CORRESPONDENCE.

ON A METHOD OF ADJUSTING TABLES OF MORTALITY.

To the Editor of the Assurance Magazine.

SIR—It is one of the properties of a table of mortality constructed upon Mr. Gompertz' celebrated formula, that the logarithms of the probability of living *one year* at successive ages form a series in *geometrical* progression. These logarithms are necessarily negative; but if their *numerical* values be denoted by p_x, then it follows, from the property in question, that log. p_x will form a series in *arithmetical* progression, *i. e.*, a series with constant differences of the first order.

Mr. Gompertz' formula, it is well known, cannot be applied, *for all ages*, to any data which has yet been obtained from actual observation, without first dividing such data into two or more portions (each comprising a certain number of consecutive ages), and dealing with each portion separately. These several divisions, in fact, form so many independent tables, in each of which the series denoted by log. p_x is in arithmetical progression, but in which the constant differences in the several series are not the same throughout.

The only objection to this mode of proceeding is, that a table so constructed must form a series more or less *discontinuous* at the points of junction of the several divisions—an objection, however, which may be obviated if we can (without too great a departure from the original data) so modify the several differences that they shall gradually merge into each other, instead of abruptly changing in passing from one division to the following. Now, let y_x denote the logarithm *of the logarithm* of the unadjusted probability of living one year, $-\Sigma y_x$ the sum of all the terms from x upwards, then, in the following table—

$$\begin{array}{llll}
\Sigma y_0 & & & \\
& \Delta\Sigma y_0 & & \\
\Sigma y_n & & \Delta^2\Sigma y_0 & \\
& \Delta\Sigma y_n & & \Delta^3\Sigma y_0 \\
\Sigma y_{2n} & & \Delta^2\Sigma y_n & \\
& \Delta\Sigma y_{2n} & & \\
\Sigma y_{3n} & & &
\end{array}$$

the symbols $\Delta\Sigma y_0$, $\Delta\Sigma y_n$, and $\Delta\Sigma y_{2n}$, will severally denote the sum of n successive terms of y_x; and if we insert $n-1$ interpolations between each of the terms Σy_0, Σy_n, &c., using differences as far as the third order, it is clear that we shall have a perfect series consisting of $3n+1$ terms, the third differences of which are constant throughout. Again: the first order of differences thus obtained is itself a series with constant *second* differences, and moreover bears this relation to the original series y_x, viz., that the sum of each consecutive n terms of the former (commencing with the first) is equal to the sum of the corresponding n terms of the latter. We have thus obtained a series, without the slightest discontinuity, which in its general features must more or less resemble the original series y_x.

The labour required in adjusting a table by this method is not, I think, greater than with the methods commonly used. The *arithmetical* values of

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the logarithms of the probabilities are of course easily obtained by subtracting the logarithm of the number surviving the year from the logarithm of the number exposed to risk. If the two last-mentioned logarithms be carried to five decimal places, we shall generally have at least three significant figures in the logarithms of the probabilities; and, retaining only that number, the logarithms of these may be rapidly run off by the table for three figures prefixed to Babbage's Logarithms, in which operation it will be sufficient to take out the logarithms to three decimal places only. Again: in interpolating the series Σy_x, it will be necessary to compute three terms only, as these will suffice to determine the first term of each of the three orders of differences—by the aid of which the remaining terms of the first order of differences (which is the series with which we are concerned) may be obtained by *two additions* only, as will appear by the following example.

In order to exhibit the results of the method here proposed, and to enable your readers to compare them with the results obtained by a method suggested by one of the latest and ablest writers on the subject, I annex an adjusted table of the Eagle experience, as given in vol. iv., page 214, of *The Assurance Magazine.* Neglecting the original data for the age of 80 and upwards, as being founded upon numbers too small to render it of any great utility, and dividing the remainder into three parts, the first comprising the ages 20 to 39, the second, ages 40 to 59, and the third, the ages 60 to 79, the preparatory table will stand thus:—

x.	Σy_x.	$\Delta\Sigma y_x$.	$\Delta^2\Sigma y_x$.	$\Delta^3\Sigma y_x$.
20	62003			
		−13870		
40	48133		−5092	
		−18962		−5117
60	29171		−10209	
		−29171		
80	0			

Interpolating in the ordinary way three terms between the ages 20 and 40, we have:—

x.	Σy_x.	y_x.	Δy_x.	$\Delta^2 y_x$.
20	62003·0000			
		−651·5587		
21	61351·4413		−·5771	
		−652·1358		−·6396
22	60699·3055		−1·2167	
		−653·3525		
23	60045·9530			
.	. . .	. .	. .	.
.	. . .	. .	. .	.
.				

