CATACLYSMIC VARIABLE STARS*

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Abstract. The properties of cataclysmic variable outbursts are reviewed, and interpreted in terms of dynamical instabilities by the red component, and subsequent evolution of the white dwarf accretion disc subject to a mass transfer burst in the transfer stream.

1. Introduction

During the past two decades many classes of eruptive star have been shown to be interacting binaries in which mass exchange onto an accreting, relatively compact companion is taking place. In these variable stars the energy liberated by accretion is commonly the dominant emission process in the system. The spectral region where the bulk of the flux is emitted depends on the mass and size of the accreting component, the rate of accretion, and the role of viscosity and angular momentum in generating an accretion disc. The cause of eruptions in the emitted flux has been ascribed to modulated accretion due either to dynamical instabilities in the cool, mass transfering star, or to intrinsic instabilities within the accretion disk.

A recent re-examination (Bath and van Paradijs, 1983) of the outburst behaviour of SS Cygni confirms that it is a relaxation oscillator. The behaviour is incompatible with simple disc-instability models, but in agreement with simple expectations of overflow instabilities by the companion. *G*, *K*, and *M* spectral-class companions are predicted to be susceptible to dynamical instabilities during mass exchange (Bath, 1975; Papaloizou and Bath, 1975). Time-dependent studies of the evolution of viscous discs indicate that the resulting overflow instabilities accurately model the eruptions of dwarf novae and certain symbiotics, and possibly also transient X-ray sources containing late spectral-type companions. The spectral evolution, eruption decay-rate and stream-impact behaviour are in accord with theories of bursting mass transfer by the companion (Mantle and Bath, 1983; Bath *et al.*, 1983; Bath, 1983). Intrinsic disc instabilities, due to the existence of multiple solutions to the local structure at ionization temperatures, do not normally produce outbursts, but rather a rapid oscillating wave which may be related to the observed flickering in dwarf novae light-curves, but does not appear related to the outbursts.

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2. Accretion as an Energy Source

Accretion, and the release of gravitational potential energy liberated by infall, plays a major role in theoretical studies of a whole range of astrophysical problems. Accretion is commonly invoked in many models of quasars and active galactic nuclei (Lynden-Bell, 1969; Lynden-Bell and Pringle, 1974; Abramowicz *et al.*, 1980) as the powerhouse which fuels these most energetic radiation sources in the Universe. It is also well established as the power source of X-ray sources (Shakura and Sunyaev, 1973; Novikov and Thorne, 1973; Rees, 1976) and, through accretion onto white dwarfs rather than neutron stars as the dominant energy generation mechanism in cataclysmic variables (Bath *et al.*, 1974; Bath and Pringle, 1981), though now only at luminosities of the order of the Sun.

The cataclysmic variable stars are well established interacting binary stars containing accreting white dwarfs (or in some cases Main-Sequence stars) encircled by angular momentum supported discs. In this review I shall describe the statistical properties of the outbursts of one of the best observed systems, discuss past work on the stability of semi-detached binary systems and recent work on disc structure in cataclysmic variables, and try to illustrate the rich rewards theoretical models may contribute to our understanding of stellar structure and of accretion disc physics.

