

similarities (chapter 6), geometric inequalities (chapter 7), circular transformations (e.g. inversions) (chapter 8) and hyperbolic geometry (chapter 9). The author proceeds carefully with a good deal of attention to rigour, although he admits to a certain amount of "hand-waving" at times. Considering the difficulty of the material and the intended audience (i.e., non-research mathematicians) this partial lapse in rigour is forgivable, even, in fact, desirable.

Some features of the book which this reviewer particularly likes are a nice introduction to geometric vectors in Chapter 2, a section on the seventeen plane crystallographic groups (all illustrated on page 78) in Chapter 3, and an impressive number of exercises throughout the whole book. On the critical side, the treatment at times seems to be a little heavy, and could possibly have been relieved by a less cumbersome notation.

All in all, the book is a good addition to the literature, and deserves serious consideration by anybody who proposes to give a university geometry course that is slanted toward high school teachers.

F. A. Sherk, University of Toronto

Class field theory, by E. Artin and J. Tate (ed.). W. A. Benjamin Inc., New York, 1968. 286 pages. \$3.95 (paper). \$9.50 (cloth).

This seems to be an almost unchanged reprint of the notes issued by the Department of Mathematics at Harvard University a number of years ago.

The dimensions of the book have been reduced in the reproduction process, and reference to Cassels and Frohlich' "Algebraic Number Theory" and Weil "Basic Number Theory" have been added in a footnote on the bottom of the first page.

In re-issuing these fine notes, it was pleasing to see that the price had not gone up for the paperback edition, although the cloth-bound edition seems astonishingly expensive by comparison.

W. Jonsson, McGill University

Proceedings. United States - Japan Seminar on Differential and Functional Equations, University of Minnesota, Minneapolis, Minnesota, June 26-30, 1967, edited by William A. Harris Jr. and Yasutaka Sibuya. W. A. Benjamin, Inc., New York, Amsterdam, 1967. 500 pages. \$8 50.

These proceedings were published in this mimeographed form less than five months after the Seminar was held as part of the United States - Japan Cooperative Science Program, under the auspices of the National Science Foundation - Office of International Science Activities and of the

Japan Society for Promotion of Science, with Masuo Hukuhara and William A. Harris as coordinators.

There were 4 sessions for 27 contributed short communications and 10 sessions for 13 major addresses. The latter are contained in Section 1 of the present Proceedings, together with the extended versions of 3 short communications prepared by invitation of the organizing committee. Sections II and III contain "essentially complete reports" and "extended abstracts", respectively, of the remaining 24 short communications (there does not seem to be much difference neither in length nor in the amount of details in the communications of these two sections). The detailed contents are the following:

Section I

- H. A. Antosiewicz, Linear Problems for Nonlinear Ordinary Differential Equations.
D. Bushaw, Dynamical Polysystems - A Survey.
Kenneth L. Cooke, Some Recent Work on Functional-Differential Equations.
Stephen P. Diliberto, New Results on Periodic Surfaces and the Averaging Principle.
Shigeru Furuya, A Nonlinear Differential Equation.
Masuo Hukuhara, Kneser Family and Boundary Value Problems.
Masahiro Iwano, Analytic Expressions for Bounded Solutions of Nonlinear Differential Equations with Briot-Bouquet Type Singularity.
Wilfred Kaplan, Analytic Ordinary Differential Equations in the Large.
Junji Kato, On the Existence of a Solution Approaching Zero for Functional Differential Equations.
Nicholas D. Kazarinoff, Recent Results and Unsolved Problems in Diffraction Theory.
Tosihusa Kimura, On the Global Theory of Algebraic Differential Equations.
W. S. Loud, Periodic Solutions of Nonlinear Differential Equations of Duffing Type.
Edward J. McShane, Optimal Control Theory and Stochastic Differential Equations.
John A. Nohel, Remarks on Nonlinear Volterra Equations.
William T. Reid, Variational Methods and Boundary Problems for Ordinary Linear Differential Systems.
Tosiya Saito, On the Flow Outside an Isolated Minimal Set.
Yoshiyuki Sakawa, Solution of an Optimization Problem in Linear Distributed-Parameter Systems.
Tatsujiro Shimizu, Numerical Study on Response Curves for Duffing's Differential Equation.
Walter Strodt, Principal Solutions of Nonlinear Differential Equations.
Shohei Sugiyama, On Some Problems of Functional-Differential Equations with Advanced Argument.
Minoru Urabe, The Newton Method and Its Applications to Boundary Value Problems with Nonlinear Boundary Conditions.
Taro Yoshizawa, Stability and Existence of Periodic and Almost Periodic Solutions.

