TESTS OF EFFECTIVE TEMPERATURE – COLOUR RELATIONS

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1. Abstract

This paper discusses some tests of synthetic spectra used in the calculation of synthetic colours, as well as some of the problems which arise in these calculations. Synthetic colours are tested in two ways. Firstly, observed angular diameters of stars are compared with ones predicted from stellar temperatures derived from synthetic colours and the infrared flux method. The average difference is less than 2%. Secondly, a very metal-poor isochrone is transformed to the M_V, B–V and M_V, V–I colour planes and compared with observations of stars in the globular cluster M92. The agreement is good in both colour–magnitude diagrams.

2. Introduction

The problem of finding the effective temperatures of stars from their colours or, conversely, finding the colours of stellar models has been studied extensively. A general approach to this problem has been to mimic the process by which the colours of stars are measured, i.e., to synthesize the spectrum of a stellar model, to multiply this spectrum by the sensitivity functions of the passbands of different photometric systems and then to normalize the results to the observational systems.

The quality of these calculations can be checked in various ways. Such checks include:

1) Comparisons of synthetic line spectra with the observed spectra of one or more "standard" stars;

2) Comparisons of synthetic fluxes with the observed fluxes of "standard" stars;

3) Comparisons of the angular diameters derived from models for various stars with their observed angular diameters;

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4) Comparisons of temperatures derived from different colours.

Furthermore, there are a number of points which must be considered in the calculation of the synthetic spectra. It is often necessary to allow for abundance changes caused by evolution, either stellar or galactic, as well as for variations of microturbulent velocity with surface gravity.

3. Comparisons of Line Spectra

The Sun is an obvious choice for tests of computed line spectra. Its effective temperature, surface gravity and chemical composition are well known. Its brightness allows it to be observed with high signal/noise and high resolution from the UV to the IR. Spectral atlases have been published by Kurucz et al. (1984) and Delbouille et al. (1973) for the 3000 Å – 12000 Å region and by the NSO for the IR. These data, obtained with ground–based telescopes, often contain both solar and telluric lines. Other infrared data have been obtained by the ATMOS experiment on the space shuttle (Farmer & Norton, 1989).

While very accurate oscillator strengths are available for some lines, e.g. O'Brian et al. (1991) and from the Oxford group (e.g. Blackwell et al. 1979), the quantity of such data is insufficient to allow us to calculate detailed line spectra which match the observations. It is necessary to alter the oscillator strengths of known atomic lines to make a good fit.

This problem is clearly seen when considering lines in the H band. Johansson & Learner (1990) reported about 360 new Fe I lines in the infrared, identifying them as transitions between $3d^64s(^6D)4d$ and $3d^64s(^6D)4f$. More than 200 of these lines coincide in wavelength with lines in the solar spectrum. These identifications have been checked by comparing the line intensities from the laboratory source with those in the solar spectrum. Only 4 of these lines deviate from this relationship by being stronger than implied from the laboratory data, thereby indicating a high probability that the identification of the remaining lines as Fe I. Few lines in the IR have measured gf values, and so the gfs of the majority of the solar Fe I and other lines must be derived from the solar spectrum.

Lines of Na I, Mg I, Al I, Si I, S I and Ca I with gf values from the Opacity Project were used in IR synthetic solar spectrum calculations. Several of these lines are much stronger in the synthetic spectrum than the observed one.

Since examples of comparisons of observed and computed solar and stellar spectra in the optical have been given elsewhere (Bell et al. 1994, Bell & Tripicco 1995), an example of the fit obtained to the solar spectrum in the IR is given in Fig. 1. It is hoped to obtain this level of fit over the J, H and K bands. Many of the solar lines were not included in the synthetic spectra calculated by Bell & Gustafsson (1989) and so the J, H and K magnitudes presented there will be too bright, probably by a few hundredths of a magnitude. The effect is probably greatest in the H band, owing to the larger number of Fe I lines there.

