Section IV

The Cometary Nucleus

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CHARACTERIZATION OF THE ROTATION OF COMETARY NUCLEI

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ABSTRACT. I review the primary methods used to determine the spin state of cometary nuclei and the pitfalls and successes experienced in their use. There are in excess of 60 reported determinations of rotational periodicities, but only a few (~4) appear to be reliable, and even these do not necessarily fully describe the true rotational state. An adequate rotational ephemeris is not, at present, available for a single cometary nucleus.

Because recent studies indicate that active cometary nuclei could be in excited spin states, I also review the theory of rigid body rotation from the point of view of remote (astronomical) observers, covering what is known of the effects of nutation on lightcurves, the influence of torques induced by jet activity, and the effects of internal energy and mass dissipation, and nuclear splitting.

The available knowledge on rotation for 8 comets, including P/Halley, is reviewed. Outstanding questions that need early resolution are: (1) Can a consensus be achieved on the rotational state of P/Halley? (2) Is it possible to accurately determine the amplitude of the transverse non-gravitational force associated with rotation? (3) Are the orientations of fan-like comas a valid indicator of the orientation of the spin vector - can definitive observational checks be made in a few cases?

Improved observational and interpretational techniques are needed to advance this field. Improved time-series and zero-date analyses are needed to connect existing and future data sets and to search for multiple periodicities in cometary lightcurves; improved sampling and extension of time-series observations with moderate- and large-aperture telescopes at good sites is needed; near-simultaneous photometric and radiometric observations made when cometary activity is low are particularly significant; well-sampled time-series imaging of near-nuclear phenomena (together with adequate software to analyze them) is essential to diagnose rotational states if nutation is present.

1. The Significance of Knowledge of the Rotational State

Determination of the intrinsic physical properties of a cometary nucleus and its surface, from remotely sensed observations, requires knowledge of its rotational state. As Sekanina (1981a) noted in an earlier review, "...it appears that the personality of a comet cannot be understood without knowledge of the spin axis" and, I might add, of the magnitude and variability of the spin.

Knowledge of the spin state allows the time history of the illumination of the nuclear

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surface to be deduced, and is therefore a key element in mapping the observable aspects of cometary activity back to specific regions of the nucleus. A spin ephemeris is needed to correlate observations of activity that are made at widely separated times. It is an essential input for characterizing the evolution of activity at a particular location.

Direct determination of the evolution of the spin state for a few active comets would lead to a better understanding of the nature of non-gravitational forces (*e.g.*, Whipple 1950, Marsden *et al.* 1973, Yeomans and Chodas 1989) and of the torques that act on the nucleus.

Knowledge of the spin state can also improve the interpretation of measurements that actively sample the cometary surface or its environment. Thus it is an important ingredient in the analysis of radar pulses reflected off the surface of cometary nuclei (*e.g.*, Kamoun *et al.* 1982, Goldstein *et al.* 1984, Ostro 1985, Campbell *et al.* 1989). Similarly, as the experience of *VEGA* and *Giotto* experimenters (Vaisberg *et al.* 1986, 1987, Trotignon *et al.* 1987, Hsieh *et al.* 1987, Simpson *et al.* 1987) shows, knowledge of nuclear spin is an important input to the interpretation of *in situ* measurements of dust and gas outflow, gradients, and other inhomogeneities in the inner coma.

If knowledge of the rotational state can be combined with measurements of the dimensions and shape of the nucleus, then, for nuclei in an excited state, it may be possible to infer details of the internal nuclear mass distribution (Belton 1990). Also, a sufficiently precise determination of the evolution of the spin state for longer period comets in the aphelion leg of their orbits could, in principle, lead to a determination of the presently unknown rate of internal energy dissipation in the nucleus (Burns and Safronov 1973, McAdoo and Burns 1974, Peale and Lissauer 1989), which is thought by some to be substantial (Wilhelm 1987).

Statistical information on the magnitudes of spin, the prevalence of excited spin states, the orientation of spin axes, and shapes for a wide population of cometary nuclei is of cosmogonic interest. From this kind of information, we may learn about aspects of the overall processes that affect the evolution of shape and size of cometary nuclei (Lamy and Burns 1972, McAdoo and Burns 1974, Keller 1987, Jewitt and Meech 1988), and relationships with other solar system objects, particularly the asteroids (Whipple 1982, A'Hearn 1988, Jewitt and Meech 1988).

Given the significance of rotation for all of the above studies, it seems remarkable that, at the present time, there is not yet even a single comet for which a reliable spin ephemeris is available.

Several reviews have been published in the past decade on this and closely related subjects (Burns and Tedesco 1979, Sekanina 1981a, Whipple 1982, Wallis 1984, Mendis *et al.* 1985, A'Hearn 1988). Only one, however, postdates the 1986 Halley apparition and reflects the enormous increase in knowledge and research activity that has been stimulated by the spacecraft encounters, the activities of the International Halley Watch, and the planning for NASA's proposed Comet Rendezvous Asteroid Flyby mission.

Recent research appears to be leading to fundamental changes in the way we look at cometary nuclei: Previously accepted ideas of the relationship of the rotation to non-gravitational orbital forces are changing. Well-used assumptions regarding the most likely shape of cometary nuclei and mode of rotation no longer appear to be appropriate (A'Hearn 1988, Sekanina 1987c, Julian 1987, Belton 1987). Previously uncontested statements such as "...the random distribution of non-gravitational accelerations and decelerations among the short period comets, which, by and large, reflect the apparently random distribution of nucleus obliquities" (*e.g.*, Sekanina 1987b), or the assumption that an active comet is most likely to be in a state of principal axis rotation, must now be carefully questioned.

2. Theoretical Basis

This section contains a description of the possible rotational states available to cometary nuclei. The development refers specifically to the spin of a rigid body and is given in some detail in an attempt to introduce some standardization in the future use of terminology. Non-rigid body rotation has been briefly mentioned in the cometary context by Bertaux and Abergel (1986) and Wilhelm (1987).

Since most observational studies are done remotely, rather from a frame of reference fixed in the nucleus itself, the phenomena are described from the point of view of a remote inertial observer. This is an important point (Watanabe 1989, Belton 1990), since for theoretical work it is very convenient to work in an accelerated frame whose axes are fixed and coincident with the principal axes of inertia (*e.g.,* Landau and Lifshitz 1960). Phenomena that appear periodic in this frame are not necessarily so when seen by an inertial observer.

Torques on cometary nuclei are expected to be small (Wilhelm 1987, Julian 1988a, Peale and Lissauer 1989) or negligible, and so the total angular momentum vector, **M**, will vary slowly or not at all. For short periods of time (*e.g.,* the time between the *VEGA* and *Giotto* flybys), **M** can be assumed constant. This suggests, following Landau and Lifshitz (1960), a basic inertial frame, OXYZ, where the Z axis is assumed parallel to **M**, and OX can be chosen for convenience. Specific regions on the nucleus can be referred to a frame Oxyz fixed in the body with the axes coinciding with the principal axes of inertia. Since the rotation of the object is a description of the changing relationship of the two coordinate systems, it is conveniently represented in terms of the Euler angles, ϕ,ψ, and θ, and their rates of change. The particular set of Euler angles used here is the same as that described by Belton (1990), and is different from the set used by Landau and Lifshitz (1960). The set used here can be obtained from Landau and Lifshitz equations by permuting the suffixes 1,2,3 to 2,3,1 in their development. This is done because of the observational experience that cometary nuclei are roughly prolate in shape, and it is the orientation of the "long," or minimum inertia, axis that is most easily specified by observation. It is therefore much more convienient to define the angle θ with respect to this axis rather than to the more difficult to observe "short," or maximum inertia, axis as is usually done in most textbooks. The maximum moment of inertia, I_s, is about the short axis, while the minimum moment of inertia, I_l is about the "long" axis. The moment of inertia about the intermediate axis is denoted as I_i.

The situation is characterized, from the *observer's* point of view, in Fig. 1. The component motions can be referred to as "ϕ motion" etc., and the periodicities associated with them as P_ϕ, P_ψ, and P_θ. P_ϕ is sometimes referred to as the period of "wobble," or "nutation," or "free-precession" period. In fact, it simply measures a particular component of the instantaneous spin and, in the most interesting case (that of minimum rotational energy), becomes the actual rotational period.

Euler angles, as defined here, are particularly useful because the description of the motion becomes very simple in important limiting cases. Thus, in the lowest rotational energy state associated with a given angular momentum, $\dot{\phi} = 2\pi/P_\phi$, $\dot{\psi} = \dot{\theta} = 0$, where P_ϕ is now the period of rotation. Other simple limiting cases are for rotation in the highest possible energy state and in the case of the symmetric rotator (see Fig. 2).

