

In its current state, I really cannot confidently recommend this book for its intended audience of those meeting real analysis for the first time. I read every page closely and felt that there was a useful and attractively readable textbook trying to get out, but that more editorial work is needed to eradicate the bugs in this first edition.

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Fourier analysis (new edition) by T.W. Körner, pp 591, £47.95 (paper), ISBN 978-1-009-23005-6 Cambridge University Press (2023)

Fourier's idea that a periodic function can be represented as a trigonometric polynomial, and his startling generalisation to non-periodic functions via the Fourier transform, are not only mathematically interesting, but also powerful tools for understanding nature.

Körner wrote this wonderful book, first published in 1988 and now reissued in the Cambridge Mathematical Library series, as a shop window displaying some of the best of Fourier analysis. He intended it to be 'accessible to an undergraduate at a British university after two or three years of study', but it will appeal to a much wider readership. The book is long but each of the 110 chapters is short, showcasing an idea, technique, application or elegant result related to Fourier analysis. The chapters are largely independent, and so after assimilating the basics (covered in the first ten pages) the reader can more or less browse the book at will. The book is broad, rather than deep, although Körner adjusts the rigour of the discussion to suit the subjects discussed. Alongside the more rigorous chapters there are many interesting applications of Fourier analysis, some historical essays, and occasional words of advice to the reader. The chapters are divided up into six parts; Fourier Series, Some Differential Equations, Orthogonal Series, Fourier Transforms, Further Developments, and Other Directions.

Much of the material that undergraduate mathematicians need to know (issues of existence, uniqueness and convergence, orthogonal polynomials and so on) is covered in the book. But the real delights are the unexpected and varied discoveries awaiting the browser. Here is a tiny sample:

Chapter 13 on Monte Carlo methods for integrating over an m -dimensional cube reveals that, in contrast to a generalised Simpson's type rule, the accuracy does not depend on the dimension of the cube.

Chapter 35 includes Liouville's own proof that 'Liouville's number' $10^{-1!} + 10^{-2!} + 10^{-3!} \dots$ is transcendental.

Chapter 90 includes Burgess Davis's remarkable probabilistic proof of Picard's theorem: If $f : \mathbb{C} \rightarrow \mathbb{C}$ is analytic and non-constant then the range of f omits at most one point of \mathbb{C} .

The proof is based on the mathematical theory of Brownian motion—a topic which links several chapters in the book. How can a probabilistic theory prove such a definite result? Because if you can show something happens with probability one, then it certainly happens *sometimes*, and in this instance that is all that is needed to complete the proof.

Chapter 95, on 'The Diameter of Stars' asks how we can measure these when the blurring due to the Earth's atmosphere is much greater than the diameters we wish to observe. In a comment which dates the book, Körner says "Soon ... we will be able to spend our way out of trouble by putting our telescope in orbit above the

atmosphere. A more elegant (and cheaper) solution has been found by Labeyrie.” This cheap solution depends, of course, on a clever use of the Fourier transform.

There are many such gems scattered throughout the book. It really is fun to read, spoiled only by a number of misprints. It's such a pity they were not corrected for this new printing, but they cannot prevent Körner's enthusiasm infecting the reader. I liked how he always keeps the reader in mind. For instance he prefaces a routine proof with the remark “It is not very interesting and I suggest the reader just skims through it.” On the other hand after presenting a handwaving argument deducing the inversion formula for Fourier integrals by analogy with Fourier series he says “the reader should spend half an hour trying to make this rigorous. Even if she does not succeed she will benefit from the experience. If she does succeed she is probably wasting her time reading this book.”

I disagree with the last sentence. I think such a reader would revel in this book, and time you enjoy wasting is not wasted time. Terence Tao (no less) says in the forward to this new printing that he ‘spent many pleasant hours as a graduate student browsing through whatever topics took my fancy...the history of the transatlantic cable, or the use of Fourier analytic ideas to locate primes in arithmetic progression...or to estimate the age of the earth.’

I cannot imagine a mathematically prepared reader failing to enjoy this book.

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Game theory basics by Bernhard von Stengel, pp 374, £34.99 (paper), ISBN 978-1-10882-423-1, Cambridge University Press (2021).

Game theory has been increasingly popular amongst economists and political scientists in recent years, but in focusing so much on the applications there is a risk of losing sight of the firm mathematical foundations upon which the theory is built. Bernhard von Stengel's textbook aims to address this by explaining the basics of game theory with clarity and rigour, prioritising mathematical precision over extensive real world examples. The book is written for undergraduates in mathematics, computer science and mathematical economics, either for self-study or as the text for a university course. It aims to cover everything a game theorist should know, including several ideas that are often omitted from other introductory textbooks, such as combinatorial games, congestion games and correlated equilibria. It is also notable for being almost entirely self-contained, and while many sections would be challenging without a reasonable degree of familiarity with linear algebra, probability theory and analysis, all of the relevant concepts are defined and explained *in situ*, from relatively basic concepts (such as matrix multiplication or the expected value of discrete random variables) up to an entire chapter dedicated to proving Brouwer's fixed-point theorem.

The author's experience and expertise as a lecturer in mathematical game theory is readily apparent at the start of every chapter, with concise, motivating introductions followed by clear and informative descriptions of the prerequisites and learning outcomes. The material itself is extremely well explained, with numerous illustrative examples included alongside the abstract definitions and proofs. I also appreciated the occasional injection of dry humour, such as the complaint about the ambiguity of the phrase ‘next Friday’ in the very first chapter, and the author's personality manages to shine through the text without affecting the clarity of the