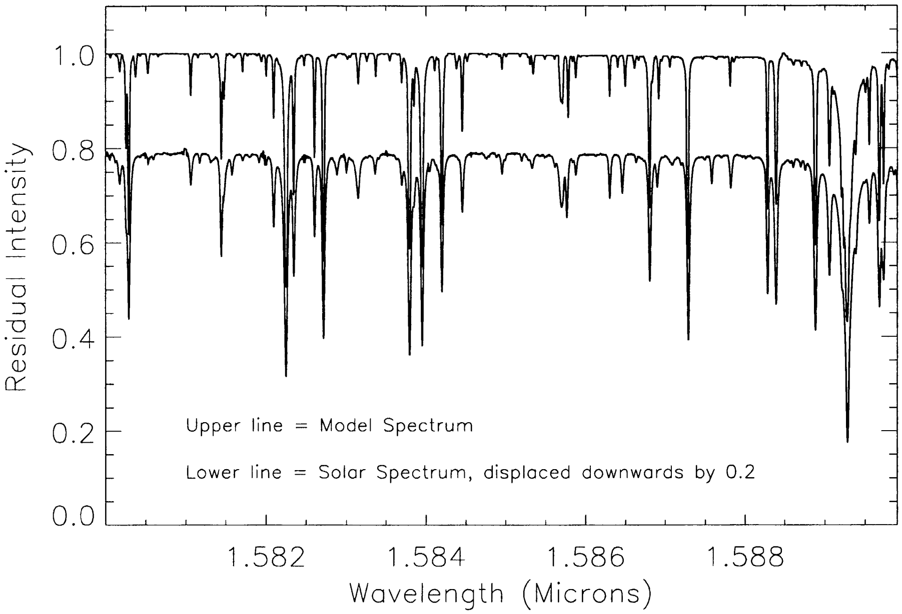

Figure 1. A comparison of observed and calculated solar spectra in the H band region. Some solar lines, e.g. that at 1.5875 μm, have not yet been identified.

4. Comparison of Fluxes

The brightness of the Sun, a virtue for getting line spectra, is a problem for measuring fluxes. The data of Neckel & Labs (1984), Lockwood et al, (1992, LTW) and Arvesen et al. (1969) disagree with one another. For this talk, I will compare calculated values with the results of LTW.

The continuous opacity used in the models includes contributions from H^-, H, electron scattering, Rayleigh scattering by H and H_2, He^-, OH, CH, AlI, SiI, MgI and Fe I. The SiI and MgI are Opacity Project data, the Fe I is that of Dragon & Mutschlecher (1980).

As a test of numerical accuracy, the OS Marcs solar model (Edvardsson, private communication) has been used to compare continuous fluxes calculated at Uppsala and at Maryland, The programs used for both opacity and flux calculations are quite independent and yet the results at 10 wavelengths between 3800 and 6000Å agree to better than 1%, with one exception. The inclusion of metal opacities from Al I, Mg I, Si I and Fe I reduces the fluxes shorward of 3800 Å by about 3%.

A Maryland Marcs solar model is fainter than the OS Marcs model at all wavelengths between 3000 and 6000 Å whereas the Holweger–Muller (1974) model is brighter than the OS Marcs model at longer wavelengths and fainter for $\lambda < 3800$ Å.

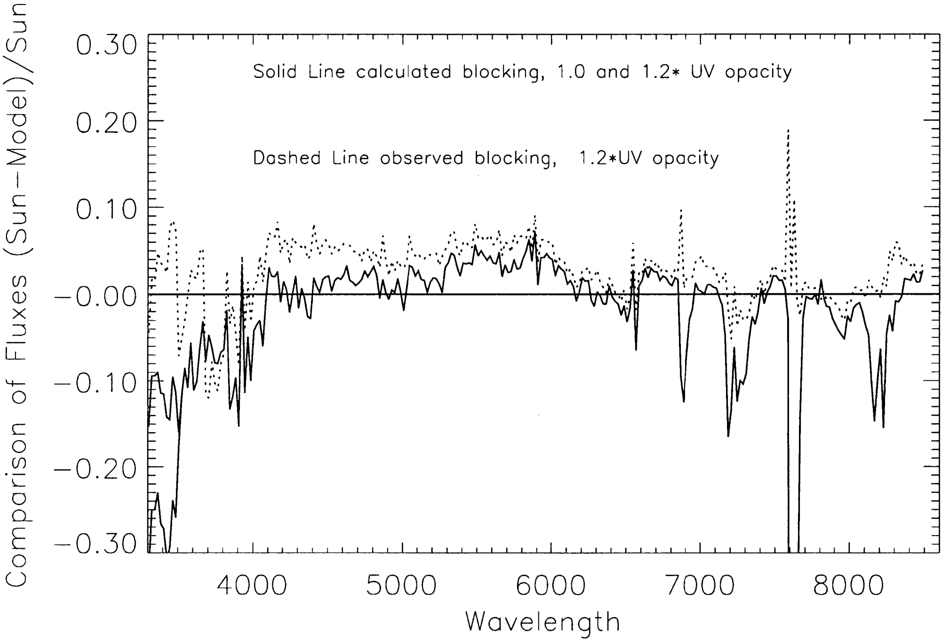

Figure 2. A comparison of the Maryland solar model and the LTW observed solar fluxes. The model fluxes (solid lines) include calculated line spectra, with the continuous opacity for $\lambda < 3500$ Å having the standard value and also being increased by 20%. The dips in the comparisons beyond 6800 Å are due to telluric lines in the observational data. The dashed line results from the model fluxes when the calculated line blocking is replaced by that derived from the Kurucz et al. solar atlas.

