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Fe Kα Lines of Seyfert I Galaxies: Thin-Torus Model

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Abstract. In order to elucidate the physics of the Fe Kα line emissions from Seyfert(-like) AGNs, we propose a thin-torus model in Kerr metric and fit it to three galaxies (i.e., NGC 3516, 3C 273, and NGC 5548). It is found that only spinning-BH galaxies radiate observable line flux from a region within tens of gravitational radii of a central BH. Besides, the observer's inclination angle is around 30°.

1. Introduction

It has already been demonstrated theoretically that the central black holes (BHs) of AGNs with Fe line emissions are very likely to be rotating (Cunningham 1975; Laor 1991). The line comes from from the fluorescence of matter within $20r_g$ ($r_g = GM/c^2$ in which M is the BH mass) in a plane from the central BH (Dabrowski 1997; Fabian et al. 2000 and references therein).

In this region, bound geodesics of particles about a spinning (Kerr) BH are not restricted only in the equatorial disk but trace out helix-like spherical orbit between the minimum and maximum latitudes about the equatorial plane of the central hole (Wilkins 1972). This sketch was luckily supported by the observation of the comparable width of the X-ray Fe line with that of the optical lines (Chen & Halpern 1989) which verified that a hot torus, but not a disk, exists in galaxies. As a result, the standard thin-disk model (Page & Thorne 1974) will not be still valid (Collin-Souffrin & Dumont 1990).

Within the framework of the generalized formulations of luminous particles in Kerr metric, this paper develops a thin-torus model to illustrate the observed Fe Kα line profiles and applies it to three galaxies: NGC 3516 (Nandra et al. 1999), 3C 273 (Yaqoob & Serlemitsos 2000), and NGC 5548 (Yaqoob et al. 2001). The work is a synthesis and an extension of previous studies (Gerlach 1971; Bardeen, Press, & Teukolsky 1972; Stewart & Walker 1973; Fang & Zhang 1975; Laor 1991; Karas, Vokrouhlicky, & Polnarev 1992; Zakharov 1994; Bromley, Chen, & Miller W. A. 1997; Dabrowski 1997; Ma 2000).

In the paper, natural units $G = c = 1$ are chosen and a geometrized radial coordinate r is in the unit of r_g; Greek indices run from 1~4; the usual Boyer-Lindquist coordinates are used: $ds^2 = -(1 - 2r/\Sigma)dt^2 - [4ar(1-\mu)/\Sigma)]dtd\phi + (\Sigma/\Delta)dr^2 + \Sigma d\theta^2 + [\Lambda(1-\mu)/\Sigma)d\phi^2$ in which $\Sigma = r^2 + \mu a^2, \mu = \cos^2\theta, \Delta = r^2 - 2r + a^2, \Lambda = (r^2 + a^2)^2 - \Delta(1-\mu)a^2$; $a = J/M^2$ is the specific reduced magnitude of the BH's angular momentum J.

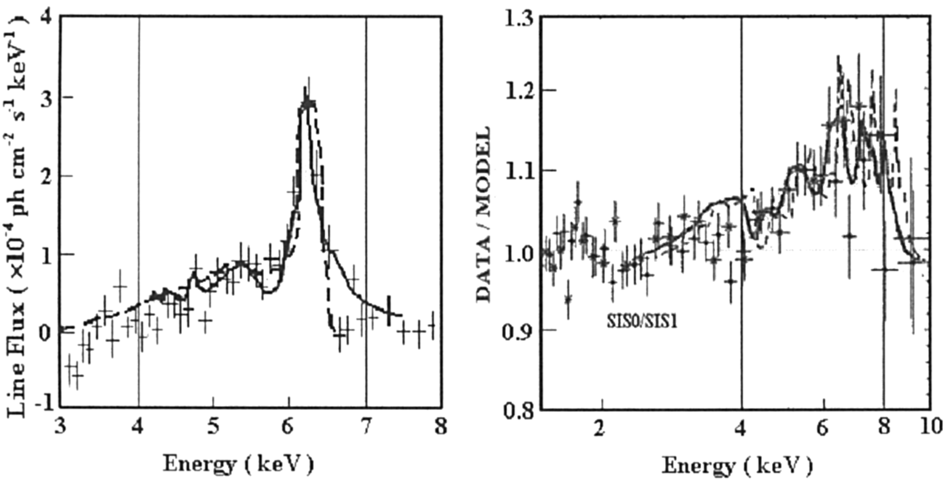

Figure 1. Best-fit theoretical and predicted profiles with $\{a, r, \theta_0\}=$ $=\{0.95 \pm 0.005,\ 14 \pm 0.05,\ 30^\circ \pm 0.5^\circ\}$ (LEFT) and $\{a, r, \theta_0\}=\{0.94 \pm 0.005,\ 4 \pm 0.05,\ 29^\circ \pm 0.5^\circ\}$ (RIGHT), overlaid directly upon the observed Fe Kα line profiles of NGC 3516 (Nandra et al. 1999) and 3C 273 (Yaqoob & Serlemitsos 2000), respectively (See text for details).