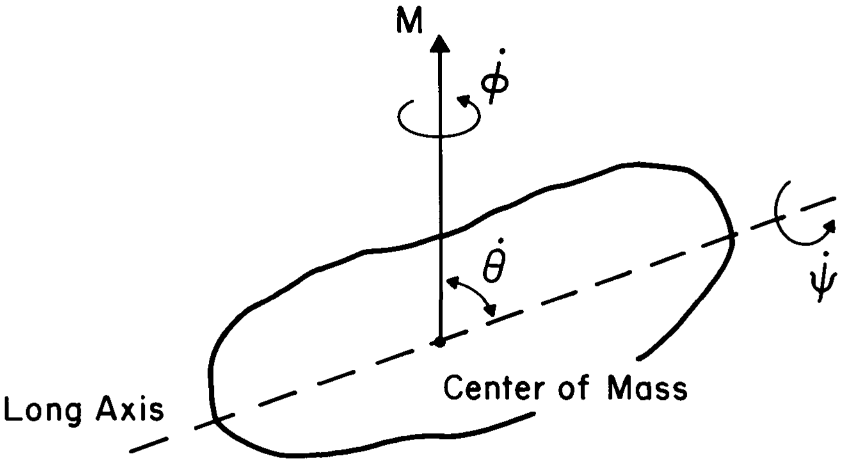

Fig. 1. The observer's view of the component rotations of a nutating cometary nucleus.

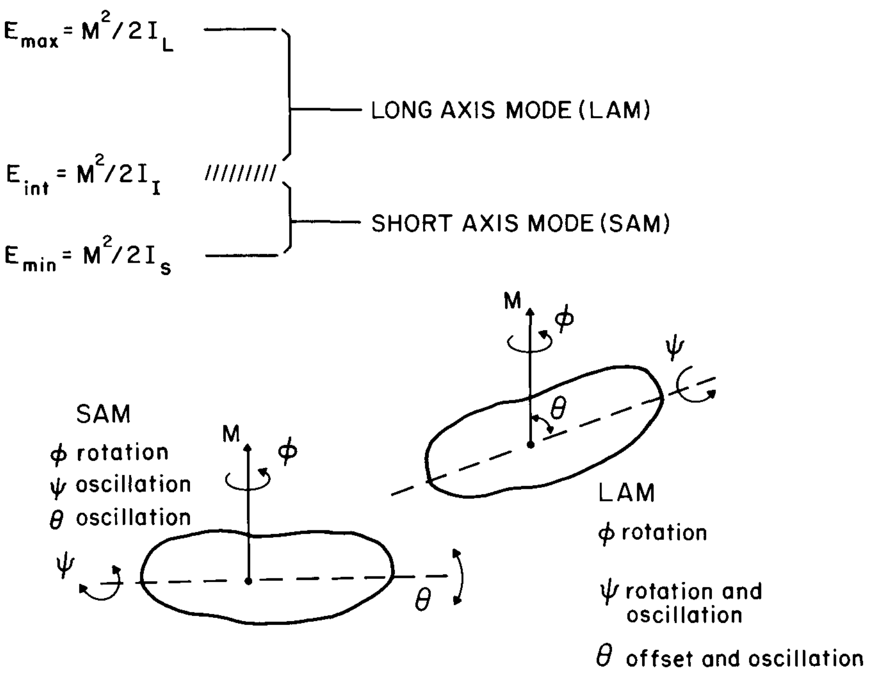

Fig. 2. Energy states and modes in a rigid rotator.

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2.1. TORQUE-FREE ROTATION AND NUTATION

The general motion, *i.e.,* the circulation of the instantaneous spin vector about **M**, of a torque-free symmetrical top is called *nutation* (Joos 1950). An alternative descriptor is *free-precession* (Sommerfeld 1952), but I will not use this, preferring to reserve the term "precession" to imply the action of torques. In this review, I extend the use of the term "nutation" to loosely describe the motion of the spin vector in a force-free asymmetric top. I also mean by *rotation* the instantaneous angular velocity (Peale and Lissauer 1989) and not a particular component of rotation around one of the principal axes as is done by several authors (Sekanina 1987c, Festou *et al.* 1987, Schulz and Schlosser 1989). These distinctions are significant, for, as several authors (*cf.* Watanabe 1989, Peale and Lissauer 1989, Belton 1990) have pointed out, conflicting definitions of terminology already exist in the literature and could lead to confusion. Nutation, as used here, is unfortunately not the same as *astronomical* nutation, which is the term for a motion of the Earth's spin axis *forced* by the Moon (*cf.* Sommerfeld 1952).

As illustrated in Fig. 2, a nucleus rotating with angular momentum **M** can be in a range of energy states $E_l < E < E_m$. When an object rotates in its lowest energy state, it is often referred to as being in a state of "principal axis rotation."

When the rotator is in an *excited* state, *i.e.,* $E > E_l$, there exists a *critical* value for the energy, $E_c = M^2/2I_i$, which divides the motion into two distinct types. When $E > E_c$, the rotator finds itself in a LAM, or "long-axis" mode, and when $E < E_c$, it is in a SAM, or "short-axis" mode (Julian 1987). SAM motion only occurs for asymmetric rotators. In both of these modes, the total spin vector circulates, often on a complex path, around the direction of **M**. What differentiates them to the outside observer is that, in a LAM, the ψ motion resolves into complete rotations about the long axis (generally with a superimposed oscillatory motion), while in a SAM, there is no complete rotation, only oscillatory motion about the long axis.

In both of these modes, the ϕ motion consists of complete rotations (often with superposed oscillations), while the θ motion is always oscillatory. In a SAM the ψ and θ motions are coupled and have the same underlying periodicity. A further distinction is that, depending on the *degree of asymmetry, i.e.,* the value of $\delta = (I_s - I_i)/I_s$, the time-average, θ_o, falls in the interval $0 < \theta_o < \pi/2$ for a LAM, while in the case of a SAM, $\theta_o \approx \pi/2$. The values of the periodicities associated with the various component motions are constrained by the internal mass distribution in the object through ratios of the principal moments of inertia. In the case of symmetric (Sekanina 1987) and nearly-symmetric LAMs:

$$P_\phi \simeq P_\psi.[(I_s - I_l)/I_l].cos\theta_o \quad (1)$$

$$P_\theta = \infty \quad (symmetric\ case) \quad (2)$$

$$P_\theta = P_\psi/2 \quad (nearly-symmetric\ case) \quad (3)$$

SAMs have similar relationships plus a further constraint connecting the oscillatory amplitudes, A_θ, A_ψ, and the principal moments (Watanabe 1989, Landau and Lifshitz 1960):

$$P_\phi = P_\psi.[(I_s - I_i)(I_s - I_l)/I_i I_l]^{1/2} \quad (4)$$

$$P_\theta = P_\psi \quad (5)$$

$$A_\theta = A_\psi.[I_l(I_s - I_i)/I_i(I_s - I_l)]^{1/2} \quad (6)$$

The motion in the state $E = E_c$ has a character very different from either of the above modes (*cf.* Landau and Lifshitz 1960), but since it is a short-lived transitory state, it is not believed to be important in observational studies of comets.

In summary, the lowest energy state reduces to a principal axis rotation with $\dot{\phi} = 2\pi/P_\phi$, $\dot{\psi} = \dot{\theta} = 0$. If the nucleus is in an *excited* state, it may be in either a LAM or a SAM. Each is characterized by two periods P_ϕ and P_ψ (and their harmonics). Expressions giving a complete description of the motion in terms of Jacobian elliptic functions can be found in Landau and Lifshitz (1960) and numerical algorithms for evaluation of these functions can be found in Press *et al.* (1986). The following approximations to the motion, subject to the constraints outlined above, may be found useful:

SAM ($\theta_\circ \sim \pi/2$):

$$\phi(t) - \phi_\circ \approx (2\pi/P_\phi)(t - t_\circ) \tag{7}$$

$$tan\psi(t) \approx A_\psi sin(2\pi/P_\psi)(t - t_\circ) \tag{8}$$

$$cos\theta(t) \approx A_\theta cos(2\pi/P_\theta)(t - t_\circ) \tag{9}$$

LAM ($\delta \sim 0$):

$$\phi(t) - \phi_\circ = (2\pi/P_\phi)[1 + \delta/2](t - t_\circ) - \delta.(P_\psi/P_\phi)sin2\omega_\circ(t - t_\circ) \tag{10}$$

$$cos\theta(t) = cos\theta_\circ[1 - \delta.P_\psi tan\theta_\circ sin\theta_\circ(1 - cos2\omega_\circ(t - t_\circ))/4P_\phi] \tag{11}$$

$$tan\psi(t) = [1 + \delta.P_\psi cos\theta_\circ/P_\phi]tan\omega_\circ(t - t_\circ) \tag{12}$$

$$\omega_\circ = (2\pi/P_\psi)[1 + \delta.(P_\psi/P_\phi)cos\theta_\circ]^{1/2} \tag{13}$$

The initial time t_o is chosen to coincide with a time when $\psi(t) = 0$. The symmetric case (*e.g.*, Sekanina 1987c) can be obtained from the above equations with $\delta = 0$.

