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A Dimension-Free Weak-Type Estimate for Operators on UMD-Valued Functions

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Abstract. Let $\mathbb{T}$ denote the unit circle in the complex plane, and let X be a Banach space that satisfies Burkholder's UMD condition. Fix a natural number, $N \in \mathbb{N}$. Let $\mathcal{P}$ denote the reverse lexicographical order on $\mathbb{Z}^N$. For each $f \in L^1(\mathbb{T}^N, X)$, there exists a strongly measurable function $\widetilde{f}$ such that formally, for all $\mathbf{n} \in \mathbb{Z}^N$, $\widehat{\widetilde{f}}(\mathbf{n}) = -i\,\mathrm{sgn}_{\mathcal{P}}(\mathbf{n})\widehat{f}(\mathbf{n})$. In this paper, we present a summation method for this conjugate function directly analogous to the martingale methods developed by Asmar and Montgomery-Smith for scalar-valued functions. Using a stochastic integral representation and an application of Garling's characterization of UMD spaces, we prove that the associated maximal operator satisfies a weak-type $(1,1)$ inequality with a constant independent of the dimension N.

1 Introduction

Fix $N \in \mathbb{N}$, and let X be a UMD space. We will use $\mathbb{T}$ to denote the unit circle in the complex plane $\mathbb{C}$. For $\mathbb{T}^N = \prod_{k=1}^N \mathbb{T}$ we consider harmonic conjugation on $L^1(\mathbb{T}^N, X)$ with respect to a certain order on the discrete dual group $\mathbb{Z}^N$. If $f \in L^1(\mathbb{T}^N, X)$, we define a maximal operator, $M(f)$, which corresponds to a pointwise summation method for the harmonic conjugate of f. We first show that the maximal function for a related continuous parameter martingale satisfies a "Good-λ" inequality. We then obtain that Mf satisfies a weak-type (1,1) estimate with constant independent of the dimension N. Our approach is similar to that used in [3] for scalar-valued functions. However, there is an important difference in the analysis of vector-valued functions, at which point Garling's characterization of UMD spaces [9] will play a crucial role.

We begin by reviewing the terminology and notation required to define these operations. Let X be a Banach space with norm $\|\cdot\|_X$. Suppose $(\Omega, \mathcal{F}, \mu)$ is a general measure space. For each $p \in [1, \infty)$, $L^p(\Omega, X)$ denotes the Banach space of strongly measurable functions $f \colon \Omega \to X$ such that $\int_\Omega \|f\|_X^p \, d\mu < \infty$ with norm $\|f\|_p = (\int_\Omega \|f\|_X^p \, d\mu)^{1/p}$. When X is the field of scalars, we simply write $L^p(\Omega)$. Also, whenever $f \colon \Omega \to X$ is strongly measurable, we define $\|f\|_{1,\infty}^* = \sup_{y>0} y\mu(\{\|f\|_X > y\})$.

In the spirit of [1], [2], [3], [4], and [10], we consider harmonic conjugation on $L^1(\mathbb{T}^N, X)$ defined with respect to the reverse lexicographical order $\mathcal{P} \subseteq \mathbb{Z}^N$:

$$\mathcal{P} = \{0\} \cup \Big(\bigcup_{i=1}^N \{(n_1, \dots, n_i, 0, \dots, 0) : n_i > 0\}\Big).$$

Define $\mathrm{sgn}_{\mathcal{P}}(\mathbf{n})$ to be 1, 0, or -1 according as $\mathbf{n} \in \mathcal{P} \setminus \{0\}$, $\mathbf{n} = 0$, or $\mathbf{n} \in (-\mathcal{P}) \setminus \{0\}$. We

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can define $\widetilde{f}$ for each X-valued trigonometric polynomial by requiring

$$\widehat{\widetilde{f}}(n_1,\ldots,n_N) = -i\,\mathrm{sgn}_{\mathcal{P}}(n_1,\ldots,n_N)\widehat{f}(n_1,\ldots,n_N). \tag{1.1}$$

Naturally, the question arises whether one can obtain the analogs of the classical theorems for harmonic conjugation on $\mathbb{T}$ due to M. Riesz, Kolmogorov, and Privalov. For each p, $1 < p < \infty$, $f \mapsto \widetilde{f}$ extends to $L^p(\mathbb{T}^N, X)$ as a strong-type (p,p) operator, *i.e*, the generalized M. Riesz theorem holds (see [5], [6], and [2]). By adapting the methods of [11], one can prove that $f \mapsto \widetilde{f}$ is weak-type $(1,1)$, thereby extending Kolmogorov's theorem. However, the problem of developing a summation procedure which would define $\widetilde{f}$ pointwise a.e. on $\mathbb{T}^N$ for all $f \in L^1(\mathbb{T}^N, X)$ was not solved. This paper proves a generalization of Privalov's theorem for $f \in L^1(\mathbb{T}^N, X)$.

