arithmetical work, but of course it would be well to have a shorter process. In this connection I should like to refer again to Table 1, because that table shows the extraordinary range of the ages at entry. The entrants come in at all ages from 11 to 50, and this illustrative fund is not the only one that has shown this marked characteristic. It is this long range of ages at entry that causes difficulty, by greatly increasing the arithmetical work, in that it necessitates so many separate sheets of factors, one for each age at entry. It may, however, frequently happen that there are only five or six entry ages, and in such cases the labour of the valuation would be very much reduced.

CORRESPONDENCE.

[We have received the following letters having relation to Mr. George King's paper.—ED. *J.I.A.*]

STAFF PENSION FUNDS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the discussion which followed the reading of Mr. King's paper on the 30th ultimo, I was prevented by want of time from mentioning the following formula, and should not now trouble you with the same but that it has been represented to me that, as an illustration of two important general principles, it might prove of interest to your readers.

The formula has reference to the special death benefit discussed by Mr. King in articles 54 to 65 of his paper. The benefit is as follows—

(*a*) The return of the whole of the contributions on death, if that event occur within the first 10 years of membership.

(*b*) A payment of one-half of the average annual salary, on death after 10 years.

The problem may be treated by means of an average entry age (if circumstances permit), or by taking groups of entry ages as advocated by Mr. Manly, or in the more thorough way, mentioned in the paper, of considering each entry age separately; but in each case the method of procedure would be the same.

It will be noticed that the benefit is a particular case of a payment assessed for the first n years as a function of the total salary, and thereafter as a function of the average salary. Now it is obvious that these forms of expression for the benefit are interchangeable; but it will be found, as was shown in Mr. Manly's earlier paper, that the form which will give the best working formula is usually, if not invariably, the one involving the total salary.

Further, as Mr. Lidstone pointed out in the discussion before mentioned, still bearing in mind the exigencies of our *practical* requirements, the more convenient course is to consider each year's contribution separately, and follow its course through the various years of assurance, rather than to fix the attention on the years of assurance, and value the benefit as it emerges in each successive year.

Proceeding therefore on these lines, we have (using Mr. King's notation)—

k'_t = the proportion of *total* salary returnable on death in the tth year

$$\text{for first 10 years } k'_t = \cdot05$$

$$\text{for the 11th year } k'_t = \frac{\cdot500}{10\cdot5} = \cdot0476$$

$$\text{for the 12th year } k'_t = \frac{\cdot500}{11\cdot5} = \cdot0435$$

and so on.

Now in regard to the salary paid in the first year after entry, the value of the return, omitting the denominator, is as follows—

$$s_x\{(\tfrac{1}{2}\overline{C}_x \times k'_1) + (\overline{C}_{x+1} \times k'_2) + (\overline{C}_{x+2} \times k'_3) + \ldots\}$$

In respect of the 2nd year's salary,

$$s_{x+1}\{(\tfrac{1}{2}\overline{C}_{x+1} \times k'_2) + (\overline{C}_{x+2} \times k'_3) + (\overline{C}_{x+3} \times k'_4) + \ldots\}$$

and so on.

The first column required is therefore—

$$\overline{C}_x \times \text{appropriate } k'_t = \text{(say) } C'_x.$$

We then require to sum the above column, and, in order to allow for the half payment over-valued in the year of death, we can use Mr. King's ingenious device, and deduct, from each figure in the resulting column, $\frac{1}{2}C'_x$, thus—

$$\sum_x^\omega C'_x - \tfrac{1}{2}C'_x = \text{(say) } M'_x.$$

The further columns required are as follows—

$$M'_x \times s_x = {}^sM'_x$$

$$\Sigma\, {}^sM'_x = {}^sR'_x.$$

In regard to past year's service, the value of the return is—

$$\text{(Total past salary)} \times \frac{M'_x}{D_x}\,*$$

and in respect of future service,

$$\text{Present salary} \times \frac{{}^sR'_x}{{}^sD_x}\{\text{where } {}^sD_x = (D_x \times s_x)\}.$$

I have used the function s_x, in preference to $\frac{s_x}{100}$, in order to shorten the explanation; in practical work, however, I think the latter is much more convenient, and as it is brought into both the numerator and denominator it is obvious that no disturbance is caused in the results whether $\frac{s_x}{100}$ or any other fraction of s_x is employed.

It has been assumed, in assessing the values of k'_t, that the average salary would be calculated on the actual salary received up to

* Strictly speaking the column M'_x used in this connection should be simply $\Sigma C'_x$ without any deduction, and not the column M'_x already referred to.

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the moment of death, but in practice it would probably be computed on the salary for each year entered upon; thus, for those dying in the 11th year, for instance, the average salary would be $\left(\sum_{x}^{x+10} s_x \div 11\right)$, and the value of k'_t would therefore be $\frac{\cdot 500}{11} = \cdot 04545$. If this condition held good, our formula would be slightly simplified, because for all years after 10 we should require no correction for the final payment, and M'_x for ages $(x+10)$ and onwards would be simply $\Sigma C'_x$.

The method proposed has the advantage of being completely in accord with that described in the postscript to Mr. King's paper (articles 178-182) for assessing the value of superannuation benefits, and is indeed obvious from a careful consideration of the explanations there given.

It seemed, however, desirable to elaborate the process a little, in order to illustrate that, with the aid of the two principles already mentioned, certain apparently complicated benefits can be reduced to a simple and orderly form.

I am, Sir,

Your obedient servant,

ERNEST C. THOMAS.

St. Mildred's House, E.C.
28 *February* 1905.