2.2. TORQUED MOTION AND PRECESSION

Applied torques cause changes in **M** and therefore in the spin state. Only torques due to cometary activity, *i.e.*, directed mass-loss, are significant; those due to gravitational interactions with planets are completely negligible. Sekanina (1979) and Whipple and Sekanina (1979) were the first to investigate the long-term evolution of the spin state of cometary nuclei under the assumption that they are in principal axis rotation and that the precession of **M** is slow. The problem involves a complex parameterization of the physical processes that occur, but was considered manageable because of the extensive information available on observational constraints such as the transverse non-gravitational force - which connects mass-loss to the rotational state - and the visual lightcurve, which, until recently, was thought to be a reliable indicator of the rate of mass-loss with heliocentric distance. We discuss the experience and limitations of this kind of physical modeling in more detail in Section 3.3.

Wilhelm (1987), Julian (1988a, 1988b), and Peale and Lissauer (1989) have investigated the expected amplitude and effects of torques on P/Halley and find that in a single perihelion passage, **M** can precess as much as 23° (Julian 1988a, 1988b) or 30° (Wilhelm 1987).

These authors assume a mass density of $\sim 0.3\text{g.cm}^{-3}$ for the nucleus, and an efficiency factor of 0.2 for converting the mass-loss flow into effective torque. Both of these parameters are poorly known at the present time (*cf.* Peale (1989) regarding the most likely value for the density of P/Halley), and each is presumably uncertain by up to an order of magnitude. Thus the typical angular precession per perihelion passage could be either much less or much more.

Wilhelm's study indicates that randomly applied torques are the most efficient in building up nutational motion. In addition, Peale and Lissauer find that nutation is easy to excite only for symmetric or nearly axisymmetric rotators - and then only around the axis of maximum inertia. This would seem reasonable from consideration of the energetics of the motion. Additional useful expressions for calculating precessional motion are presented by Peale and Lissauer.

Any nutation that is seen today cannot be primordial motion because any reasonable estimate of the damping time is far too short (Jewitt and Meech 1988, Peale and Lissauer 1989). It is also unlikely that the basic underlying rotation is primordial if the comet has experienced many close perihelion passages. The possibility of nuclear spin-up (Whipple 1961, 1978, 1982), or spin-down (Wallis 1984), and the precession of spin axes seem to indicate that the distribution of rotational periods and axes will have been modified over cometary lifetimes. Nevertheless, Ferrin (1988) has proposed that Halley and other comets are near to their original primordial rotational momentum since they are found to fall near the "primordial line" defined in a plot of log(angular momentum/mass) versus log(mass) by many objects in the solar system.

Although possibly not primordial, nutational motion could be common in active nuclei as a result of a stochastic buildup over many perihelion passages (Belton 1987 , Peale and Lissauer 1989) balanced by damping due to internal stress-induced friction. The available calculations (*e.g.,* Wilhelm 1987) show jet activity can induce nutation at each perihelion passage even though the amplitudes of induced wobble and the rate at which the motion is damped are very poorly understood.

If the torques due to directed mass-loss have a component parallel to the spin axis, then "spin-up" can occur as discussed by Whipple (1961, 1978, 1982), and noted by Samarasinha *et al.* (1986). Whipple proposed that this process may lead to the observed propensity of cometary nuclei to split. The critical period at which this might happen has been investigated by Whipple (1982), Jewitt and Meech (1988), and Weidenschilling (1981). Weidenschilling finds that the shortest rotational period to be expected for an intrinsically weak (deformable) object before it can lose mass due to centripetal forces is $\sim 6.6\rho^{-1/2}$ hr (where ρ is in g.cm^{-3}). The observed boundary of short periods could define a lower bound on the density for objects with no internal strength. In Whipple's investigation, a density of 1.3 g.cm^{-3} implies that the shortest period should be 5.8 hr, which he notes is already longer than the periods of four comets he has studied. Jewitt and Meech note that one estimate of the density for P/Halley (0.1 to 0.2 g.cm^{-3}) places several of the comets whose periods are known with some precision near to the centripetal limit.

Wallis (1984) has argued that active surface processes on cometary nuclei will redistribute mass away from the poles across the surface of the nucleus and ultimately lead to a systematic drain of angular momentum from the system, much in the way that Dobrovolski and Burns (1984) have proposed for asteroids. Wallis asserts that nuclei should "spin-down" rather than "spin-up."

2.3. DISSIPATIVE MOTION

2.3.1. *Internal Dissipation.* As the nucleus nutates, it is subject to changing internal stresses. It therefore flexes, and friction converts the energy of rotation into heat. This process has been studied by Burns and Safronov (1973) and by McAdoo and Burns (1974). Burns and Safronov find that the characteristic time, τ, for an oblate rotator to relax to a state of principal axis rotation is given by:

$$\tau = \mu Q/(\rho K_3^2 r^2 \omega^3) \tag{14}$$

where ω is the angular velocity, μ is the rigidity of the nucleus material, ρ is its density, Q is a dimensionless measure of internal energy dissipation per rotation cycle, and K_3 is a shape factor depending on the principal moments of inertia. Although worked out only for an oblate rotator, other shapes should have similar relaxation times. Jewitt and Meech (1988) and Peale and Lissauer (1989) have considered the values typical for cometary nuclei. The latter suggest

$$\tau = 10^6 Q \quad years \tag{15}$$

for cometary nuclei with the properties of P/Halley. Typical values for Q in the case of asteroids are 10^2 to 10^3 (Burns and Safronov 1973), although there seems little experimental basis for a rational choice of value for this parameter. For cometary nuclei, Peale and Lissauer recommend $Q \leq 10^2$ and even consider a case where $Q \leq 1$. Thus any nutation that may exist today is probably not primordial. However, τ is clearly much longer than the typical periods associated with periodic comets (in contrast to an assumption made by Wilhelm (1987)), presenting the possibility that nutational states, built up as a result of net torques applied effectively randomly at successive perihelion passages (Belton 1987, Peale and Lissauer 1989), may be common in short-period active comets.

2.3.2. *Mass-Loss.* Cometary nuclei dissipate substantial amounts of mass in a more or less continuous fashion during their active phases. While undirected mass-loss is not expected to lead to dramatic changes in the spin (Peale and Lissauer 1989), Wallis (1984) has argued that the details of the process will lead to a systematic drain of angular momentum. Sekanina (1981b) points out that only a fraction of the escaping mass exerts a force on the nucleus - mainly the gas. The dust, or gas released from dust, that is already in the coma does not contribute to the net torque. Sekanina gives an extended discussion of the processes that lead to momentum transfer with the nucleus in the general mass-loss process. Keller (1987) and Jewitt and Meech (1988) have briefly discussed anisotropic mass-loss and its possible effects on the shape, and therefore the moments of inertia, of the nucleus. One can imagine a cycle of events in which preferential mass-loss takes place near the equator, leading to a roughly prolate object in LAM rotation. Simultaneously, energy dissipation causes the nucleus to nutate and ultimate evolve into a SAM rotator - following a scenario similar to one sketched out by Lamy and Burns (1972). The process then possibly repeats through further cycles. Jewitt and Meech point out that for such effects to be important, the lifetime of the nucleus against sublimation must be much shorter than its dynamical lifetime in the inner solar system. Unfortunately, both of these time-scales are hard to estimate. Keller doubts that this process can be significant and that, in the case of P/Halley, its current shape was determined at the time of its formation.

Nuclei also lose what presumably are substantial fractions of their mass in discrete events - when the nucleus is observed to *split*. The observational and kinematic characteristics of this process are described by Sekanina (1982). Peale and Lissauer (1989) find that splitting can lead to LAM nutation in some cases, but no detailed study is presently available on the exchange between rotational and translational energy that might take place in this process. Belton (1987) and Peale and Lissauer (1989) have both pointed out that this process may be important in understanding the generation of nutational energy in active cometary nuclei.

2.4. RELATIONSHIP OF THEORY TO OBSERVATION

Before concluding this synopsis of the theory, it is of interest to explore the relationships between the above theoretical results and the periodicities that are measured by remote astronomical observers.

2.4.1. *Inactive Cometary Nuclei.* In this case, the drives for nutational motion are absent, and it can reasonably be assumed that the nucleus has reached its lowest energy state and is in principal axis rotation. If the object resembles an oblate spheroid of constant surface albedo, there will be no rotational modulation of the scattered sunlight. Minor irregularities in shape and albedo will be responsible for whatever variability is seen and, since such irregularities will not necessarily have any particular geometric symmetry, periodicities, P_n, at all harmonics ($P_n = P_{spin}/n$) of the spin may appear in the lightcurve.

