neglect the order unity fraction of  $P_{rad}$  needed to balance the effective gravity, then  $P_{rad} \approx P_{wind}$ , and so the mass loss rate may be estimated simply by

$$\dot{M} \approx \frac{L_*}{c\Delta v} \approx N_{thick} \frac{L_*}{c^2}, \quad (6)$$

where, in the latter expression, we identify  $N_{thick} \approx c/\Delta v$  as the total number of optically thick lines. As seen from eq. (2) this latter form gives just the *same scaling as occurs without line overlap* in the usual CAK formalism. Thus the effectively gray approach extends this CAK formalism to the overlapping line case with  $\Delta v < v_{\infty}$ , which allows one to achieve wind momentum factors  $P_{wind} \approx v_{\infty}/\Delta v > 1$ .

Note that, as in the CAK case, the number of thick lines  $N_{thick}$  (and hence  $\Delta v$ ) is not known *a priori*, but rather itself depends on the wind solution, including the mass loss rate. Thus, although physically instructive, the above heuristic expressions do not yet allow one to determine a value for the mass loss rate in terms of given stellar and line list parameters. For this, one must obtain self-consistent solutions for the effectively gray transfer. FC describe numerical techniques for obtaining such solutions, and then apply these to numerical computations of the wind velocity and mass loss rate.

A recent paper by Gayley, Owocki, and Cranmer (1994; see also poster paper in these proceedings) shows how the transfer in thick winds with  $P_{wind} \gg 1$  can be modelled using a non-isotropic diffusion approximation. The derived analytic mass loss expression is, in fact, quite similar to the original CAK formula, modified to account for the angular dependence of the radiation field resulting from the non-isotropic opacity. Gayley and Owocki (1994) have recently extended this diffusion treatment of the line-transfer to a computation of the instability growth rate in such multi-line-scattered winds. The results imply that WR winds can be quite unstable, and this



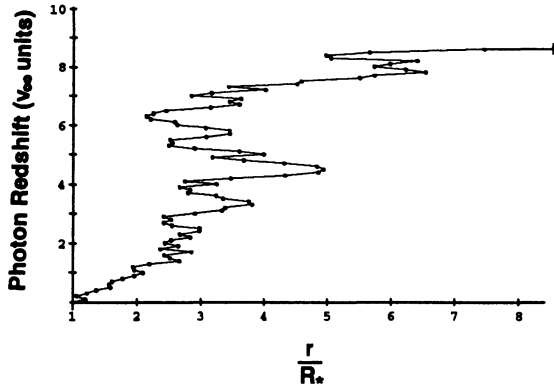


Fig. 2. Simulated photon redshift in the comoving frame as a photon executes a random walk through the wind. The adopted line density is  $v_\infty/\Delta v = 10$  which, because of the random walk character, implies the photon will interact with roughly  $10^2$  different lines during escape. Since the comoving redshift between scatterings is  $v_\infty/10$ , this implies a total comoving redshift of  $10 v_\infty$ .

suggests a possible formation mechanism for the extensive structure inferred from variability of optical emission lines from these winds (Robert 1991; Moffat *et al.* 1993). One can also view this multi-line scattering that occurs in a WR wind as a random walk over wind *velocity*. To achieve the rms target velocity  $v_\infty$  in random increments of  $\Delta v$  requires stepping through  $(v_\infty/\Delta v)^2$  lines. Since the expected net redshift from each line interaction is of order  $\Delta v/c$ , photons undergo a cumulative redshift  $\Delta E/E \approx (v_\infty^2/\Delta v^2)\Delta v/c$  over the course of their escape. The associated radial momentum deposition factor is  $P_{rad} \approx (\Delta E/E)(c/v_\infty) \approx v_\infty/\Delta v$ , as found above.