Which series, it is evident, may be continued to any extent required, by continually adding the constant second difference, 6396, to the last term of the first order Δy_x, and then adding the several terms of the latter to the corresponding terms of y_x. Thus, as before stated, each term of the adjusted series required is obtained by two additions only, it being useless

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to continue the series Σy_x, which is introduced merely for the purpose of *explaining* the process.

The next step consists in taking out the numbers corresponding to the logarithms in the column y_x, by which we obtain the logarithm of the probability of living one year, at all ages, in its negative form, and the logarithms of the numbers living at each age may then be found by the continued subtraction of these from the logarithm of the number fixed for the radix of the table.

The following table contains the numbers living, the decrements, and also the expectation of life, at every age from 20 upwards. By comparing the expectation with that deduced by Mr. Jellicoe's method, it will be found that, upon the whole, the former adheres rather more closely to the original data than the latter. It may also be remarked, that although from the age of 80 upwards the decrements are deduced entirely by a continuation of the law according to which the preceding portion of the table is formed, and are consequently entirely independent of the original observations, yet even at these extreme ages a very close connection is observable between them.

I am, Sir,

Your very obedient Servant,

M.

London, 28th November, 1856.

Adjusted Mortality Table.—Eagle Experience.

Age.	Living.	Decrements.	Expectation of Life.	Age.	Living.	Decrements.	Expectation of Life.	Age.	Living.	Decrements.	Expectation of Life.
20	9215·0	94·6	38·447	47	6578·9	117·0	21·256	74	2230·2	194·5	6·859
21	9120·4	93·8	37·842	48	6461·9	119·7	20·631	75	2035·7	191·8	6·466
22	9026·6	93·1	37·230	49	6342·2	122·5	20·012	76	1843·9	187·8	6·087
23	8933·5	92·5	36·612	50	6219·7	125·5	19·395	77	1656·1	182·6	5·720
24	8841·0	92·1	35·991	51	6094·2	128·6	18·784	78	1473·5	176·0	5·367
25	8748·9	91·8	35·364	52	5965·6	132·0	18·179	79	1297·5	168·1	5·027
26	8657·1	91·6	34·734	53	5833·6	135·3	17·578	80	1129·4	158·8	4·701
27	8565·5	91·5	34·099	54	5698·3	138·9	16·984	81	970·6	148·3	4·388
28	8474·0	91·6	33·462	55	5559·4	142·5	16·396	82	822·3	136 5	4·089
29	8382·4	91·9	32·822	56	5416·9	146·3	15·814	83	685·8	123·9	3·804
30	8290·5	92·1	32·181	57	5270·6	150·3	15·239	84	561·9	110·5	3·532
31	8198·4	92·6	31·537	58	5120·3	154·1	14·672	85	451·4	96·6	3·274
32	8105·8	93·2	30·891	59	4966·2	158·3	14·112	86	354·8	82·7	3·029
33	8012·6	93·8	30·243	60	4807·9	162·2	13·560	87	272·1	69·1	2·798
34	7918·8	94·6	29·597	61	4645·7	166·4	13·016	88	203·0	56·1	2·580
35	7824·2	95·6	28·948	62	4479·3	170·4	12·481	89	146·9	44·2	2·375
36	7728·6	96·7	28·301	63	4308·9	174·5	11·955	90	102·7	33·7	2·183
37	7631·9	97·8	27·653	64	4134·4	178·3	11·438	91	69·0	24·6	2·003
38	7534·1	99·1	27·006	65	3956·1	182·0	10·931	92	44·4	17·2	1·836
39	7435·0	100 6	26·358	66	3774·1	185·4	10·434	93	27·2	11·4	1·680
40	7334·4	102·1	25·713	67	3588·7	188·6	9·947	94	15·8	7·2	1·536
41	7232·3	103·8	25·068	68	3400·1	191·4	9·471	95	8·6	4·2	1·403
42	7128·5	105·7	24·426	69	3208·7	193·7	9·006	96	4·4	2·4	1·281
43	7022 8	107·7	23·787	70	3015·0	195·5	8·553	97	2·0	1·1	1·170
44	6915·1	109·7	23·149	71	2819·5	196·4	8·111	98	·9	·6	1·069
45	6805·4	112·1	22·514	72	2623·1	196·8	7·681	99	·3	·2	·978
46	6693·3	114·4	21·884	73	2426·3	196·1	7·264	100	·1	·1	·897