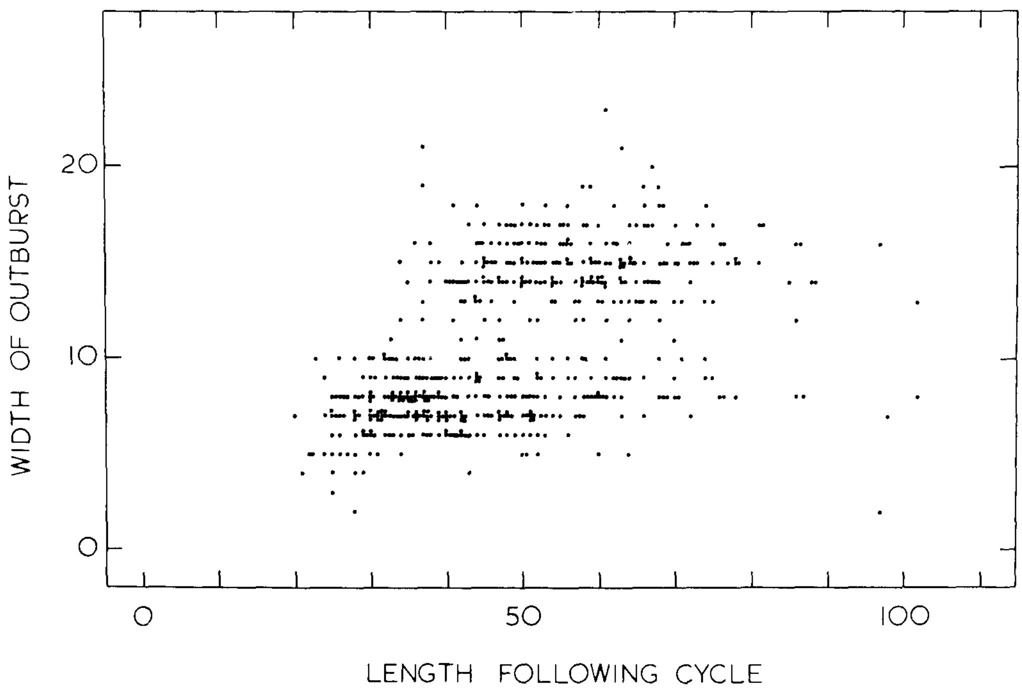

Fig. 1. Width-of-outburst vs outburst cycle-time of the following quiescent period. Note evidence for a relaxation cycle, and a bimodal distribution of burst widths.

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3. Outbursts of SS Cygni

Dwarf novae are commonly considered to conform to a relation between outburst amplitude and period. Payne-Gaposchkin (1977) pointed out that no striking relation for dwarf novae as a class exists but that there is a larger proportion of large ranges for longer cycles.

Within any individual system, the range of outburst behaviour permits one to search for a similar relation, but now obeyed by individual outbursts. The main question is then whether any correlation exists between the previous cycle-time and the outburst energy (storage process), or between the following cycle-time and outburst energy (relaxation oscillator). Examination of the light curve of SS Cygni indicates a clear correlation of the latter-type (Kruytbosch, 1928; Sterne and Campbell, 1934; Brecher *et al.*, 1977; Bath and van Paradijs, 1983). There is no correlation of the former type. The behaviour is illustrated in Figures 1 and 2 where outburst energy is measured by the outburst width in days at a magnitude level of 10.0 mag., and the outburst cycle-length is measured as the time between successive outburst rises at the same magnitude level. Covering the period 1897 to 1979, with a gap in the data between 1933 and 1940, the correlation coefficients are 0.516 ± 0.031 between width and following cycle-time but only 0.089 ± 0.047 between width and preceding cycle-time. The outburst behaviour is, on average, a relaxation process in which the energy liberated within the accretion disc at outburst is approximately related to the time which must elapse before a second outburst can occur.