SECTION II

- Fred Brauer, A Class of Nonlinear Eigenvalue Problems.
Jagdish Chandra and B.A. Fleishman, Comparison of Positive Solutions of Nonlinear Equations.
Charles C. Conley, Invariant Sets in a Monkey Saddle.
W.A. Harris, Jr. and Jon W. Tolle, A Nonlinear Functional Equation.
C.J. Himmelberg and F.S. van Vleck, Some Remarks on Filippov's Lemma.
Allan M. Krall, Differential-Boundary Equations and Associated Boundary Value Problems.
A. C. Lazer, On Schauder's Fixed Point Theorem and Forced Second-Order Nonlinear Oscillations.
R.K. Miller, An unstable Nonlinear Integrodifferential System.
Emilio O. Roxin, On Varaiya's Definition of a Differential Game.
J.D. Schuur, Estimates of the Exponential Growth of Solutions of Linear Third Order Differential Equations.
George R. Sell and Yasutaka Sibuya, Behaviour of Solutions Near a Critical Point.
Ken-Iti Takahasi, A Note on the PLK-Method.

SECTION III

- Craig Comstock, On Boundary Layers and Almost Characteristic Boundaries.
R.J. Hanson, Simplification of Second Order Systems of Ordinary Differential Equations with a Turning Point.
Philip Hartman, On Homotopic Harmonic Maps.
Harry Hochstadt, Inverse Problems Associated with Second-Order Differential Operators.
Po-Fang Hsieh, On an Analytic Simplification of a System of Linear Ordinary Differential Equations Containing a Parameter.
V. Lakshmikantham and S. Leela, Almost Periodic Systems and Differential Inequalities.
Donald A. Lutz, Linear Perturbations of Irregular Singular Systems.
Kenjiro Okubo, Two Point Connection Problems for a System of First Order Linear Ordinary Differential Equations with a Singular Point of Rank Two.
David L. Russell, An Extended Block-Diagonalization Theorem.
George Seifert, Stability and Uniform Stability in Almost Periodic Systems.
G. Stengle and S. Khabbaz, An Application of K-Theory to the Global Reduction of Matrix Valued Functions and Differential Equations.
H.K. Wilson and D.V. Ho, On the Existence of a Similarity Solution for a Compressible Boundary Layer.

The main fields were Control Theory, Functional Differential Equations, (containing also Integral Equations!), Stability Theory, Boundary Value and Variational Problems, Dynamical Systems, and Analytic Theory. It might seem desirable that such seminars and publications in the future should embrace also Functional Equations in their modern sense, as defined, for instance, by M. Kuczma, (A Survey of the Theory of Functional Equations, Publications Fac. Electrotechn. Univ. Belgrade, vol. 130, 1964), E. Hille (Topics in Classical Analysis, Lecture on Modern Mathematics, vol. 3, pp. 1-57, Wiley, New York, 1965), R. Bellman (Functional

Equations, Handbook of Mathematical Psychology, vol. 3, pp. 487-513, Wiley, New York, 1965) and J. Aczel (Lectures on Functional Equations and Their Applications, Academic Press, New York, 1966; First, German, edition Birkhauser, Basel, 1961). (The nearest to the topic of functional equations in this sense in the present Proceedings is the paper of W.A. Harris Jr. and J.W. Tolle on a nonlinear finite difference equation!).

J. Aczel, University of Waterloo

Linear Algebra, 3rd edition (Die Grundlehren der mathematischen Wissenschaften, Vol. 97), by W.H. Greub. Springer-Verlag, New York, Inc. 1967. ix + 434 pages. \$9.80 U.S.

Multilinear Algebra (Die Grundlehren der mathematischen Wissenschaften, Vol. 136), by W.H. Greub. Springer-Verlag, New York, Inc. 1967. x + 224 pages. \$8.00 U.S.

These two books are an expansion of the author's Linear Algebra 2nd edition, with the material concerning tensor algebras and multilinear functions now appearing in a separate volume. This edition of Linear Algebra contains many new topics while preserving the axiomatic spirit of earlier ones. Chapters V (Algebras) and VI (Gradations and homology) are entirely new and prerequisites for Chapter II of Multilinear Algebra where the tensor products of vector spaces with additional structure (e.g., algebras, graded algebras and vector spaces, differential spaces) are discussed. Other new additions include a chapter of facts about polynomial algebras over a field (to be generalized in the second volume) which is immediately used in the new concluding chapter entitled "Theory of a linear transformation". Semisimple transformations are introduced there and proof offered that every linear transformation is the sum of a semisimple and nilpotent one. The restriction of the 2nd edition to vector spaces of finite dimension has been removed in the 3rd except for those theorems which do indeed fail when the dimension is infinite.

Multilinear Algebra opens with a decree that all fields are "... fixed, but arbitrarily chosen..." throughout chapter I. In no other chapter is the field mentioned although restrictions are obviously needed. (e.g., the definition of the alternator on page 84 requires the underlying field to have characteristic greater than the length of the tensor products). On the other hand, by persistent use of the universal properties of tensor products in proofs the condition of finite dimensionality needed in the 2nd edition is now avoided. Chapters V and VI develop the exterior algebra and related themes and should prove useful as a reference for differential geometers. Tensor methods are used to obtain proofs of several classical results (e.g., the Cayley-Hamilton theorem). The polynomial algebra in n indeterminates over a field is constructed and shown isomorphic to the symmetric tensor algebra of an n -dimensional vector space over the same field.

The carefully chosen exercises in each book are rewarding but not trivial or excessively difficult.