A direct comparison of fluxes of the Maryland Marcs solar model (T_{eff} = 5780K) and the LTW observational data shows that the model is too bright in the UV (Figure 2). One way of obtaining better agreement is increasing the model's continuous opacity shortward of 3500 Å by about 30%. If we measure the observed line blocking from the Kurucz et al. (1984) solar atlas and combine these data with the continuous fluxes of the solar model, the resulting fluxes are still brighter than the Sun in the UV. An increase in the continuous opacity of about 20% is needed to get agreement.

However, if the current Dragon & Mutschlecner opacity of Fe I is too small, an increase by over a factor of ten would be needed to solve the problem. It would be very helpful to have Opacity Project data for Fe I.

Balachandran and I are studying this problem further by making fits to OH lines in the 3100 Å region. Since the solar oxygen abundance is known accurately and the oscillator strengths of the OH lines are believed to be accurate to 10–20%, they allow us to check the continuous opacity to that accuracy.

Short & Lester (1996) have also found that an additional UV opacity source is needed in models of both the Sun and Arcturus. They suggest diatomic molecules may be the source of this opacity.

5. Details of the calculations

Since errors of 1% in fluxes correspond to errors of 0.01 in magnitudes, it is necessary to treat the hydrogen lines, at least in the hotter stars, with care. In work on the Balmer lines, I include lines up to H_{20} and, when appropriate, add the line absorption coefficients of the two nearest lines. I do not include the blueward component of H_{20} and alter the Balmer limit from 3646 Å to 3676.36 Å. Similar approximations are made for other hydrogen series.

Replacement of the Van der Waals pressure broadening treatment by the theory of Anstee & O'Mara (1995) will be a welcome improvement, in view of the work of Anstee, O'Mara & Ross (1997) in resolving a conundrum in the solar abundance of iron – see Blackwell et al. (1995a, b) and Holweger et al. (1995).

6. Abundance effects

In some circumstances, it is necessary to include abundance changes resulting from stellar evolution or to use non-solar abundance ratios in order to best match stellar spectra.

For example, observational work (e.g. Kjaergaard et al. 1982) shows that C is depleted, N enhanced and $^{12}C/^{13}C$ decreased in Population I giant stars. The same effect is seen in globular cluster giants, the C depletion increasing with evolution on the RGB and with decreasing [Fe/H]. This has an effect on the DDO colour index C(42–45), since CH lines occur in the 42 passband. Tripicco & Bell (1995) showed that this could cause an error of up to 250K in T_{eff} if the C depletion was not included in Population I giant star models. The strength of the CO bands will also be affected by this evolutionary change.

Recent work on isochrones for metal poor stars (VandenBerg et al. 1997) have used abundances $[\alpha/Fe]$ of 0.0, 0.3, and 0.6, where the "α" elements include O, Ne, Mg, Si, S, Ar, Ca and Ti, while Al and Mn are depleted, with [Al/Fe] = – [α/Fe] and [Mn/Fe] = – 0.5 [α/Fe]. While evolutionary abundance changes in globular cluster giants make the results somewhat uncertain, a value of [α/Fe] = 0.3 seems most appropriate for the metal poor globular clusters. Such a value should be used in color calculations, since Stetson et al. (1997) have found that cool giant models with [Fe/H] = –2.14 and [α/Fe] = 0.6 are somewhat bluer than those with [α/Fe] = 0.0.

7. Comparison of Angular Diameters

Bell & Gustafsson (1989, BG89) presented temperature scales for G and K dwarf and giant stars. They used their stellar temperatures and relative absolute fluxes to deduce angular diameters for a sample of 95 stars. The new data on stellar diameters presented by Mozurkewich (1997) at this meeting gives an excellent opportunity to compare observed and predicted diameters.

The BG89 temperatures were derived from tables of infrared colours as well as from the infrared-flux-ratio-method (IRFM, Blackwell & Shallis 1977). The IRFM finds stellar temperatures from the ratio of the apparent integrated flux of a star to its apparent K-band flux, which is, in effect, a K-band bolometric correction. The ratio is also, of course, the ratio of absolute integrated flux to absolute K-band flux, which can be found from model atmospheres.