2. Thin-torus model and its applications

There are a few the same assumptions in the thin-torus model as those suggested previously: (1) The torus is treated as a thin-shelled ensemble of particles at r surrounding a BH (Gerlach 1971); (2) The Fe line emission is monochromatic (Laor 1991; Dabrowski 1997) with the rest energy of each emitted photon as 6.4 keV; (3) Photons emitted by particles are homogeneous and are free to reach the observer and thus only half of them can be emitted outwards (Fang & Zhang 1975).

The dimensionless relative flux of the thin torus is then expressed as:

$$\mathrm{F_{line}}(\nu,\ \mathrm{r_0},\ \theta_0,\ \mathrm{r},\ \mathrm{a}) = \frac{\mathrm{r_0^2}}{\mathrm{h}\nu_\mathrm{E}} \sum_\mathrm{q} \int \mathrm{d}\nu \mathrm{d}\Omega \cdot \mathrm{I} \cdot (\frac{\nu}{\nu_\mathrm{E}})^3 \cdot \cos\alpha \qquad (1)$$

where ν and ν_E are the observed photon frequency and emitter's rest frequency, respectively, r_0 is the distance between the BH and the observer, θ_0 is the inclination angle of the observer, $h\nu_E = 6.4$ keV. The integration over the element of the solid angle $d\Omega = \frac{1}{2} \sin 2\alpha d\alpha d\beta$ covers the image of the spherical ring in the observer's sky plane, in which $\alpha = -s_\phi / \sin\theta_0$ and $\beta = \sqrt{s_\theta^2 + a^2 cos^2\theta_0 - s_\phi^2 cot^2\theta_0}$ are impact parameters of the observer related to the photon's constants of motion (Karas, Vokrouhlicky, & Polnarev 1992); $I = \delta(u^\mu s_\mu - h\nu_E)$ (ergs $\mathrm{s^{-1}cm^{-2}sr^{-1}eV^{-1}}$) is the specific intensity; $(\frac{\nu}{\nu_e})^3$ arises because along the entire trajectory of the photon $I_\nu/\nu^3 = I_{\nu e}/\nu_e^3$ is an invariant resulting from the conservation of photons in a flux tube together with conservation of volume in phase space. The expression of s_μ and u^μ are as follows (Zakharov 1994; Ma 2000):

$$\left.\begin{array}{l}
s_t = -h\nu \\
s_\theta^2 = \frac{(Mh\nu)^2[\Sigma\Lambda + 4ar\Sigma x - (\Sigma - 2r)x^2/(1-\mu)]}{\Lambda(1-2r/\Sigma) + 4(ar)^2(1-\mu)/\Sigma} \\
s_\phi = Mh\nu\rho\cos\beta\sin\theta\sin\theta_0 \\
(\Sigma/E)u^t = -a[a(1-\mu) - p] + [(r^2+a^2)/\Delta](r^2 + a^2 - ap) \\
(\Sigma/E)^2(u^r)^2 = R(r) = -(1/E^2 - 1)r^4 + (2/E^2)r^3 - \\
\qquad -[a^2(1/E^2 - 1) + (p^2+q)]r^2 + 2[(a-p)^2 + \\
\qquad +q]r - a^2 q \\
(\Sigma/E)^2(u^\theta)^2 = \Theta(\mu) = 4\mu\{a^2(1/E^2-1)\mu^2 - [a^2(1/E^2 - \\
\qquad -1) + (p^2+q)]\mu + q\} \\
(\Sigma/E)u^\phi = -[a - p/(1-\mu)] + a/\Delta(r^2 + a^2 - ap) \\
p_\pm = [-B_2 \pm (B_2^2 - 4B_1B_3)^{1/2}]/(2B_1) \\
q = -\frac{r}{\Delta^2}\{[r(r-2)^2 - a^2]p^2 + 2a[r(3r - \\
\quad -4) + a^2]p - [(r^2+a^2)^2 - 4a^2r]\} \\
E^2 = \frac{r^2(r^2-2r+a^2)}{r^4 + (a^2-p^2-q)r^2 + 2[(a-p)^2+q]r - a^2q}
\end{array}\right\} . \qquad (2)$$

where $B_1 = A_2/A_1 - A_5$, $B_2 = A_3/A_1 - A_6$, $B_3 = -A_4/A_1 + A_7$; $A_1 = 5r^4 - 16r^3 + 6(a^2+2)r^2 - 8a^2r + a^4$, $A_2 = \Delta^2(5r^3 - 16r^2 + 12r - 2a^2)$, $A_3 = 4a\Delta^2(6r^2 - 6r + a^2)$, $A_4 = 2\Delta^2(3r^4 + 4a^2r^2 - 6a^2r + a^4)$, $A_5 = r(r-2)^2 - a^2$, $A_6 = 2a[r(3r-4) + a^2]$, $A_7 = (r^2+a^2)^2 - 4a^2r$.

