Canad. Math. Bull. Vol. **16** (3), 1973

A CONSTRUCTION OF SUBEQUALIZERS

BY
FRED COPPOTELLI

Given a pair of functors $F, G: \mathbf{A} \to \mathbf{B}$, Lambek defines [1] the subequalizing category, $\mathbf{E}$ of (F, G) as the category with objects, ordered pairs (A, b) with $A \in |\mathbf{A}|$ and $b: FA \to GA$ a morphism of $\mathbf{B}$. The morphisms of $\mathbf{E}$ from (A, b) to (A', b') are ordered triples (b, a, b') where $a: A \to A'$ is a morphism of $\mathbf{A}$ and $G(a)b = b'F(a)$. Lambek obtains the comma category of a pair of functors $F_0: \mathbf{A}_0 \to \mathbf{B}$, $F_1: \mathbf{A}_1 \to \mathbf{B}$ as the subequalizing category of the pair of functors $F_0P_0, F_1P_1: \mathbf{A}_0 \times \mathbf{A}_1 \to \mathbf{B}$, where P_i is the projection $\mathbf{A}_0 \times \mathbf{A}_1 \to \mathbf{A}_i$, and asks for a construction of the subequalizing category in terms of the comma category. The construction follows.

Given $F: \mathbf{A} \to \mathbf{C}$, $G: \mathbf{B} \to \mathbf{C}$, the comma category F/G has "projections" $P_F: F/G \to \mathbf{A}$, $P_G: F/G \to \mathbf{B}$. (These are the outside arrows in the diagram consisting of three pullbacks used to define F/G.) When $\mathbf{A} = \mathbf{B}$ so that we have a pair of functors $F, G: \mathbf{A} \to \mathbf{C}$ we have $P_F, P_G: F/G \to \mathbf{A}$. The subequalizing category of the pair (F, G) is seen to be the equalizer of P_F and P_G, $\mathbf{E} \xrightarrow{E} F/G \underset{P_G}{\overset{P_F}{\rightrightarrows}} \mathbf{A} \underset{G}{\overset{F}{\rightrightarrows}} \mathbf{C}$. The functor part of the subequalizer is $P_FE = P_GE$, and the natural transformation has the same description as in [1].

Reference

1. J. Lambek, *Subequalizers*, Canad. Math. Bull. **13** (1970), 337–349.

Virginia, Polytechnic Institute and State University,
Blacksburg, Virginia

https://doi.org/10.4153/CMB-1973-068-4 Published online by Cambridge University Press