If the nucleus is prolate with constant surface albedo, then, except for pole-on configurations where no rotational modulation would be seen, the second harmonic of the spin period should dominate the lightcurve because of the two-fold symmetry of the object about the rotation axis. The dominant "periodicity" will be exactly one-half of the true rotational period, and the amplitude, or range, of the variability will be a measure of the projected axial ratio on the sky. As in the case of an oblate object, irregularities, or higher degrees of symmetry, in shape or albedo may superpose other harmonics into the lightcurves. It is possible to separate out some of the effects of geometry and albedo by comparing the phase of the variability in radiometric and optical lightcurves taken at the same epoch (Campins *et al.* 1987, Millis *et al.* 1988, A'Hearn *et al.* 1989). If they are in phase, then the variability is most likely dominated by changes in the projected geometrical cross-section of the nucleus on the sky, while if they are 180° out of phase, contrasts in surface albedo are more likely to be responsible. In the three comets (P/Arend-Rigaux, P/Neujmin 1, P/Tempel 2) where such data are available, only the former situation is observed.

By following the changing amplitude of the second harmonic as the orbital configuration evolves, it may, in some cases, be possible to estimate the gross shape of the object, particularly if the direction of the rotational axis is known. Jewitt and Luu (1989) have begun such an investigation in the case of P/Tempel 2. Sufficiently detailed photometry (not yet available) can also be used to determine the direction of the spin axis. Such photometric determinations are common in the study of asteroids (*e.g.,* Pospieszalska-Surdej and Surdej 1985, Weidenschilling *et al.* 1987, Drummond *et al.* 1988). The information that can be deduced from the harmonic content in optical lightcurves is treated by Ostro and Connelly (1984). Lebofsky *et al.* (1986) and Brown (1985) cover the information available in thermal radiometry.

As the above discussion indicates, a definitive interpretive step from an observed periodicity to the determination of the rotation period is not always guaranteed. This explains why different authors, often with similar data sets, may often advocate periods that, while different, are harmonically related. A classic case is that of P/Arend-Rigaux outlined in Section 6.4 below.

2.4.2. *Active Cometary Nuclei at Large Heliocentric Distances.* Active cometary nuclei may be in an excited rotational state at any point in their orbit. Whether any nutation has an appreciable amplitude depends on the competition of torques induced near perihelion and internal dissipation. In this case, the above theory indicates that the rotation will be characterized by two periodicities, P_ϕ and P_ψ(or P_θ). How these periodicities manifest themselves in the observed lightcurve is not obvious. Festou *et al.* (1987), Watanabe (1989), Peale and Lissauer (1989), and Belton (1990) have all made exploratory calculations for the case of P/Halley. If, near aphelion, the comet shows no residual activity, or its effects can be removed, the situation should be as described in Section 2.4.1., but with the added complication of a second period. For prolate objects the second harmonics of P_ϕ should dominate for most orbital configurations. In addition, harmonics of P_ψ and "beats" between P_ϕ and P_ψ may be detected.

If residual activity cannot be removed (*cf.* Roemer 1966, Millis *et al.* 1988), then the periodic signature will also depend on the way active areas are illuminated by the Sun and the relative contributions of light directly reflected from the nucleus and that scattered by coma particulates. To separate these effects and infer the rotational state almost certainly requires information beyond that present in the lightcurves alone. Examples of the complexity of lightcurves at large heliocentric distance can be found in the post-perihelion photometric observations of P/Halley by West and Jorgensen (1989) and West (1990) when the comet was at 8.5 and 10.1 AU, but still active.

2.4.3. *Active Cometary Nuclei Near Perihelion.* In this case, the light reflected from the nucleus is an insignificant contribution to the total light. Nevertheless, the variability should be greatest in a small region centered on the nucleus. Ground-based and orbital observations relate to material, or fragments of material, that have been removed from the surface by the action of the Sun. If a single discrete region on the surface dominates the activity, then the periodicities that are seen will depend crucially on its location with respect to the principal axes of inertia, the direction of the angular momentum vector, and the particular spin state. For a nucleus in principal axis rotation, there will be essentially no modulation if the active region is near to the rotation axis, although fan-like coma features may appear (Sekanina 1987a). If the active region is elsewhere on the surface, then, depending on the orbital configuration, the rotation may not only be evidenced by a periodic lightcurve dominated by the first harmonic of P_{spin}, but also by repeating patterns of discrete events (jets, halos, etc.). In some orbital configurations, it is possible that the active area does not see the Sun for long periods when it obviously will not contribute to the variability of the lightcurve. If several areas are simultaneously active, then the observer may experience a range of harmonics of the rotational period.

If the nucleus is in an excited state, then harmonics of the two fundamental periods and their beats may be present, depending on the location of active areas with respect to the principal axes of inertia. It may be possible to distinguish between SAM and LAM by following the evolution of activity with changing orbital configuration. SAM rotation, as in the case of principal axis rotation, should show modulation of the activity on orbital time-scales (*e.g.*, some regions active before perihelion but not afterwards). In LAM rotation, the large excursions of the instantaneous spin axis ensure that, regardless of the orbital geometry, the entire surface is illuminated in time-scales comparable with the spin period.

Near-nucleus images may contain detailed information on the geometric location of the active areas on the surface which can be used to diagnose the observed periodicities. The shape of features (Sekanina 1987a, 1987b, Sekanina *et al.* 1987, Sekanina and Larson 1984, Keller and Thomas 1988) may help pinpoint information on the orientation of the instantaneous projection of the spin axis, or, in some cases, **M**, on the sky at particular times. The shape of jet features may also help place constraints on the magnitude of the spin vector.

In summary, the theoretical interpretation of observed periodicities is fraught with difficulties and complications. An observed periodicity, even if its signature is clear and indisputable, is not usually the true period of rotation of the nucleus. If the comet is inactive, the observed period is most likely to be $P_\phi/2$, but if it is active, P_ϕ. However, this simplification is not assured. Discovery of harmonically unrelated multiple periodicities in cometary lightcurves is as important to the interpretation of observations as the recognition of harmonic relationships. It seems essential that a wide variety of techniques be simultaneously applied, particularly in the case of active comets. Each method gives its own particular account of the phenomenon, and a definitive result for the true state of spin is likely only if all aspects of the observations are satisfied. The baffling case of P/Halley, briefly discussed in Section 6.7, illustrates how difficult it is to obtain a definitive, or even a consensus, result.

3. Observational Methods for Determining Periodicities and Spin Axis Orientations

There are basically four methods in use to diagnose the rotational state of cometary nuclei: the search for periodicities in *time-series* of some continuously varying property of the comet; the "*zero-date*" method, in which series of discrete events are examined for periodicities; *physical modeling* of time-dependent properties (*e.g.*, lightcurves, or non-gravitational forces) that depend upon nuclear spin; and analysis of *sequences of images* of near-nucleus structures shaped by rotation, or direct images of the nucleus itself. All except the zero-date method can also be used to estimate the orientation of the spin axis.

Each approach has its strengths. However, experience has shown that all of these approaches can suffer from problems of ambiguity. All are susceptible to interpretive errors when the sample of data is inadequate. Physical modeling is particularly susceptible to weaknesses in the underlying physical assumptions.

3.1. TIME-SERIES

The goal is to obtain a homogeneous set of time-series of measurements of some quantity that varies continuously as a result of rotation and derive periodicities, and, if possible, information on the orientation of the spin axis. A broad selection of techniques is in use to analyze such data, including *Phase-Dispersion Minimization* and *String-Length Minimization*, *Least-Squares Fits of Harmonic Functions*, and *Fourier Transform* methods. There are also several photometric techniques for determining spin axis orientations (*e.g.*, Drummond *et al.* 1988), which have not yet been applied to cometary nuclei. Broadband optical or thermal infrared magnitudes reflect the changing projection of the geometric cross-section of the rotating nucleus on the sky, or the quantity of dust liberated from active areas. Spectrophotometric data, from UV to radio wavelengths, on molecular and atomic emissions reflect the changing net production rates of volatile species. The overall length, T_o, of a time-series roughly sets the basic precision of a period determination, $\delta P \sim P^2/2\pi T_o$, and also an approximate upper limit, $P<T_o/2$, on the range of periods that can reasonably be determined. The mean sampling rate, $(\Delta t_{obs})^{-1}$, sets an approximate lower limit to the length of periods, $P>2\Delta t_{obs}$, that can be estimated with typical data sets.