2 The Martingale Representations and The Maximal Operator

Let $\mathcal{F}_0 = \{\varnothing, \mathbb{T}^N\}$ while for $1 \le k \le N$, let $\mathcal{F}_k = \sigma\{e^{i\theta_1},\ldots,e^{i\theta_k}\}$, the σ-algebra generated by the first k coordinate functions. Whenever $\mathcal{F}$ is a sub-σ-algebra of $\mathcal{F}_N$, we denote the conditional expectation with respect to $\mathcal{F}$ by $\mathbb{E}(\cdot|\mathcal{F})$. Let $f \in L^1(\mathbb{T}^N, X)$. For $k = 0,1,2,\ldots,N$, define $f_k = \mathbb{E}(f|\mathcal{F}_k)$. Letting $d_0 = f_0 = \int_{\mathbb{T}^N} f\,dm$ while $d_k = f_k - f_{k-1}$ for $k = 1,\ldots,N$ gives a martingale difference decomposition, $f = \sum_{k=0}^N d_k$. Suppose f is an X-valued trigonometric polynomial given by

$$f = \sum_{j_1,\ldots,j_N} x_{j_1,\ldots,j_N} e^{ij_1\theta_1}\cdots e^{ij_k\theta_N},$$

where $x_{j_1,\ldots,j_k}$ is nonzero for only finitely many indices. Then, for $k = 1,\ldots,N$ we have the following Fourier expansion for d_k:

$$d_k = \sum_{\substack{j_1,\ldots,j_k \\ j_k \neq 0}} x_{j_1,\ldots,j_k} e^{ij_1\theta_1}\cdots e^{ij_k\theta_k}. \tag{2.1}$$

If one applies the martingale decomposition to $\widetilde{f}$, the terms $\widetilde{d}_k$ have Fourier expansion

$$\widetilde{d}_k = \sum_{\substack{j_1,\ldots,j_k \\ j_k \neq 0}} -i\,\mathrm{sgn}(j_k) x_{j_1,\ldots,j_k} e^{ij_1\theta_1}\cdots e^{ij_k\theta_k} \tag{2.2}$$

where $\mathrm{sgn}(\cdot)$ denotes the usual signum on $\mathbb{Z}$. For an arbitrary $f \in L^1(\mathbb{T}^N, X)$, one can interpret the expansions (2.1) and (2.2) as formal representations of d_k and $\widetilde{d}_k$ respectively.

We now define the maximal operator we wish to study:

$$Mf = \sup_{1\le m\le N} \Big\| \sum_{k=1}^m \widetilde{d}_k \Big\|_X. \tag{2.3}$$

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In the natural manner, the proof of a weak-type $(1,1)$ estimate for this maximal function will imply corresponding pointwise convergence results. The remainder of this paper is devoted to proving such an estimate.

A priori, the operator $f \mapsto Mf$ may be considered as a collection of N ergodic Hilbert transforms composed with a maximal martingale operator. However, this perspective is not productive since it is not necessarily true that a composition of weak-type operators will satisfy a weak-type estimate. Using an adaptation of techniques from [3] we will prove the following theorem.

Theorem 2.4 *Suppose X is a UMD space. For all $f \in L^1(\mathbb{T}^N, X)$ let Mf be defined as in (2.3). Then, there exists $C > 0$ such that for all $f \in L^1(\mathbb{T}^N, X)$,*

$$\|Mf\|^*_{1,\infty} \le C\|f\|_1. \tag{2.4.1}$$

Furthermore, the constant C is independent of N.