The point which is quite critical for the application of the IRFM is the accuracy of the K-band flux. Two factors affecting this quantity are the filter profile of the K passband, which will affect the derivation of the K-band flux from stellar models, and the conversion of the K-band apparent magnitudes of stars to fluxes. This latter conversion is based either upon the K-band flux radiated by a Vega model and the angular diameter of Vega or upon absolute measurements of the flux from Vega or the use of solar analogues. BG89 used the fluxes from the Dreiling & Bell (1980) model of Vega.

Megessier (1995) has argued that the calibration of K band fluxes should be based upon measurements using either solar analogue stars or terrestrial light sources, which give larger values than do models. However, as discussed below, it is unclear if the use of solar analogues gives reliable results. No specific criticism can be made of the absolute measurements of Vega fluxes that would make them in better agreement with the Vega model.

The measurements of the infrared fluxes of solar analogues (Wamsteker 1981, Campins et al. 1986) are based upon the flux data of Labs & Neckel (1968). Vernazza et al. (1976) have questioned the way in which Labs & Neckel combined the relative observations of solar central intensity of Peyturaux (1952) and Pierce (1954) between 1.0 and 2.5 μm with their own absolute measurements for $\lambda < 1.25$ μm. Vernazza et al. quote Labs & Neckel – this fitting was done "not by fitting Pierce's data at about 1.0 μ to our data but rather by using the model–distribution between 1.0 and 1.5 μ as a linking medium between both observational data sets. By this procedure, Pierce's values at around 1.0 μ become about 6% higher than our values." (The model distribution referred to is from Gingerich & de Jager 1968). Vernazza et al. present an alternative way of fitting the two data sets together by studying the region of overlap. This uncertainty in fitting the different data sets leads directly to uncertainty in the absolute infrared fluxes from solar analogs.

The ground–based measurements, where Vega is compared with terrestrial light sources, have to be corrected for extinction. This is complicated by water vapour lines in the terrestrial spectrum. Examination of the telluric line spectrum in the NSO solar atlas shows a rich H_2O spectrum, with some lines being as much as 40% deep in the 3.8 μm region observed by Selby et al. (1983) while the line at 4.9163 μm, which is 80% deep, may have affected the 4.92 μm observations of Mountain et al. (1985). However, an underestimate of the extinction, which is possible since Selby et al. and Mountain et al. assumed that the extinction between 1 and 2 airmasses could be extrapolated to give that between 0 and 1 airmasses, would make their IR fluxes even brighter than those predicted by the Dreiling & Bell (1980) Vega model.

In order to compare angular diameters from Mozurkewich (1997, M97) with those of BG89, we need to convert the M97 uniform-disc diameters to limb-darkened ones. The U.S. Navy Prototype Optical Interferometer project has derived limb-darkened diameters of 5.58 and 6.87 milli-arc seconds (mas) for α Cas and α Ari, respectively (Hindsley, private communication). The ratios of these diameters to the M97 uniform-disc diameters measured at 0.8 μm are 1.073 and 1.037, respectively, for an average ratio of 1.055. Hanbury Brown et al. (1974) found the very similar ratio of 1.044 for their data. The ratio of 1.055 has been applied to the M97 data for the 17 stars in common with BG89. The details of the comparison are given in Table 1. The mean ratio of angular diameters then becomes(BG89)/(M97) = 1.017. While the uncertainty in the limb-darkening correction makes it unprofitable to explore the consequences of the ratio departing from unity, the agreement is very gratifying.

Using the same limb-darkening correction for Vega as for the K giants, the M97 limb-darkened diameter for Vega is about 1.4% larger than that of Hanbury Brown et al. (1974), being 3.285 versus 3.24 mas, with an error of 0.015 versus 0.07.

TABLE 1. Comparison of Angular Diameters

Star		Sp T	ϕ(Obs, 0.8 μm)	err	ϕ(BG89)	ϕ(BG89)/ϕ(Obs)	T
617	α Ari	K2 III	6.754	0.013	6.91	1.02	45
1409	ϵ Tau	G9.5 III	2.648	0.016	2.64	0.985	49
1457	α Tau	K5 III	20.776	0.036	20.62	1.01	39
3249	β Cnc	K4 III	5.108	0.037	5.17	1.025	40
3748	α Hya	K3 II–II	9.536	0.043	9.44	0.99	41
3873	ϵ Leo	G1 II	2.612	0.036	2.72	1.04	53
4301	α UMa	K0 IIIa	6.962	0.038	6.79	0.975	46
4335	ψ UMa	K1 III	4.077	0.025	4.18	1.025	45
4932	ϵ Vir	G8 III	3.252	0.018	3.30	1.015	50
5340	α Boo	K2 IIIp	21.061	0.067	21.07	1.00	43
5602	β Boo	G8 IIIa	2.455	0.026	2.61	1.06	49
5681	δ Boo	G8 III	2.731	0.018	2.80	1.025	48
5854	α Ser	K2 III	4.717	0.014	4.96	1.05	45
6220	η Her	G8 IIIb	2.627	0.034	2.61	0.99	49
6418	π Her	K3 IIab	5.177	0.042	5.52	1.075	41
6623	μ Her	G5 IV–V	1.994	0.020	1.99	1.00	55
8684	μ Peg	G8 III	2.469	0.021	2.47	1.00	50