In a typical ground-based study, the sampling follows a regular pattern characterized by the diurnal and lunar cycles. Superposed on this are irregular gaps caused by weather patterns, the vagaries of observing schedules, and finally the details of observing procedures, calibrations, etc., at the telescope. The regular patterns produce "alias" periodicities in the periodograms, while the irregularities (particularly long gaps in the data) often impose a myriad of "sidelobe" and "spurious" periodicities. These superposed periodicities must be discriminated against in the analysis of the periodogram to yield the underlying periodicities of physical significance. The primary tool for accomplishing this is the *spectral window* (Deeming 1975a, 1975b), which depends only on the sampling and defines the pattern of aliases, spurious periodicities, and sidelobe periodicities in the periodogram. It is common practice to use the information in the spectral window in the application of Fourier techniques (Roberts *et al.* 1987, Belton 1990), but not in connection with phase or string-length minimization techniques - even though there is no reason why this should not be done.

Manfroid *et al.* (1983) have compared the results of analyses based on the above methods using data characteristic of singly periodic phenomena. They find that no method is clearly superior to another and view the Fourier methods as complementary. They did not, however, consider the application of the CLEAN algorithm, which automatically makes full use of the information in the spectral window.

3.1.1. *Phase-Dispersion and String-Length Methods.* Stellingwerf's (1978) phase-dispersion minimization method is thought to be good for analyzing irregularly spaced data with non-sinusoidal variability (Millis *et al.* 1988). However, its chief attraction appears to be its simplicity and ease of application. According to A'Hearn *et al.* (1989), there is an important bias in this method which should be accounted for when attempting to identify the physically meaningful periodicity in the periodogram. At shorter periods, the bins are narrower in time, and so the observations in each bin tend to be taken closer together in time, artificially reducing the dispersion of the data values and systematically amplifying the depth of response.

A further problem is that wide gaps in the data make the phase-dispersion/period plot very rich in sidelobes and other spurious responses. Stellingwerf also notes that subharmonics will also appear in the periodogram and suggests how to identify these.

Dworetsky's (1983) method is a modern version of the "string-length" method of Lafler and Kinman (1965). Jewitt and Meech (1985) have made wide use of this method, which is popular for the same reasons as Stellingwerf's method.

A basic problem with past applications of these methods to cometary nuclei is that no attempt has been made to use the information available in the spectral window to understand the pattern of aliases and other responses and to "pre-whiten" the data of the primary periodicity in order to look for any other periodicities that may be present.

3.1.2. *Least-Squares Fit of Harmonic Series.* This technique is popular because of its physical and intuitive simplicity, but also because it directly yields the amplitudes of individual harmonics for interpretive purposes. It is an important tool in the analysis of asteroidal lightcurves (*e.g.,* Barucci *et al.* 1989, Ostro and Connelly 1984). It nevertheless suffers from the same problems of aliases, sidelobes, and spurious periodicities seen in the other techniques. In this method, it is a simple matter to "pre-whiten" the data in successive iterative steps and examine the periodograms for multiple periodicities (*e.g.,* Belton *et al.* 1981).

3.1.3. *Fourier Series.* Fourier methods based on the discrete transform method of Deeming (1975a, 1975b) are in wide use (*cf.* Ponman (1981) and Kurtz (1985) for useful discussions of these methods). Scargle (1982) has shown that, with a minor renormalization, the discrete transform is equivalent to a least squares fit of a harmonic series. Recently Roberts *et al.* (1987) have shown, at least in principle, that it is possible to remove the sidelobes and aliases from the discrete transform and construct a "clean" version with the use of the CLEAN algorithm (Högbom 1974). Belton and Gandhi (1988) have reported a modification to Roberts *et al.*'s approach that improves its stability when applied to "real" (noisy) data. Called WindowCLEAN, Belton (1990) describes the application of this technique to an analysis of P/Halley data.

When applying the Fourier method, it is very important to ensure that the transform is calculated with enough resolution. If the responses are not resolved, then spurious results will be obtained (*e.g.,* compare Leibowitz and Brosch (1986a) and Belton (1990)). Also, it is good practice (for all of the above methods) to remove all trends from the data before computing a periodogram. This pre-whitens the data of very long periodicities that might show with very large amplitude. The convolution of these non-physical responses with the spectral window will spread a large amplitude pattern of sidelobes and aliases throughout the periodogram that will mask any pattern of responses associated with physically meaningful periodicities.

For very short or very long periods (relative to a single day), it may be possible to sample with equal spacing. In this case, the MEM (Maximum Entropy Method) algorithm may be the best method to use (Percy 1977), but this has yet to be applied to cometary data.

3.2. ZERO-DATE METHODS

Whipple (1982) has discussed the pitfalls of this method, noting that "...a period calculated by this method is almost exactly correct or completely wrong." Basically, it considers time sequences of events (*e.g.,* recurring jets, halos, envelopes, and plasma knots, etc.) in the coma or tail from which "zero-dates," *i.e.,* the initiation times of activity at the surface of the nucleus, can be calculated if the characteristic velocity associated with the phenomenon is known or can be deduced from the data. In the form used by Whipple, the time intervals between observations of similar events are assumed to be an integral number of periods. This is tantamount to assuming that a single active area is responsible for the events. If this assumption is incorrect, then the calculated periods will be too short. Whipple notes that his formulation of the method is devastated if several active areas are responsible for what is observed or if incorrect velocities are assumed. There are only a few checks of the validity of the 47 periods calculated by Whipple (1982) using this technique. Comparisons (*cf.* Section 6) with periods determined with photometric time-series for the cases of P/d'Arrest, P/Halley, P/Tempel 2, and P/Encke are all unfavorable to this method. The results of studies that are based on Whipple's 47 periods should therefore be carefully reappraised with these limitations in mind.

A new approach to zero-dating has been proposed by Schulz and Schlosser (1989). The sequence of time intervals between events is examined for evidence of periodicity. A comet in principal axis rotation with period P which has n areas currently active will show a pattern of n distinct time intervals δt_i. If all of the events that occur have, in fact, been observed, then the intervals, plotted as a function of event number, will display a periodic pattern. The pattern will repeat after the number of intervals equals the number of discrete events n. The rotation period can be estimated from the mean interval between events:

$$\overline{\delta t} = (P/n) + \epsilon \qquad (16)$$

The correction term ϵ is small as long as $P << T_o$, where T_o is the extent of the observing period.

If the comet is nutating, the motion is characterized by two periods and, depending on exactly where the active areas are located with respect to the principal axis of inertia (Watanabe 1989, Belton 1990), the pattern of intervals could be characterized by either of these periods, their harmonics, or their "beats." The distribution of time intervals obviously becomes quite complex. Periodicities as a function of event number still occur (I have ascertained this by calculating a few sample situations), but the relationship between the fundamental rotation periods and the repetition period or the mean interval between events is now obscure. In such cases, it is probably best to use event strings simply as constraints on models of the rotation that have been developed from other considerations.

Observational programs are usually less than perfect, and zero-dates of some events in the interval T_o will, of course, be missing. If there are enough events to define a recurring pattern and the nucleus is in principal axis rotation, then it should be possible to account for the missing events by inspection. If this is not the case and there are no clear periodicities, or there are an insufficient number of events, then the zero-date method cannot be expected to give much useful insight *except by chance.* Its use under such conditions is obviously inadvisable.

3.3. PHYSICAL MODELING

A cornerstone of Whipple's (1950) icy conglomerate model of the cometary nucleus is the explanation of secular effects in the orbital motion of P/Encke by non-gravitational forces that originate at an active region through directed mass-loss. The sense and magnitude of the tangential component, A_2, is visualized to be the direct result of the systematic deviation of reaction forces away from the comet-Sun line due to a coupling of nuclear spin and the thermal hysteresis of the nuclear surface material. Whipple and Sekanina (1979) and Sekanina (1979, 1981a, 1984, 1985c, 1985d, 1986) have developed this concept into a "precessional" model for cometary nuclei that can be applied in those cases in which the long-term evolution of the transverse non-gravitational force and a visible lightcurve are available. The model has been applied to P/Encke, P/Kopff, P/Giacobini-Zinner, and P/Solá, yielding information on the rotation, size, and shape of their nuclei. In this multi-parameter physical model, the nucleus is assumed to be in a state of principal axis rotation with the spin axis susceptible to slow precession under the torques associated with directed mass-loss. To minimize calculational complexity, it is assumed that the nuclear shape can be modeled by an oblate spheroid. Mass-loss is parameterized in terms of the visual lightcurve and modeled by an active region displaced in longitude from the subsolar point, or by specifying the location of specific active areas on the surface. The entire problem is constrained by the observed evolution of the transverse non-gravitational force.

Unfortunately, many of the assumptions that underlie this model are now in question. Also, in the few cases where checks of the model predictions for nuclear shape or rotation are available, disagreement is the case (*cf.* Section 6).