Proof It suffices to consider the case where f is a finite sum of characters with coefficients in X such that $d_0 = 0$. Thus f has the expansion

$$f(e^{i\theta_1}, \dots, e^{i\theta_N}) = \sum x_{j_1,\dots,j_N} e^{ij_1\theta_1} \cdots e^{ij_N\theta_N} = \sum_{k=1}^{N} d_k,$$

only finitely many of the coefficients $x_{j_1,\dots,j_N}$ are nonzero. In this case, we extend f and $\widetilde{f}$ to $\mathbb{C}^N$ as follows:

$$f(r_1e^{i\theta_1}, \dots, r_Ne^{i\theta_N}) = \sum x_{j_1,\dots,j_N} r_1^{|j_1|} e^{ij_1\theta_1} \cdots r_N^{|j_N|} e^{ij_N\theta_N};$$

$$\widetilde{f}(r_1e^{i\theta_1}, \dots, r_Ne^{i\theta_N}) = \sum -i \operatorname{sgn}_{\mathcal{P}}(j_1, \dots, j_N) x_{j_1,\dots,j_N} r_1^{|j_1|} e^{ij_1\theta_1} \cdots r_N^{|j_N|} e^{ij_N\theta_N}.$$

Thus, we take f and $\tilde{f}$ to be functions harmonic on $\mathbb{C}^N$.

We now introduce two continuous parameter martingales with continuous paths. For $1 \le n \le N$, let $c_{n,t} = a_{n,t} + ib_{n,t}$ where $\{a_{n,t}\}_{n=1}^N \cup \{b_{n,t}\}_{n=1}^N$ denote $2N$ independent Brownian motions starting at 0 such that $\mathbb{E}(a_{n,t}^2) = \mathbb{E}(b_{n,t}^2) = t$ for $n = 1, \dots, N$. Define stopping times by $\tau_n = \inf\{t : |c_{n,t}| \ge 1\}$. We will say $(n,t) < (m,s)$ if either $n < m$ or $n = m$ while $t < s$. In this case, the following equations define stochastic processes with time parameter $\mathcal{T} = \{1, \dots, N\} \times [0, \infty)$,

$$\begin{aligned} F_{n,t} &= \sum_{k=0}^{n-1} d_k(c_{1,\tau_1}, c_{2,\tau_2}, \dots, c_{k,\tau_k}) + d_n(c_{1,\tau_1}, c_{2,\tau_2}, \dots, c_{n,t\wedge\tau_n}) \\ \widetilde{F}_{n,t} &= \sum_{k=0}^{n-1} \widetilde{d}_k(c_{1,\tau_1}, c_{2,\tau_2}, \dots, c_{k,\tau_k}) + \widetilde{d}_n(c_{1,\tau_1}, c_{2,\tau_2}, \dots, c_{n,t\wedge\tau_n}). \end{aligned} \tag{2.4.2}$$

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Note that because $d_0 = \widetilde{d}_0$ we may start these summations at $k = 0$ for notational convenience. Since the two dimensional Browninan motion meets the boundary of the circle almost surely, we may also define

$$F_\infty = \sum_{k=0}^{N} d_k(c_{1,\tau_1}, c_{2,\tau_2}, \ldots, c_{k,\tau_k}) = f(c_{1,\tau_1}, c_{2,\tau_2}, \ldots, c_{N,\tau_N}).$$

We can consider these as processes with continuous time parameter by using the order preserving bijection $\phi\colon (\mathcal{T} \cup \{\infty\}) \to [0, N]$ given by $\phi\big((n,t)\big) = n - 1 + \frac{t}{t+1}$ while $\phi(\infty) = N$. Thus, each process has a continuous parameter and because f is harmonic, the processes have continuous paths. As in the corresponding case for scalar-valued harmonic conjugation treated in [3], the processes $F_{n,t}$ and $\widetilde{F}_{n,t}$ are martingales with respect to the filtration generated by $\{c_{n,t}\}_{n=1}^{N}$.