Eq.(1) tells that the dimensionless flux profile $\mathrm{F} = \frac{\mathrm{r}_0^2}{\mathrm{h}\nu_\mathrm{e}}\mathrm{F}_\mathrm{line}$ versus the shift $\frac{\nu}{\nu_e}$ depends on three parameters: the radial position r of emitters, the spin a of the BH, and the inclination angle θ_0 of the observer. In numerical calculations, an effective range of the three-parameter set $\{r,\ a,\ \theta_0\}$ is covered: $\{0\sim40,\ 0\sim1,\ 0\sim90°\}$ with steps of $\{0.1,\ 0.01,\ 1°\}$.

With the progress of steps, F changes continuously which results in a relationship of one-to-one correspondence between one profile and one set of three parameters $\{a,\ r,\ \theta_0\}$. Therefore, it is possible to fit the theoretical profiles to every observed Fe K-line one of all detectable Seyfert(-like) galaxies with lead pursuit approach. Fig.1 and Fig.2 give three examples, respectively.

In the three pictures of the two figures, there show two kinds of curves: theoretical profiles and predicted ones. The former is the calculated ones from Eq.(1) directly and the latter is the convolving results of the calculated ones with the instrument responses, which are approximated by the Gaussians in *ASCA* and *Chandra* campaigns (Yaqoob 2002).

In the left picture of Fig.1, the parameter set of the best-fit theoretical and predicted profiles is $\{a, r, \theta_0\}$=$\{0.95 \pm 0.005,\ 14 \pm 0.05,\ 30° \pm 0.5°\}$ (Ma 2002a), overlaid directly upon the observed line profile of iron Kα emission from NGC 3516 (Nandra et al. 1999). Observed data are shown with *crosses*. Theoretical and predicted profiles are coincident with each other and indicated by the same solid curve. The broken line is the original fit with the thin-disk model for a central Kerr BH.

In the right picture of Fig.1, the parameter set of the best-fit theoretical and predicted profiles is $\{a, r, \theta_0\}$=$\{0.94 \pm 0.005,\ 4 \pm 0.05,\ 29° \pm 0.5°\}$ (Ma 2002b), overlaid directly upon the observed line profile of iron Kα emission from

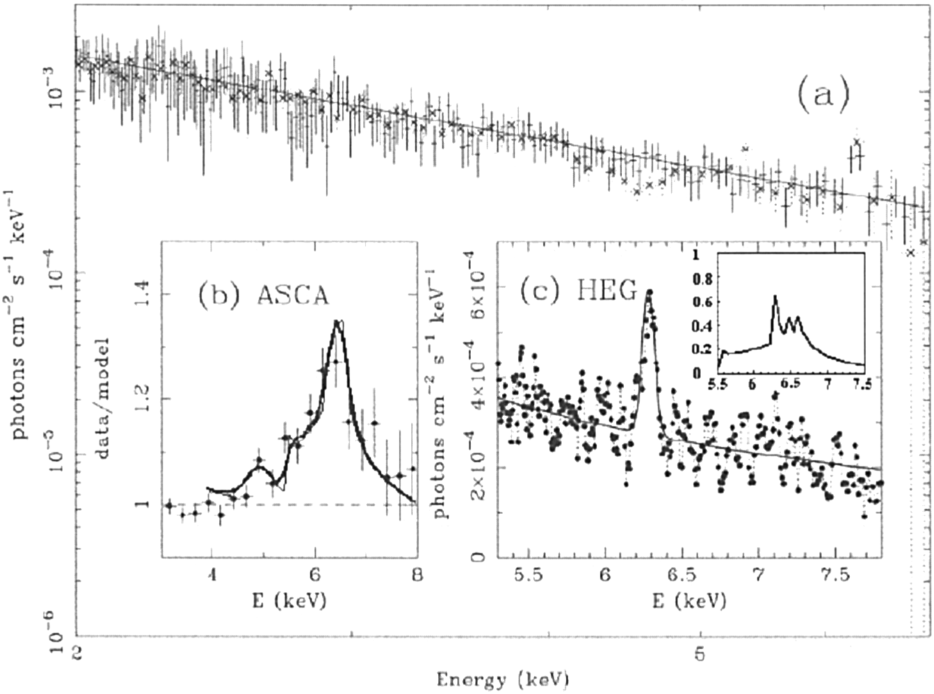

Figure 2. Best-fit theoretical and predicted profiles with $\{a, r, \theta_0\}=$ $=\{0.96 \pm 0.005,\ 7.5 \pm 0.05,\ 28° \pm 0.5°\}$ overlaid directly upon the observed Fe Kα line profile of NGC 5548 (Yaqoob et al. 2001) (See text for details).