A'Hearn *et al.* (1985) have found that the total gas production does not necessarily follow the run of the visual lightcurve in the case of P/Encke. Sekanina (1986), in a recalculation of his P/Encke model, finds that this change leads to drastic changes in the predictions of the model. Yeomans and Chodas (1989) have made calculations that indicate that the apparent magnitude and sense of A_2 may be strongly affected by other aspects of the mass-loss process and not be strongly indicative of nuclear rotation after all. Jewitt and Luu (1989) point out that the assumption regarding the likely shape of nuclei as oblate is not borne out in time-series photometry, or direct imaging, of P/Halley, P/Arend-Rigaux, P/Neujmin 1, and P/Tempel 2. Finally, the results for P/Halley indicate that even the assumption of principal axis rotation may be suspect for many active comets. As A'Hearn (1988) has pointed out, the basis for the precessional model needs a complete rethinking. Existing results should be viewed with caution.

Sekanina (1979, 1987a, 1987b, 1988b, 1989) has proposed that the "fan-shaped" structures seen in the coma of many comets are indicative of the orientation of the spin axis of the nucleus. The connection between the spin axis and what is observed is made through a simple physical model in which the active area(s) is situated near the spin axis. The method is designed for analyzing observations of fans in comets that are positioned at larger heliocentric distances than the Earth, and he has determined the spin axis orientations for several comets - P/Encke, P/Tempel 2, Borrelly, P/Schwassmann-Wachmann 3, Pons-Winneke.

There are, as yet, few definitive checks on Sekanina's hypothesis. In the case of P/Tempel 2, there is consistency with the photometry of Jewitt and Luu (1989); in the case of P/Encke, there is disagreement (*cf.* Section 6.3) with the photometry of Jewitt and Meech (1987). Since Sekanina's method can be applied to many comets (fan-like comae are quite common), it is important that a definitive confirmation of his hypothesis be obtained in a few cases.

The evolution of the *shape* of near-nucleus features has been used in some cases to constrain determinations of nuclear rotation (Larson and Minton 1972, Rettig *et al.* 1987, Keller and Thomas 1988, Hoban *et al.* 1988). In some applications it is necessary to assume an outflow velocity for the coma material, and this is a major impediment to determining precise periods.

Physical modeling of cometary activity can, in principle, yield detailed information about the rotational state of the nucleus. It has not, however, given reliable results in those cases for which independent checks are available. This is presumably a reflection of the validity of the physical assumptions that must be made for the method to work. For comets where the rotational state is already known with some precision, physical modeling can be expected to be much more dependable, and its application in such cases may provide deep insights into cometary processes.

A case in which physical modeling has had success is that of P/Halley, where a combination of direct images of the nucleus and an application of the theory of rigid body rotation have yielded insights into the rotational state (Sagdeev *et al.* (1989) and Belton (1990)). Least-squares fits to the observed directions of the "long" nuclear axis allow the determination of the orientation of the spin axis as well as estimates of P_ϕ and P_θ. Assumptions have to be made about the ratio of moments of inertia if the direct images are used alone, but this can be avoided if independent data (*e.g.,* ground-based observations) constraining the values of P_ϕ and P_θ are combined into the solution.

3.4. IMAGING

Ground-based imaging of continuum or molecular emissions (*e.g.,* A'Hearn *et al.* 1986) provides a wealth of data about cometary processes and, when obtained as an extended time-series, contains detailed information on periodicities connected with rotation (*e.g.,* Bobrovnikov 1931, Celnik *et al.* 1988, Larson and Minton 1972, Larson and Sekanina 1985, Larson *et al.* 1987b, Sekanina and Larson 1984, 1986a, Samarasinha *et al.* 1986, Hoban *et al* 1988). Care must be exercised with images of phenomena associated with coma ions. In such cases, periodicities may have more to do with interactions of the comet with magnetic structures in the interplanetary medium than with nuclear rotation. The deduction of information about the rotational state from such images has been attempted by many authors - primarily for the case of P/Halley (*e.g.,* Whipple 1978, 1980, Schulz and Schlosser 1989, Sagdeev *et al.* 1986b, 1989, Samarasinha *et al.* 1986, Schlosser *et al.* 1986, Sekanina 1987b, Sekanina and Larson 1984, 1986b, Storrs *et al.* 1986, Watanabe 1988, 1989). However, there are no cases among these studies of a definitive, stand-alone, determination of the rotational state of a comet. In active comets, images provide an aspect of the phenomenon that is complementary to the information latent in photometric time-series. This technique should therefore be used in combination with other techniques for it to realize its full potential.

4. Search for Multiple Periodicities

Claims for the detection of multiple periodicities have been made so far only for P/Halley (Schutz and Schlosser 1989, Celnik and Schmidt-Kaler 1987, Sagdeev *et al.* 1989, Belton 1990). All of these results are controversial.

As we have seen, the typical gaps in ground-based data produces a rich spectrum of sidelobes around the primary periodicity and its aliases in periodograms, which serve to obscure weaker signals of other periodicities that may be present. So far, there have been no published attempts (except for P/Halley) to look further than the primary period, even though there are no technical impediments to this. Once the primary periodicity has been found, it is usually a simple matter to remove its contribution to the variability of the data in a "whitening" process. The pre-whitened data are then examined for further periodicities using the preferred analysis technique.

Using phase-dispersion minimization, Stellingwerf (1978) gives an example of multiple period determination in the lightcurve of a Cepheid variable. Reed and Welch (1988) describe a whitening process that uses least-squares fits of harmonic functions. Belton *et al.* (1981) describe the determination of multiple periods from photometric time-series data on Neptune using Fourier techniques. Finally, Belton (1990) describes the "whitening" process implicit in the WindowCLEAN algorithm.

5. Synodic and Sidereal Periods

The periods obtained from remote observation are apparent, or synodic, periods. They differ from the true sidereal periods by an amount that reflects the components of the rate of angular motion of the Sun and the Earth with respect to the object, in the direction of its instantaneous spin axis. For observations of cometary activity, the relative position of the Earth is presumably unimportant; only the relative motion with respect to the Sun matters. In the case of reflected light, both the motion of the Sun and the Earth are involved. For comets in principal axis rotation, the effects can easily be computed once the orientation of the spin axis is known. In the case of a nucleus in a SAM or LAM state, the instantaneous spin vector is no longer fixed, and methods for correcting for synodic effects have not been worked out.

An illustration of the approximate magnitude of the correction is given in Belton (1990): he finds for P/Halley that the correction is as much as ± 0.4 d for a 7.4 d period. The magnitude of the corrections are proportional to the square of the estimated period.

6. Individual Comets

Whipple (1982) gives periods for 47 comets derived by his zero-date method (*cf.* Section 3.2) and the direction of the spin axis for 7 comets. Sekanina (1981) lists 12 comets. A'Hearn (1988) has questioned the validity of these periods.

Less controversial (but nevertheless often riddled with conflict and uncertainty) are the results on the few comets whose "nuclei" have been studied by optical and IR photometry. P/Arend-Rigaux, P/Neujmin 1, P/Halley, P/Encke, P/Tempel 2, and P/d'Arrest. Comet IRAS-Araki-Alcock, which was intensely studied by a wide variety of techniques during its close passage to the Earth in 1983, should also be included in this group.

With the exception of P/Halley, P/Encke, and IRAS-Araki-Alcock, all of these comets are relatively inactive and the *assumption* that they are in a state of principal axis rotation is probably valid. However, caution should rule, since present day inactivity does not exclude the possibility that there could be residual nutation, still undamped, that was excited during an earlier phase of activity.

6.1. P/d'ARREST

Using Fourier analysis of a photometric time-series, Fay and Wisniewski (1979) found a period of 5.17 ± 0.01 hr for this comet. The photometric amplitude was 0.15 mag, which is believed to be the signature of the nucleus. Using his zero-date method, Whipple (1982) - see Whipple (1981) for an earlier estimate for the period - found periods of 6.7 or 7.9 hr - but defered to Fay and Wisniewski's value.

Leibowitz and Brosch (1986b) have reanalyzed the Fay and Wisniewski data with the intent of calculating the statistical significance of the period. In conflict with Fay and Wisniewski, they found a period of 1.3 hr and estimated a probability greater than 20% that the response was a chance occurrence.

6.2. P/NEUJMIN 1

Wisniewski *et al.* (1986) found periodic variability in this comet and proposed a periodicity of 25.34 hr. The lightcurve with this period has 4 maxima per period and was chosen because it minimizes the scatter about the mean curve.

Campins *et al.* (1987) found the thermal IR and optical lightcurves were in phase, indicating that the changing projected shape of the nucleus was probably responsible for the variability. Because the second harmonic (not the 4th) is more likely to be dominant, the rotational period is probably 12.67 hr as suggested by A'Hearn (1988). The projected axial ratio was inferred to be $\approx 1.45 : 1$.

Using Dworetsky's method, the charge-coupled device (CCD) photometry of Jewitt and Meech (1988) yielded a periodicity of 6.34 ± 0.05 hr, with a photometric amplitude of 0.5 ± 0.1 mag. This corresponds to a period of 12.68 hr.

