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CONVOLUTIONS OF DISTRIBUTIONS WITH EXPONENTIAL AND SUBEXPONENTIAL TAILS: CORRIGENDUM

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The proof of Lemma 2.3(ii) as originally given is incomplete since we cannot recursively apply (2.1) with a fixed t_0. This is corrected below. In addition, we are able to extend the result so the conclusion is that $H \in \mathscr{L}_\alpha$. The statement of Lemma 2.3(ii) is thus as follows:

Assume $\lambda_n \geq 0$ *and* $\sum_{n=0}^{\infty} \lambda_n (\overline{F}(0))^n < \infty$. *If* $F \in \mathscr{L}_\alpha$ *and* $H = \sum_{n=0}^{\infty} \lambda_n F^{*n}$, *then* $H \in \mathscr{L}_\alpha$.

PROOF. We may assume without loss that $\lambda_1 > 0$; the proof otherwise differs slightly. Following Embrecht and Goldie's (1980) proof of (2.1), we obtain by recursion, for $t \geq nt_0$,

$$\overline{F}^{*n}(t-u) \leq (1+\varepsilon)e^{\alpha u}\overline{F}^{*n}(t).$$

Hence, for $t \geq nt_0$,

$$\begin{aligned}\overline{H}(t-u) &\leq (1+\varepsilon)e^{\alpha u}\sum_{j=0}^{n}\lambda_j\overline{F}^{*j}(t) + \sum_{j=n+1}^{\infty}\lambda_j\overline{F}(0)^{j-1}\overline{F}(t)\\ &\leq (1+\varepsilon)\left(1+\frac{\delta(n)}{\lambda_1\overline{F}(0)}\right)e^{\alpha u}\overline{H}(t),\end{aligned}$$

where

$$\delta(n) = \sum_{j=n+1}^{\infty}\lambda_j\overline{F}(0)^j.$$

Therefore,

$$\limsup_{t\to\infty}\frac{\overline{H}(t-u)}{\overline{H}(t)} \leq (1+\varepsilon)\left(1+\frac{\delta(n)}{\lambda_1\overline{F}(0)}\right)e^{\alpha u}.$$

Since both ε and n are arbitrary,

$$\limsup_{t\to\infty}\frac{\overline{H}(t-u)}{\overline{H}(t)} \leq e^{\alpha u}.$$

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If $\alpha = 0$, then it follows that $H \in \mathscr{L}_0$ since $\overline{H}$ is nonincreasing. Assuming $\alpha > 0$, choose ε to satisfy $(1 - 2\varepsilon)e^{\alpha u} > 1$. Following the proof of Lemma 2.3(ii), we may find an increasing sequence t_n such that for $t \geq t_n$,

$$\overline{F}^{*n}(t-u) \geq (1-2\varepsilon)e^{\alpha u}\overline{F}^{*n}(t).$$

Then for $t \geq t_n$,

$$\begin{aligned}\overline{H}(t-u) &\geq (1-2\varepsilon)e^{\alpha u}\sum_{j=0}^{n}\lambda_j\overline{F}^{*j}(t) + \sum_{j=n+1}^{\infty}\lambda_j\overline{F}^{*j}(t)\\ &\geq \left((1-2\varepsilon)e^{\alpha u} - ((1-2\varepsilon)e^{\alpha u} - 1)\frac{\delta(n)}{\lambda_1\overline{F}(0)}\right)\overline{H}(t).\end{aligned}$$

Thus

$$\liminf_{t\to\infty}\frac{\overline{H}(t-u)}{\overline{H}(t)} \geq \left((1-2\varepsilon)e^{\alpha u} - ((1-2\varepsilon)e^{\alpha u} - 1)\frac{\delta(n)}{\lambda_1\overline{F}(0)}\right).$$

Again, since both ε and n are arbitrary,

$$\liminf_{t\to\infty}\frac{\overline{H}(t-u)}{\overline{H}(t)} \geq e^{\alpha u},$$

and this proves $H \in \mathscr{L}_\alpha$.

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