3C 273 (Yaqoob & Serlemitsos 2000). Observed data are shown with *crosses*. Theoretical and predicted profiles are indicated by the dash line and the solid one, respectively.

In Fig.2, the parameter set of the best-fit theoretical and predicted profiles is $\{a, r, \theta_0\}=\{0.96 \pm 0.005,\ 7.5 \pm 0.05,\ 28° \pm 0.5°\}$, overlaid directly upon the observed line profile of iron Kα emission from NGC 5548 (Yaqoob et al. 2001). Fig.2(a) is the original *Chandra* HETG spectra from the HEG (solid error bars) and MEG (crosses for data points with dotted error bars). In Fig.2(b), the observed data are shown with *dotted crosses* (for *ASCA* SIS0+SIS1) whileas the theoretical and predicted profiles are indicated by the thick solid line and the thin solid one.

In Fig.2(c), the observed data are shown with *linked stars* (for HEG) and the solid line in the spectrum is the original best-fitting model, which consists of a power-law continuum and a narrow Gaussian. In the inset of Fig.2(c), theoretical and predicted profiles are coincident with each other and indicated by the same solid curve. Although the three model parameters are set constant as $\{a,\ r,\ \theta_0\} = \{0.96,\ 7.5,\ 28°\}$, the numerical step of the emission energy is selected as 64 eV, about 1.7 times as large as the instrument response FWHM=38eV. Different from the broad *ASCA* line profile in Fig.2(b) with no obvious signature of a narrow core, the inset presents the narrow core at 6.3 keV. This result verifies that shown by the observation in Fig.2(c).

3. Results and Discussion

The thin-torus model and its application to three example AGNs suggest that (1) the line emission is related not only to the invariant photon density and the photon four-momentum in the frame comoving with the particles, but also to the four-velocity components of particles; (2) it is not the photons in a few orbits, but particles emitting photons in all possible orbits, are responsible for line emissions.

According to these results, it is easy to explain why MCG-6-30-5 (Tanaka et al. 1995), and even NGC 3516 (Nandra et al. 1999) are all fit to both a Kerr-BH system and a Schwarzschild-BH system simultaneously with the commonly accepted thin-disk model. In the thin-disk model, the only effect caused by the spin of the central BH is the difference of the minimal deduced radial positions of the emitted photon. As a result, the predicted profiles with either a rotating BH or a nonrotating one are much similar to each other.

The application of the thin-torus model to the three sample galaxies reveals that there are several features which cannot be dealt with by the thin-disk model: (1) the central BH of each AGN is clearly rotating rapidly ($a \sim 1$); (2) the thin torus emitting Fe line is only located within a few tens of r_g; (3) the observer's inclination angles are around the moderate one, $\sim 30^\circ$. (4) From Eq.(2), the extension of the torus to both sides of the equatorial plane of the BH is about $\pm 50^\circ$, suggesting that the Fe K line emission originates from a thin torus extending from the polar angle of $\sim 40^\circ$ to that of 140°.

It deserves to be mentioned here that the predicted profiles is not completely fitted to the observed ones. The differences may be brought about by the factors both extrinsically and intrinsically. The former is caused by the data processing in observations while the latter should be caused by some divergences from the model assumptions as follows: (1) the thickness of the torus may not be so thin as to be neglected; (2) photons emitted by luminous torus particles may not be entirely homogeneous; (3) besides the monochromatic Fe Kα, other components like Fe Kβ, and/or Ni Kα (Wang, Zhou,, & Wang 1999) may also contribute to the emission; (4) besides the stable spherical particles, radially oscillating particles (Stewart & Walker 1973; Chandrasekhar 1983) may also play a role to radiate photons. (5) The selection of numerical steps may also contribute to the discrepancy between the observed profile and the predicted one (Yaqoob 2002).

Although there exist these uncertainties mentioned above, we hope they are the proper explanations for the differences between the observed profiles and the predicted ones. Besides the thin-disk model, The thin-torus one presented in this paper can also be used as a new tool to provide the information more definitely about BHs in more extragalactic sources in the near future.

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