6.3. P/ENCKE

In this comet, Jewitt and Meech (1987) observed periodic variability with a photometric amplitude >0.4 mag. in time-series CCD photometry when the comet was near aphelion. They adopted a period of 22.43 hr based on an analysis using Dworetsky's method. Jewitt and Meech found a projected axial ratio of 2:1 for this nucleus, and a strong phase coefficient reminiscent of the darkest asteroids. In a more recent study, Luu and Jewitt (1990) have discussed photometry obtained when the comet was near aphelion and have determined that the data could be satisfied by periodicities of $P = 15.08$ hr or $P/2 = 7.54$ hr, or $3P/2 = 22.62$ hr. The most likely value of the rotational period is 15.08 hr, since it is this value that gives a double-peaked lightcurve, expected if the nucleus was observed. This result, which appears to be secure, is inconsistent with the results of Whipple and Sekanina's (1979) precessional model. These latter authors found a period of 6.55 hr, using the zero-date method, and inferred a spin-up rate of 21 min/century.

Sekanina (1988d, 1988e) has followed the evolution of the orientation of the comet's sunward fan-like coma to find the orientation of the spin axis and believes that he has established the precession of the spin axis (1°/year) "beyond doubt." He also deduced that the sense of rotation is prograde. The pole position is in conflict with the photometry of Jewitt and Meech (1987), and Sekanina (1988e) has questioned their interpretation of the variability, basing his argument on the fact that the comet was *intrinsically* brighter at larger heliocentric distances in Jewitt and Meech's photometry. However, as Jewitt and Meech pointed out, this was probably the result of nothing more than a large phase coefficient expected of very dark surfaces - now considered the norm for cometary nuclei.

6.4. P/AREND-RIGAUX

Jewitt and Meech (1985) found periodicities of 9.58 and 6.78 hr in their limited data set on this comet and were not able to decide between them. These periods are obviously aliased by the diurnal sampling. The photometric range in their observations was ≈ 0.3 mag.

Wisniewski *et al.* (1986) proposed a period of 27.31 hr, which minimizes the scatter around a complex lightcurve that has four cycles per period and a photometric range of ≈ 0.6 mag.

Millis *et al.* (1988) showed that variability of the nuclear contribution to the lightcurve in the thermal IR was in phase with the optical variability, indicating that it is controlled by the shape of the object. The dominant periodicity in the lightcurve was determined to be 6.73 hr through an application of the phase-dispersion minimization method. This is interpreted as the second harmonic of the rotation period. The combination of IR and optical measurements yielded a visual geometric albedo of 0.028 and an effective radius of 5.15 ± 0.2 km. From the amplitude of the optical lightcurve, Millis *et al.* found a projected axial-ratio of 1.6 : 1.

All of the above data is explained by a dominant periodicity near 6.73 hr, which, as Millis *et al.* have pointed out, is probably the second harmonic of the rotational period of 13.47 hr.

6.5. P/GIACOBINI-ZINNER

Leibowitz and Brosch (1986b) employed a differential line-continuum photometric technique in their study of this comet. They found a periodicity of 9.5 hr and proposed that the variability, with an amplitude of ±0.2 mag., is the signature of the nucleus. The periodicity appears to refer to the second harmonic, since the lightcurve has but a single cycle for this period. If this is, in fact, the case, then the period is probably ≈19 hr. Leibowitz and Brosch believe that their technique smooths out the effects of global changes in brightness due to changing distances, flares, bursts, etc., leaving only the signature of the nucleus. They also noted that their results disagree with those of Sekanina (1985d). They suggest that the precessional model does not give unique solutions and that there are other possible solutions besides the one found by Sekanina.

Sekanina's (1985d) parameterization of this erratic comet is somewhat startling, if not extreme. He found the nucleus to have an axial ratio of 8.3:1 and a period of rotation of 1.66 hr.

6.6. P/TEMPEL 2

Using his zero-date method, Whipple (1982) found a period of 4.8 hr for this comet.

Jewitt and Meech (1988) found possible periodicities at 8.9±0.1 and 7.5±0.1 hr (obviously aliased by diurnal sampling) in variability that ranged over 0.3 mag. Since they were unsure that they had directly measured the nucleus, they did not claim a rotation period.

Wisniewski (1988) reported aperture photometry on this comet. He found a very regular two-cycle lightcurve with a range of 0.5 mag and a periodicity of 8.97 hr.

A'Hearn *et al.* (1989) made both optical and N-band (10μ) observations of this object. The IR and optical lightcurves are in phase, showing that the shape of the object is probably responsible for the variability. The primary periodicity, determined with the phase-dispersion algorithm, was found to be 4.45 hr., from which A'Hearn *et al.* deduced a rotational period of 8.9 hr. Application of the standard thermal model yielded a projected axial ratio of ≈ 1.9 : 1, a visual geometric albedo of 0.022, and an effective radius at maximum light of 5.9 km. A'Hearn *et al.* estimated an approximate size for the nucleus of 16 x 8.5 x 8.5 km.

Jewitt and Luu (1989) made a comprehensive photometric study of the comet while it was largely inactive at heliocentric distances between 1.4 and 4 AU. Using Dworetsky's "string-length" method, they found a periodicity of 8.95±0.01 hr. Their photometry is consistent with a spinning prolate nucleus with axes in the ratio 1.9:1:1.

Sekanina (1987b) has analyzed the orientations of fan-like coma in this comet and found that they are consistent with a well-defined spin axis at $(\alpha, \delta) = (147°, +55°)$. This result is consistent with the photometry of Jewitt and Luu.

6.8. P/HALLEY

In an early study, Whipple (1980) estimated the period of this comet to be near 10.3 hr, using his zero-date method.

Several groups of investigators (Sagdeev *et al.* 1986*a*, 1986*b*, Bertaux and Abergel 1986, Wilhelm *et al.* 1986, Kaneda *et al.* 1986*a*, 1986*b*, Vaisberg *et al.* 1986, 1987, Trotignon *et al.* 1987, Keller and Thomas 1988) associated with the *VEGA*, *Suisei*, *Sakegaki*, and *Giotto* missions have found that the nucleus rotates with a period of approximately 2.2 days.

However, ground-based and orbital time-series studies of ultraviolet, optical, infrared, and radio wave observations have shown strong evidence for a periodicity near 7.4 days (Millis and Schleicher 1986, Stewart 1987, Festou *et al.* 1987, McFadden *et al.* 1987, Meech and Jewitt 1987, Neckel and Münch 1987, Schleicher *et al.* 1986, Williams *et al.* 1987, Schloerb *et al.* 1987, Sterken *et al.* 1987, Colom and Gerard 1988). In several ground-based photometric investigations (West 1990, West and Jorgensen 1989, Belton *et al.* 1986, Leibowitz and Brosch 1986a, Cochran and Barker 1986, Kosai 1986, Sekanina 1985*a*, Morbey 1985) there is no convincing evidence for either a well-defined period at all, or a period near 2.2 days. Time-series of images of near-nucleus activity, or of features in the coma or tail, apparently show evidence for both 2.2-day (Sekanina and Larson 1984, 1986*a*, Schlosser *et al.* 1986, Celnik 1986, Larson *et al.* 1987*b*, Rettig *et al.* 1987) and 7.4-day (Samarasinha *et al.* 1986, Hoban *et al.* 1988, Watanabe 1988) periodicities. In other observational studies, the investigators (Beisser and Boehnhardt 1987, Larson and Sekanina 1987*a*, Celnik and Schmidt-Kaler 1987, Schulz and Schlosser 1989) concluded that both periods may be present simultaneously.

Several groups of investigators (Whipple 1983, Sekanina and Larson 1986*a*, Grun *et al.* 1986, Wilhelm *et al.* 1986, Samarasinha *et al.* 1986, Sekanina 1988a, Keller and Thomas 1988, Sagdeev *et al.* 1989, Belton 1990) have estimated the direction of the pole of rotation. In other studies (Sekanina and Larson 1984, Larson *et al.* 1987*b*, Keller and Thomas 1988), there is strong evidence, from both 1910 and 1986 data, that the sense of nuclear spin is "direct" with respect to the orbital motion and, from the curvature of near-nucleus jets, that the *total* spin vector is near 2 days. The sense of spin appears to have been confirmed by Vaisberg *et al.* (1986) through measurements, using the SP-1 dust detectors on the *VEGA* spacecraft, of mass dispersion in a jet emanating from the nucleus.

There have been several attempts (Sekanina 1987c, Julian 1987, Belton 1987, Wilhelm 1987, Festou *et al.* 1987, Kamel 1988, Möhlmann 1989, Sagdeev *et al.* 1989, Watanabe 1989, Abergel *et al.* 1989) to rationalize these periods in terms of the free nutational motion of a rigid body but these models either have been refuted (Smith *et al.* (1987), but see also Sekanina (1988a)) or have failed to satisfy ground-based observations (Belton 1990). The rotation is, at present, universally considered to be in an excited state, with advocates for both SAM and LAM. The contending models are illustrated in Fig. 3 and labelled with the proposed component periods.