Now consider maximal functions for these continuous parameter objects,

$$F^* = \sup_{s\in[0,N)} \|F_{\phi^{-1}(s)}\|_X, \quad \text{and} \quad \widetilde{F}^* = \sup_{s\in[0,N)} \|\widetilde{F}_{\phi^{-1}(s)}\|_X. \tag{2.4.3}$$

With this notation, the remainder of the proof divides into proving the following estimates:

$$\|Mf\|_{1,\infty}^* \le \|\widetilde{F}^*\|_{1,\infty}^*; \tag{2.4.4}$$

$$\|\widetilde{F}^*\|_{1,\infty}^* \le C\|F^*\|_{1,\infty}^*; \tag{2.4.5}$$

$$\|F^*\|_{1,\infty}^* \le \|F_\infty\|_1; \tag{2.4.6}$$

$$\|F_\infty\|_1 = \|f\|_1. \tag{2.4.7}$$

The inequality (2.4.6) follows from Doob's Maximal inequality for continuous time martingales [8]. Observe the following lower estimate for $\widetilde{F}^*$:

$$\widetilde{F}^* = \sup_{(n,t)\in\mathcal{T}} \|\widetilde{F}_{n,t}\|_X \ge \sup_{1\le n\le N} \|\widetilde{F}_{k,\tau_k}\|_X = \sup_n \Big\| \sum_{k=1}^{n} \widetilde{d}_k(c_{1,\tau_1}, \ldots, c_{m,\tau_m}) \Big\|_X.$$

From this, (2.4.4) follows directly using the uniform distribution of complex Brownian motion starting at the origin and hitting $\mathbb{T}$. Similarly, (2.4.7) follows from the uniform distribution of complex Brownian motion starting at the origin and hitting $\mathbb{T}$. All that remains is the proof of (2.4.5).

3 A Good-λ Inequality and the Proof of (2.4.5)

Remark 3.1 Lemma (3.4) is our version of a "Good-λ" inequality. In the case of scalar-valued functions treated in [7] and [3], if F is real-valued, one uses the complex analytic function $G = F + i\widetilde{F}$, and the identity $|G|^2 = |F|^2 + |\widetilde{F}|^2$. However, for vector-valued functions there is no corresponding relation between F and G. Thus, our approach uses

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Lemma (3.2) as a means to deal with $\widetilde{F}^*$ directly. The proof of Lemma (3.2) is a natural adaptation of Garling's arguments in [9] for harmonic conjugation on $\mathbb{T}$. We include the details for the completeness of the exposition.

Lemma 3.2 *Suppose that X is a UMD space and that ν_1 and ν_2 are stopping times taking values in $\mathcal{T}\cup\{\infty\}$ such that $\nu_1 \le \nu_2$ a.e. Letting F and $\widetilde{F}$ be as above, then for each $1 < p < \infty$ there exists C_p independent of N so that the following holds:*

$$C_p^{-1}\|F_{\nu_2} - F_{\nu_1}\|_p \le \|\widetilde{F}_{\nu_2} - \widetilde{F}_{\nu_1}\|_p \le C_p\|F_{\nu_2} - F_{\nu_1}\|_p. \tag{3.2.1}$$

Proof of Lemma (3.2) By the construction of the process, when we apply Ito's formula for stochastic integrals, we obtain the following:

$$F_{n,t} = \sum_{k=1}^{n-1}\left(\int_{(k,0)}^{(k,\tau_k)} \frac{\partial d_k}{\partial x_k}(c_{1,\tau_1},\dots,c_{k,s_k})\,da_{k,s_k} + \int_{(k,0)}^{(k,\tau_k)} \frac{\partial d_k}{\partial y_k}(c_{1,\tau_1},\dots,c_{k,s_k})\,db_{k,s_k}\right)$$
$$+ \int_{(n,0)}^{(n,t\wedge\tau_n)} \frac{\partial d_n}{\partial x_n}(c_{1,\tau_1},\dots,c_{n,s_n})\,da_{n,s_n} + \int_{(n,0)}^{(n,t\wedge\tau_n)} \frac{\partial d_n}{\partial x_n}(c_{1,\tau_1},\dots,c_{n,s_n})\,db_{n,s_n}.$$

Note that the second order terms vanish due to the harmonicity of f. Since $\nu_2 \ge \nu_1$, we find the following representation:

$$F_{\nu_2} - F_{\nu_1} = \sum_{k=1}^{N}\left(\int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial d_k}{\partial x_k}\,da_{k,s_k} + \int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial d_k}{\partial y_k}\,db_{k,s_k}\right). \tag{3.2.2}$$

Further note that the natural analog of (3.2.2) holds for $\widetilde{F}$. Observe that d_k and $\widetilde{d}_k$ satisfy the Cauchy-Riemann equations applied to coordinate k, *i.e*,

$$\frac{\partial d_k}{\partial x_k} = \frac{\partial \widetilde{d}_k}{\partial y_k} \quad\text{and}\quad \frac{\partial d_k}{\partial y_k} = -\frac{\partial \widetilde{d}_k}{\partial x_k}. \tag{3.2.3}$$

