The Bulletin of Symbolic Logic
Volume 25, Number 4, December 2019

THESIS ABSTRACTS

The Association for Symbolic Logic publishes abstracts of recent PhD theses in logic. The aim of this activity is to publish abstracts for the majority of recent PhD theses in logic world wide and submitted abstracts will therefore only be edited to ensure that they fall within the general area of logic and are appropriate in terms of length and content. This section will provide a permanent publicly accessible overview of theses in logic and thus make up for the lack of central repository for the theses themselves. The Thesis Abstracts Section is edited by Christian Rosendal. Any abstract should formally be submitted by the thesis advisor though it is expected to usually be prepared by the candidate. For detailed instructions for preparation and submission, including the required TeX template, please consult the link below. http://aslonline.org/LogicThesisAbstracts.html

Marija Boričić, ***Probability Sequent Calculi and Entropy Based Nonclassical Logics Classification***, Faculty of Mathematics, University of Belgrade, Serbia, 2016. Supervised by Nebojša Ikodinović and Zoran Ognjanović. MSC: 03B48, 03B50, 03B05, 03B55, 94A17, 37A35. Keywords: probability logic, sequent calculus, model, soundness, completeness, nonclassical logics, fuzzy logics, cut-elimination, classical propositional logic, many-valued propositional logics, Lindenbaum–Tarski algebra, partition, logical system, uncertainty measurement, entropy, classification.

Abstract

After a brief introductory survey, the thesis is divided into two parts. The first part deals with the presence of probability in logic (see [3–7, 9, 10]), and the second one is devoted to the application of the concept of entropy in classification of many-valued logics (see [1, 2, 8]).

The basic idea, dominant in the first part of the thesis, is to enrich the Gentzen's sequent calculus LK for propositional classical logic by a kind of probability operator defined over the sequents $\Gamma \vdash \Delta$ in order to express the fact that "the truthfulness probability of $\Gamma \vdash \Delta$ belongs to the interval $[a, b] \subseteq [0, 1]$". We introduce the following four systems: $LKprob$, $LKprob(\varepsilon)$, $NKprob$, and $LKfuzz$. The basic form of sequents in $LKprob$ is $\Gamma \vdash_a^b \Delta$ with the above given intended meaning. The system $LKprob(\varepsilon)$ is focused on the Suppes' forms $\Gamma \vdash^n \Delta$ enabling to formalize the sentence "the truthfulness probability of $\Gamma \vdash \Delta$ belongs to the interval $[1 - n\varepsilon, 1] \subseteq [0, 1]$", for some $n \in \mathbf{N}$ and a small real $\varepsilon > 0$. The system $NKprob$ presents a natural deduction counterpart of the sequent calculus $LKprob$. The models founded on Carnap–Popper–Leblance probability semantics are defined for each of these calculi and accompanied by the corresponding soundness and completeness results. Finally, the calculus $LKfuzz$ is introduced with a more general form of the sequents $\Gamma \vdash_x \Delta$, where x is an element of a finite lattice, with the aim to describe a fuzzification of LK. The meaning of $\Gamma \vdash_x \Delta$ is that "x is the fuzziness measure of $\Gamma \vdash \Delta$". Models for $LKfuzz$ are given with soundness and completeness results, and a proof-theoretical treatment of $LKfuzz$ includes the cut-elimination theorem.

The second part of the thesis explores the fact that each logical system associated with the partition induced by the corresponding Lindenbaum–Tarski algebra makes it possible to define its entropy. We define the entropy of a logical system based on geometric distribution of measures over matching partition of set of formulae. This definition enables the classification of many-valued propositional logics according to their entropies. Asymptotic entropy approximations for some infinite-valued logics are proposed as well. The considered

1079-8986/19/2504-0002
DOI:10.1017/bsl.2019.33

classification examples include Lukasiewicz's, Kleene's and Priest's three-valued logics, Belnap's four-valued logic, Gödel's and McKay's m-valued logics, and Heyting's and Dummett's infinite-valued logics.

[1] M. Boričić, *On entropy of a logical system*. ***Journal of Multiple–Valued Logic and Soft Computing***, vol. 21 (2013), no. 5–6, pp. 439–452.

[2] ———, *On entropy of a propositional logic*, this Bulletin, vol. 20 (2014), no. 2, p. 225. Logic Colloquium 2013, Abstract.

[3] ———, *Hypothetical syllogism rule probabilized*, this Bulletin, vol. 20 (2014), no. 3, pp. 401–402. Logic Colloquium 2012, Abstract.

[4] ———, *Models for the probabilistic sequent calculus*, this Bulletin, vol. 21 (2015), no. 1, p. 60. Logic Colloquium 2014, Abstract.

[5] ———, *Inference rules for probability logic*. ***Publications de l'Institut Mathematique***, vol. 100 (2016), no. 114, pp. 77–86.

[6] ———, *Suppes–style rules for probability logic*, this Bulletin, vol. 22 (2016), no. 3, p. 431. Logic Colloquium 2015, Abstract.

[7] ———, *Suppes–style sequent calculus for probability logic*. ***Journal of Logic and Computation***, vol. 27 (2017), no. 4, pp. 1157–1168.

[8] ———, *A note on entropy of logic*. ***Yugoslav Journal of Operations Research***, vol. 27 (2017), no. 3, pp. 385–390.

[9] ———, *Natural deduction probabilized*, this Bulletin, vol. 23 (2017), no. 2, p. 259. Logic Colloquium 2016, Abstract.

[10] ———, *Sequent calculus for classical logic probabilized*. ***Archive for Mathematical Logic***, vol. 58 (2019), pp. 119–138.

Abstract prepared by Nebojša Ikodinović and Marija Boričić.
E-mail: marija.boricic@fon.bg.ac.rs
URL: https://uvidok.rcub.bg.ac.rs/handle/123456789/1698

Jun Le Goh, ***Measuring the Relative Complexity of Mathematical Constructions and Theorems***, Cornell University, USA, 2019. Supervised by Richard A. Shore. MSC: 03B30, 03D30. Keywords: computable reducibilities, reverse mathematics.

Abstract

We investigate the relative complexity of mathematical constructions and theorems using the frameworks of computable reducibilities and reverse mathematics.

First, we study the computational content of various theorems with reverse mathematical strength around Arithmetical Transfinite Recursion (ATR_0) from the point of view of computable reducibilities, in particular Weihrauch reducibility. We show that it is equally hard to construct an embedding between two given well-orderings, as it is to construct a Turing jump hierarchy on a given well-ordering. We obtain a similar result for Fraïssé's conjecture restricted to well-orderings.

We then turn our attention to König's duality theorem, which generalizes König's theorem about matchings and covers to infinite bipartite graphs. We show that the problem of constructing a König cover of a given bipartite graph is roughly as hard as the following "two-sided" version of the aforementioned jump hierarchy problem: given a *linear* ordering L, construct either a jump hierarchy on L (which may be a pseudohierarchy), or an infinite L-descending sequence. We also obtain several results relating the above problems with choice on Baire space (choosing a path on a given ill-founded tree) and unique choice on Baire space (given a tree with a unique path, produce said path).

Next, we investigate three known ways to formalize the notion of solving a problem by applying other problems in series: the compositional product, the reduction game, and the step product. We clarify the relationships between them by giving sufficient conditions for them to be equivalent. We also show that they are not equivalent in general.