CORRESPONDENCE

(*To the Editors of the* Journal of the Institute of Actuaries)

SIRS,

At a time such as the present when a change in the mortality basis of valuation of life assurance policies is being considered by many offices, it may be opportune to draw attention to a feature of the "Z" method of valuing endowment assurances.

The function Z_m is usually taken as $Z_m = c^{-55} \times c^m$. It could be put in the form $Z_m = A + Bc^m$, where A and B have any reasonable values: as long as c is left unaltered, identical mean ages will be brought out. Two advantages would appear to ensue.

(1) An additional "easy" value can be obtained, e.g. whereas the usual method is to make $Z_{55} = 1$ and all other values are "odd", the constants A and B could be so chosen that, say, $Z_{55} = 1$ and $Z_{65} = 2$. Where individual values of the Z constant are placed on each card a large saving of work might ensue, bearing in mind the modern tendency towards the issue of policies for odd amounts assured that cannot be placed in a small table, e.g. the sums assured corresponding to a premium of £1 a month.

(2) The table of Z_m for the A 1924–29 table published in *J.I.A.* vol. LXIV, p. 478, shows that the value of Z_m varies from 0·110 at age 35 to 9·120 at age 75, while if the Z method is used for the comparatively few policies outside that range there is a very wide variation in the value of the function. The constants A and B can be chosen so that the curve of Z_m as m increases is much "flatter", and there would in practice be less possibility of errors in calculating the constants. The very small and the very large values of Z would be to a certain extent avoided.

If A is made equal to $(c^{10}-2)/(c^{10}-1) = 0{\cdot}50495$ and $B = c^{-55}/(c^{10}-1)$, with $\log_{10} c = 0{\cdot}048$, we have

$$Z_{35} = 559 \text{ per } 1000 \qquad Z_{65} = 2000 \text{ per } 1000$$

$$Z_{55} = 1000 \quad ,, \quad ,, \qquad Z_{75} = 5020 \quad ,, \quad ,,$$

Thus compared with the published values Z_{35} is much increased and Z_{75} much diminished without affecting the accuracy of the results to be obtained.

A disadvantage lies in the calculation of the mean age for the group. If $\frac{1000 Z_m}{SA}$ is calculated, there is no difference, but if it is desired to use

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logarithms to base c the total Z must first be adjusted by subtracting the total sum assured multiplied by A. As A is constant for all groups, the operation would take only a few minutes.

In connexion with the value of c chosen for the A 1924–29 table, viz. $\log_{10} c = 0{\cdot}048$, as a small difference in the value of c will not affect the results appreciably, it might be of practical advantage to make $c^{10} = 3$, i.e. $\log_{10} c = 0{\cdot}047712$. The resulting value of c would be 1·1161, viz. only 0·0008 less than the value of c used in the paper, without falling below the values of r published in the paper. Taking A as zero and $B = c^{-55}$ (i.e. the ordinary practice) with the adjusted value of c we would have:

$$Z_{35} = \tfrac{1}{9}, \quad Z_{45} = \tfrac{1}{3}, \quad Z_{55} = 1, \quad Z_{65} = 3, \quad Z_{75} = 9.$$

These are simple values and much more work might be saved though, of course, Z_{40}, Z_{50}, Z_{60} and Z_{70} would show *no reduction in labour*.

If A were made equal to $\frac{1}{2}$ and B therefore made equal to $\dfrac{c^{-55}}{2}$, we should have $Z_{45} = \frac{2}{3}$, $Z_{55} = 1$, $Z_{65} = 2$ and $Z_{75} = 5$. As the values 1, 2 and 5 are "easy", it would perhaps be better to make $B = \dfrac{c^{-50}}{2}$ and therefore $Z_{50} = 1$, $Z_{60} = 2$ and $Z_{70} = 5$, because there is usually a greater sum assured at age 60 than any other maturity age. If $A = \frac{1}{2}$, the adjustment to the Z's, to allow the use of logarithms to base c for the purpose of calculating the mean ages, is much simplified.

I am, Sirs, etc.

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[Mr Hedley's suggestions are quite practical, and it is clear that the alteration of the value of $\log_{10} c$ from Mr Lidstone's 0·048 to $0{\cdot}047712 = (\log_{10} 3)/10$ cannot sensibly affect the resulting mean ages. It is perhaps probable that "odd" sums assured arising from a monthly premium table would be most conveniently provided for by a special table of Z's per £1 of premium for the usual maturity ages. Opinions may differ as to whether the advantages secured by bringing in the additional constant A are or are not bought too dearly at the expense of increased work and liability to error, and loss of simplicity, in the course of the actual valuation—arising from the necessity of deducting $A\Sigma S$ from ΣZ before proceeding to find the mean ages by the logarithmic method. If the ages are found, as originally proposed, by inverse entry of the Z-table this point does not arise.

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It is suggested that Mr Hedley's interesting letter may with advantage be read in conjunction with the short and practical Paper by Mr John Inglis, F.F.A., A.I.A. [*T.F.A.* vol. XVI (1937), p. 113] of which mention is made on p. 92 of the present number. He gives, p. 120, a new table for finding mean ages, according to the Z method, by Coote's logarithmic plan, and points out, para. 5, p. 114, that the new table is virtually a table of Z, so that the ordinary table of Z can be used in the same way, in place of tables in Coote's form.—Eds. *J.I.A.*]