Let $\{a'_{k,t}\}_{k=1}^N \cup \{b'_{k,t}\}_{k=1}^N$ denote a collection of $2N$ independent real Brownian motions which are also independent of $\{a_{k,t}\}_{k=1}^N \cup \{b_{k,t}\}_{k=1}^N$ and satisfy $\mathbb{E}({a'_{n,t}}^2) = \mathbb{E}({b'_{n,t}}^2) = t$ for $k = 1,\dots,N$. From Garling's characterization of UMD spaces, [9, Theorem 2′], and (3.2.3) it follows that there exists a constant c'_p which depends only upon X and p such that the following inequalities hold:

$$\|F_{\nu_2} - F_{\nu_1}\|_p = \left\|\sum_{k=1}^{N}\left(\int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial d_k}{\partial x_k}1_{[\nu_1,\nu_2]}\,da_{k,s_k} + \int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial d_k}{\partial y_k}1_{[\nu_1,\nu_2]}\,db_{k,s_k}\right)\right\|_p$$
$$= \left\|\sum_{k=1}^{N}\left(\int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial \widetilde{d}_k}{\partial y_k}1_{[\nu_1,\nu_2]}\,da_{k,s_k} + \int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} -\frac{\partial \widetilde{d}_k}{\partial x_k}1_{[\nu_1,\nu_2]}\,db_{k,s_k}\right)\right\|_p$$
$$\le c'_p\left\|\sum_{k=1}^{N}\left(\int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial \widetilde{d}_k}{\partial y_k}\,da'_{k,s_k} + \int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} -\frac{\partial \widetilde{d}_k}{\partial x_k}\,db'_{k,s_k}\right)\right\|_p$$
$$= c'_p\left\|\sum_{k=1}^{N}\left(\int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial \widetilde{d}_k}{\partial x_k}\,d(-b'_{k,s_k}) + \int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial \widetilde{d}_k}{\partial y_k}\,da'_{k,s_k}\right)\right\|_p.$$

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But, $\{a'_{k,t}\}_{k=1}^N \cup \{-b'_{k,t}\}_{k=1}^N$ is again a collection of real Brownian motions which are independent of $\{a_{k,t}\}_{k=1}^N \cup \{b_{k,t}\}_{k=1}^N$. Thus, [9, Theorem 2′] further implies the following:

$$\|F_{\nu_2} - F_{\nu_1}\|_p \le (c'_p)^2 \left\| \sum_{k=1}^N \left(\int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial \widetilde{d}_k}{\partial x_k}\, da_{k,s_k} + \int_{(k,0)\vee\nu_1}^{(k,\tau_k)\wedge\nu_2} \frac{\partial \widetilde{d}_k}{\partial y_k}\, db_{k,s_k} \right) \right\|_p .$$

Thus, we have proved the left-hand inequality in (3.2.1) with $C_p = (c'_p)^2$; the right-hand inequality follows from a virtually identical argument. ■

Remark 3.3 That (2.4.1) holds with constant independent of N will depend heavily on the fact that C_p in (3.2.1) is independent of N. This is because the constants which arise in the sequel will be determined by C_p for $p = 2$ and $p = 4$.

Lemma 3.4 *With the notation as above, let $\alpha \ge 1$ and $\beta > 1$. Then there exists $c = c(\alpha, \beta)$ such that whenever $\lambda > 0$ satisfies*

$$P(\widetilde{F}^* > \lambda) \le \alpha P(\widetilde{F}^* > \beta\lambda), \tag{3.4.1}$$

then,

$$P(\widetilde{F}^* > \lambda) \le cP(cF^* > \lambda). \tag{3.4.2}$$

Proof of Lemma (3.4) Let ν_1 and ν_2 be given by

$$\nu_1 = \inf\{(k,t) : \|\widetilde{F}_{k,t}\|_X > \lambda\} \quad \text{and} \quad \nu_2 = \inf\{(k,t) : \|\widetilde{F}_{k,t}\|_X > \beta\lambda\},$$

respectively, with the convention that $\inf(\varnothing) = 0$. Using the notation of (3.2), we find that if λ satisfies (3.4.1), then