Reviews of the details of many of the above determinations can be found in Sekanina (1988a) and Belton (1990).

At present, there is little consensus on the actual mode in which the nucleus of P/Halley finds itself. Divergent opinion appears to be rooted in uncertainties surrounding the validity of the assumptions that underpin the derivation of the periodicities that have been proposed.

It seems clear that further detailed discussion of the *VEGA/Giotto* images is needed, with a focus on the accuracy with which the orientation of the nuclear axis can be determined. Belton (1990) has claimed that there are several periodicities, alternate to the 2.2-d period, latent in the *VEGA/Giotto* observations, He has also questioned the reality of the 2.2-d periodicity in the few ground-based studies that have reportedly detected it.

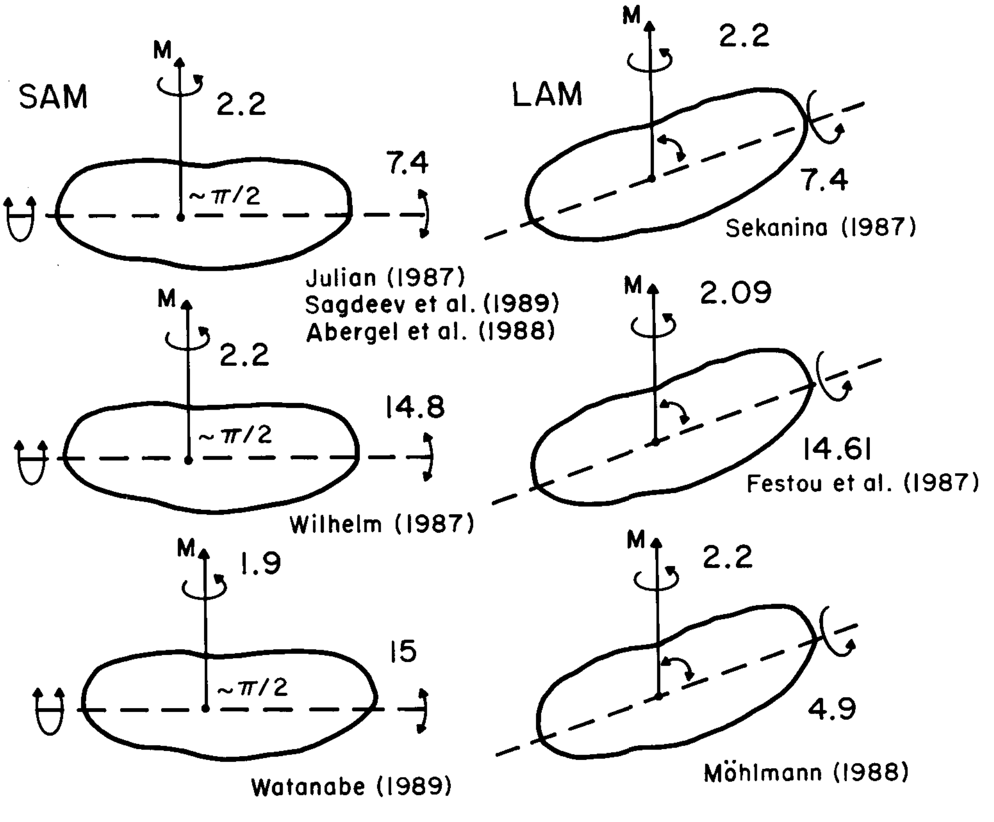

Fig. 3. Proposed models for the rotation of P/Halley. The component rotation periods are given in days.

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6.9. COMET IRAS-ARAKI-ALCOCK

This comet, which passed close to the Earth in 1983, was heavily studied with a wide variety of observational techniques. Sekanina (1988c) determined the spin vector by combining the observations of Feldman *et al* (1983), Whipple and Schild (1983), Goldstein *et al.* (1984), Storrs *et al.* (1986), and Watanabe (1987).

Sekanina found prograde rotation with a sidereal period of 2.14 d. This result is, with one exception, in agreement with the less precise estimates of the original investigators. Watanabe finds $18<P<170$ hr; Feldman *et al.* find $P>27$ hr, or $P\approx 2$ or 4 days; and Goldstein *et al.* find $1<P<2$ d. Only in the case of Whipple and Schild, who proposed 8.7 hr for the rotation period, is there a conflict.

Sekanina found the position of the spin axis in the direction $(\alpha,\delta) = (255°,-15°)$.

7. Comparisons

Whipple (1982) attempted the first comparison between the rotational properties of cometary nuclei and asteroids. He concluded, on the basis of a list of 47 periods, that the mean period of cometary nuclei, near 15 hr, was more than twice that of small ($\sim$ 1 km) asteroids and that cometary nuclei had a flatter distribution of periods. He interpreted this as an indication that the collisional history of cometary nuclei must have been much less extreme - perhaps they formed in a more quiescent region than was the case for the asteroids. He found no indication of spin-up with "age," but did find an indication that brighter comets had longer periods. Farinella *et al.* (1985) have performed a similar comparison and find no clear evidence to differentiate between asteroids and cometary nuclei on the basis of rotation alone. They compared periods of comets with those of 21 Apollo-Amor asteroids and 68 main-belt asteroids. No difference was found between cometary nuclei and the main-belt sample. However, they did find a difference with Apollo-Amors', which have a tail of fast rotators, unlike cometary nuclei.

Most of the above findings depend on the validity of the periods in Whipple's list and, as we have seen, where checks are available, the values of Whipple's periods have not been supported. Jewitt and Luu (1988) conclude, in their investigation of the relationship between photometric range and rotation period, that the number of reliable periods is presently too small to detect any significant correlations. A similar cautious view, based on similar grounds, has also been advocated A'Hearn (1988).

8. Summary and Conclusions

Precise knowledge of the rotation state of cometary nuclei is fundamental information for:

- Understanding the sources of cometary activity;
- Interpreting *in-situ* and remote observations of the coma and nuclear surface;
- Understanding the role of internal dissipation and gaining insights into the nucleus structure;
- Rationalizing cosmogonical problems associated with the evolution of nuclear shape, nuclear splitting, physical relationships to asteroids, and aging.

Although there are in excess of 60 reported rotational periods, only a few - P/Neujmin 1, P/Encke, P/Arend-Rigaux, and P/Tempel 2 - can be considered reasonably secure at this time. Even in these cases, information on the full spin state is incomplete. An adequate rotational ephemeris does not appear to be available for a single comet at this time. Particularly disturbing is the lack of a consensus regarding P/Halley's period, for which a vast amount of data exists. It would seem essential for confident future progress in this field that an early rationalization of the problem of P/Halley be accomplished.

The assumption that cometary nuclei are in their lowest rotational energy state has been undermined by the claim that the nucleus of P/Halley is nutating. The theoretical base for the interpretation of the lightcurves of nutating objects requires development, as does our theoretical understanding of the spin of episodically torqued, dissipative rotators.

The character of rotational studies of cometary nuclei has changed substantially in the few years since Whipple's 1982 review. Much of this has been due to the stimulus of activities surrounding the P/Halley encounters, the International Halley Watch, and the determination of NASA to pursue a Comet Rendezvous Asteroid Flyby Mission opportunity.

There now appears to be a clear sense that precise spin ephemerides will be key elements in understanding surface activity on cometary nuclei, and there is new promise for learning, in those cases where the nucleus is in an excited state, something about interior structure. It has become clear that the acquisition of photometric and radiometric time-series on comets, whether active or not, is of great importance to these studies. The greater emphasis that is being placed on photometric observations of periodic comets near aphelion, even though the comet is likely to be very faint, is clearly paying off, as are improved sampling and attempts to acquire extended observing runs.

Improvement is needed in techniques for data analysis. Particularly significant would be the use of a periodogram analysis technique that would allow sensitive searches for multiple periodicities. While extended photometric time-series can provide adequate precision in the determination of intrinsic periodicities, ambiguities latent in the interpretation of such data appear to demand that time-series imaging of the near-nuclear regions of comets, during their active phases, also be obtained. Since data on sequences of discrete events may be best analyzed with the zero-date method, or some allied technique, there is some urgency in gaining a deeper understanding of how the vagaries of observational sampling affect this method (Whipple 1982).

Finally, highly parameterized physical modeling of non-gravitational accelerations, cometary lightcurves, and nuclear geometry to yield rotational properties appears to give unreliable results. However, when applied to comets whose rotation is already understood, such modeling efforts may ultimately yield special and valuable insights into the nature of non-gravitational forces and the relationship between mass-loss and cometary lightcurves.

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