Figure 2 illustrates the comoving-frame redshift for a typical photon as it diffuses in radius due to multi-line scattering. The photon track shown is a characteristic result of a simple Monte Carlo calculation for a wind velocity law  $v(r) = v_\infty(1 - R_*/r)$  and a constant line spacing  $\Delta v = v_\infty/10$ . Each of the nodes shown represents scattering in one line. In this specific case, the photon escapes only after interacting with nearly 90 lines, resulting in cumulative redshift of nearly  $9v_\infty/c$ . These are near the statistically expected values of  $(v_\infty/\Delta v)^2 = 100$  line scatterings resulting in a total redshift  $v_\infty^2/c\Delta v = 10v_\infty/c$ , as given by the above random walk arguments.

### 3. Conclusion

A central question for this approach to modeling WR winds is: How valid is the FC picture of lines being statistically distributed across the flux spectrum in such a way that their cumulative effect can be treated as effectively gray? This cannot hold in the strictest sense across the entire spectrum, because observations of WR stars in the optical and near UV clearly show distinct, well-separated lines, with no evidence of the very strong line overlap

postulated for the wind driving. On the other hand, these spectral regions represent only a small fraction of the radiative energy flux within the wind, and so the key issue is whether there exists sufficient density of lines spread across the *flux-carrying frequencies* at each radius, and whether correlations between these lines may be disregarded.

The calculation by Lucy & Abbott (1993) suggests that this may indeed be the case. They show, in fact, that there is a tendency for the ionization balance to adjust so that there are always lots of lines near the peak of the local photon energy spectrum, even as this spectrum shifts from higher to lower energy with the outward decrease in wind temperature. This spatial gradation tends to fill in any large gaps between lines, so that photons are essentially trapped within the wind. Much as in the effectively gray picture described above, wind photons thus must undergo a spatial diffusion involving multiple line scatterings to escape the envelope. Moreover, correlations between lines are reduced when photons propagate between different ionization stages, although multiplet structure will always yield some correlation. The many advantages of the effectively gray picture motivate further work to quantify the degree to which it can be applied to line distributions obtained from detailed treatments of the ionization and excitation balance.

### Acknowledgements

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**DISCUSSION:**

**Schmutz:** I would like to point out that with today's line lists we do not find a Poisson distribution of the lines. Instead, each depth point has a set of lines that has wavelength regions with a low density of lines. Figure 5 of Lucy & Abbott (1993) illustrates this fact clearly. The flux at the photosphere is flowing in the regions with low line density. Thus there, the grey approximation breaks down and there, realistic models do not yield enough flux to accelerate a massive WR wind.

**Cassinelli:** Since many of us here have some training in radiation transfer, I think the effect of optical depth,  $\tau$ , can be explained as follows. As  $\tau$  increases the mean intensity increases as according to the familiar formula  $J = 3H(\tau + 2/3)$ , where  $4\pi H$  is the flux, which is constant in Stan's "equivalent gray case". However there are some consequences to having  $J$  increase. The momentum problem has not only been a puzzle in line driven winds but in the dust driven winds of cool giants as well. In that field one often also sees the formula,  $\dot{M} v_{\infty} = L/c \tau$ , that Stan presented. However in the case of dust the Planck function  $B$  is proportional to  $J$ , so if  $\tau$  increases the temperature rises and the grains on the inner side of the shell evaporate. Even in the hot stars case as  $\tau$  increases both  $J + T$  increase, but that doesn't appear to present a problem. In any case, I think it is more useful to think of the effect of  $\tau$  in terms of  $J$ , instead of trying to envision the effects on the scattering of a single photon.

**Owocki:** I agree. The increase in  $J$  you describe is exactly the wind or line blanketing effect I mentioned in my talk, when I tried to emphasize the importance of including this in the ionization and excitation balance that determines the opacity. As for the single photon picture mentioned in the previous comment, this too can be useful, but one must be careful to distinguish between the radial momentum such a photon can impart (which increases with  $\tau$  of the shell), and the energy it can give, which of course is limited to  $h\nu$ .