8. Comparison of isochrones and colour magnitude diagrams

One test of synthetic photometry is to ask whether different colours give the same T_{eff} for a star. An alternative test is to ask if observations of the same cluster in different colours give colour magnitude diagrams which match the same isochrone.

This comparison has been carried out for M92, using an M_V, B–V diagram consisting of mean points for the fainter stars from Stetson & Harris (1988) and the brighter stars from Sandage (1970) and an M_V, V–I diagram consisting of mean points from Johnson & Bolte (1997). These are compared with isochrones calculated by VandenBerg for [Fe/H] = –2.14 and [α/Fe] = 0.3. (These isochrones are intended for comparison with HST observations of globular clusters – see Stetson et al. 1997). VandenBerg (private communication) has found that these isochrones gives a very good fit to the Frogel et al. (1981) red giant branch for M92 in the M_V, V–K plane.

The stars and the isochrone are compared in the M_V, B–V and M_V, V–I colour magnitude diagrams in Figure 3. The zero points of the colours have been found by using the Vega fluxes of Hayes (1985) to predict colours for Vega and the bolometric corrections found using a solar model and a

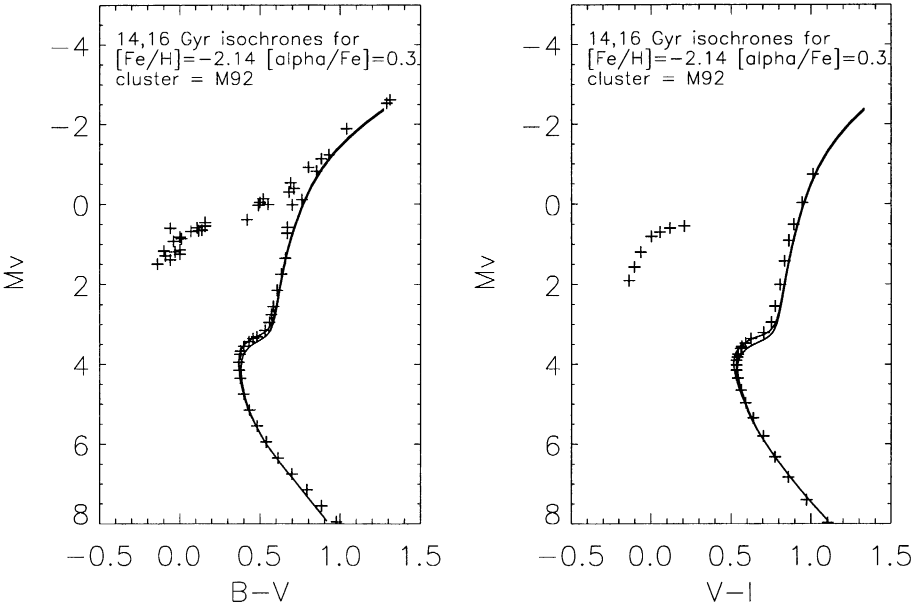

Figure 3. A comparison of the colour magnitude diagrams of M92 in M_V, B–V and M_V, V–I with transformed VandenBerg 14 and 16 Gyr isochrones, computed for [Fe/H] = -2.14, [α/Fe] = 0.3

bolometric correction of -0.12 for the Sun. The distance modulus used for M92 is $V-M_V = 14.65$. Reddening corrections of E(B-V) = 0.02 and E(V-I) = 0.026 have been applied.

In the M_V, B–V diagram, the fit to the 14 Gyr isochrone is very good from the tip of the RGB down to $M_V = 6.0$, after which the stars are redder and/or brighter than the isochrone. In the M_V, V–I diagram, the 14 Gyr isochrone is slightly bluer than the stars at $M_V = 6.0$ as well as at the turn off, while the reverse is true on part of the subgiant branch and part of the giant branch.

9. Acknowledgments

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Discussion of this paper appears at the end of these Proceedings.