$$\begin{aligned} \mathbb{E}(1_{\widetilde{F}^*>\lambda}\|F_{\nu_2} - F_{\nu_1}\|_X^2) = \|F_{\nu_2} - F_{\nu_1}\|_2^2 &\ge C_2^{-2}\|\widetilde{F}_{\nu_2} - \widetilde{F}_{\nu_1}\|_2^2 \\ &\ge C_2^{-2}(\beta\lambda - \lambda)^2 P(\widetilde{F}^* > \beta\lambda) \\ &\ge C'\lambda^2 P(\widetilde{F}^* > \lambda). \end{aligned} \tag{3.4.3}$$

On the other hand,

$$\mathbb{E}(1_{\widetilde{F}^*>\lambda}\|F_{\nu_2} - F_{\nu_1}\|_X^4) = \|F_{\nu_2} - F_{\nu_1}\|_4^4 \le C_4^4\|\widetilde{F}_{\nu_2} - \widetilde{F}_{\nu_1}\|_4^4 \le C''\lambda^4 P(\widetilde{F}^* > \lambda). \tag{3.4.4}$$

Hence, by [12, V.8.26] there exists $c > 0$ satisfying

$$P(\widetilde{F}^* > \lambda) \le cP(c\|F_{\nu_2} - F_{\nu_1}\|_X > \lambda).$$

At this point, (3.4.2) follows by noting that $\|F_{\nu_2} - F_{\nu_1}\|_X \le 2F^*$. ■

Proof of (2.4.5) We observe that since $A = \|\widetilde{F}^*\|_{1,\infty}^* < \infty$, we may choose $\lambda_0 > 0$ such that $2\lambda_0 P(\widetilde{F}^* > 2\lambda_0) \ge A/2$. Meanwhile, $\lambda_0 P(\widetilde{F}^* > \lambda_0) \le A$. Thus, we have

$$P(\widetilde{F}^* > \lambda_0) \le \frac{A}{\lambda_0} \le 4P(\widetilde{F}^* > 2\lambda_0).$$

Therefore, (3.4.1) holds with $\alpha = 4$ and $\beta = 2$. Letting $c = c(4,2)$, one can show that (3.4.2) implies that (2.4.5) holds with $C = 4c^2$. ■

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References

[1] N. Asmar, E. Berkson and T. A. Gillespie, *Distributional control and generalized analyticity.* Integral Equations Operator Theory **14**(1991), 311–341.

[2] N. Asmar, B. P. Kelly, and S. Montgomery-Smith, *A note on UMD spaces and transference in vector-valued function spaces.* Proc. Edinburgh Math. Soc. **39**(1996), 485–490.

[3] N. Asmar and S. Montgomery-Smith, *Dimension-free estimates for conjugate maximal functions and pointwise convergence.* Studia Math., to appear.

[4] E. Berkson, T. A. Gillespie and P. S. Muhly, *Generalized analyticity in UMD spaces.* Ark. Mat. **27**(1989), 1–14.

[5] J. Bourgain, *Some remarks on Banach spaces in which martingale difference sequences are unconditional.* Ark. Mat. **21**(1983), 163–168.

[6] D. Burkholder, *A geometric condition that implies the existence of certain singular integrals of Banach-space-valued functions.* In: Proceedings, Conference on Harmonic Analysis in Honor of A. Zygmund, Chicago, 1981 (eds. W. Becker *et al.*), Wadsworth, Belmont, CA, 1983, 270–286.

[7] D. Burkholder, R. F. Gundy and M. L. Silverstein, *A maximal characterization of the class H^p.* Trans. Amer. Math. Soc. **157**(1971), 137–153.

[8] J. L. Doob, *Stochastic Processes.* Wiley Publications in Statistics, New York, 1990.

[9] D. J. H. Garling, *Brownian motion and UMD spaces.* In: Probability and Banach spaces, Springer Lecture Notes in Math. **1221**(1986), 36–49.

[10] H. Helson, *Conjugate series in several variables.* Pacific J. Math. **9**(1959), 513–523.

[11] B. P. Kelly, *Distributional controlled representations acting on vector-valued functions spaces.* Doctoral Dissertation, University of Missouri, 1994.

[12] A. Zygmund, *Trigonometric Series.* 2nd edition (2 vols.), Cambridge University Press, 1959

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