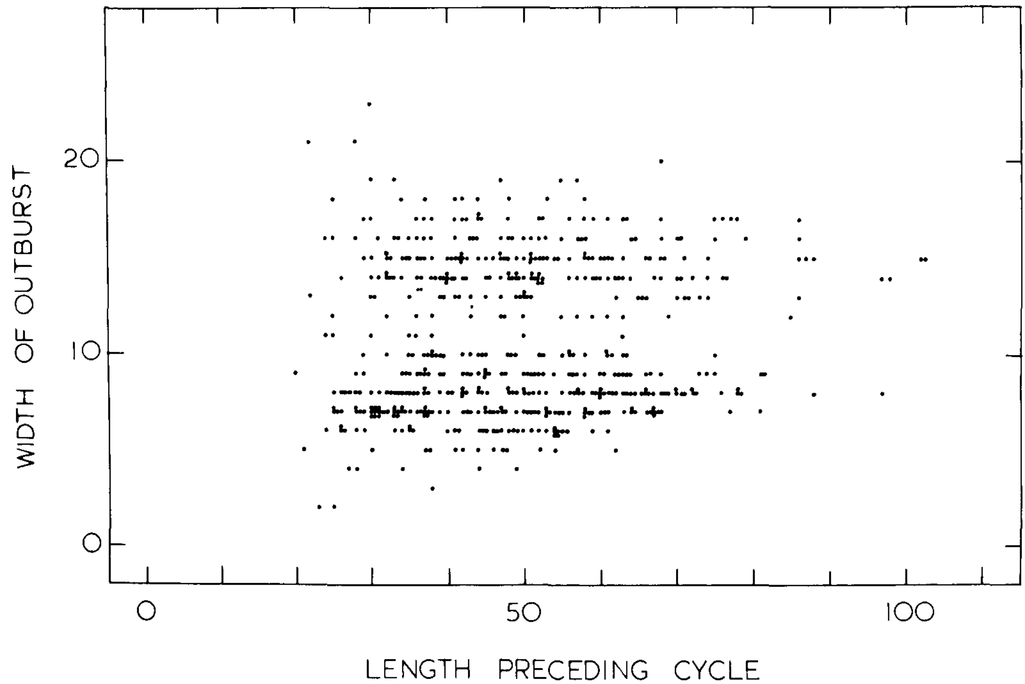

Fig. 2. Width-of-outburst vs outburst cycle-time of the previous quiescent period.

This is in direct conflict with the assumptions of simple disc instability models (e.g., Osaki, 1974). Osaki takes as his starting point the assumed existence of a correlation between outburst amplitude and cycle-length *of the preceding cycle*. Matter stored in the disc in the quiescent inter-outburst state is assumed to be dumped down onto the white dwarf in the succeeding outburst. The main assumption of this class of model is that the mass exchange rate is, at least roughly, constant. Then the preceding cycle time τ is related to the transfer rate $\dot{m}$ and the mass ΔM stored in time τ by

$$\tau = \frac{\Delta M}{\dot{m}} . \tag{1}$$

The outburst energy is

$$E = \frac{GM_1 \Delta M}{R_1} , \tag{2}$$

where the symbols have their usual meaning and the subscripts 1 refers to the white dwarf and 2 to the companion. Osaki points out that such a model obeys a period/amplitude relation, but the fact that it is not a relaxation process is not recognised. Osaki's relation is

$$\tau(\text{previous}) = \left(\frac{R_1}{GM_1 \dot{m}}\right) E . \tag{3}$$

An analogous relation to (3) can be derived for overflow instabilities of the type described by Bath (1977). Following an overflow instability driven by ionization zones in the red component, the envelope must relax and recover thermal equilibrium before a second instability can take place. The thermal relaxation time between outbursts is

$$\tau = \frac{\mathcal{R}\bar{T}\Delta M}{\Delta L} , \tag{4}$$

where $\bar{T}$ is the mean temperature of the thermally disturbed region suffering a luminosity deficit ΔL, of mass ΔM. The instability is confined to a region of scale-height dimensions z, in the vicinity of the zero-gravity tidal point (the inner Lagrangian point; Papaloizou and Bath, 1975). Therefore,

$$\Delta L \approx \frac{L_2 z^2}{R_2^2} \approx \left(\frac{\mathcal{R}\bar{T} L_2 R_2^2}{GM_2}\right),$$

i.e.,

$$\tau \approx \frac{GM_2 \Delta M}{L_2 R_2} . \tag{5}$$

On the average, ΔM will equal the mass transfered in the previous burst and models indicate that $\Delta L \simeq L_2$. Thus we obtain a cycle-time/outburst energy relation

$$\tau(\text{following}) \approx \left(\frac{M_2}{M_1}\frac{R_1}{R_2}\right)\frac{E}{L_2}\ . \tag{6}$$

Expression (6) is in quantitative agreement with the behaviour of SS Cygni. The known distance and orbital elements of SS Cygni allow E, M_1, and M_2 to be measured. If we assume white-dwarf and Main-Sequence structure for stars 1 and 2, respectively, then R_1, R_2, and L_2 can be estimated. The deduced range of outburst cycle times is 20–100 days, as observed.

4. Stellar Stability in Semi-Detached Systems

Two unique features of cataclysmic variables affect the stability properties of the mass-losing component. The first is the gravitational potential in which the star is distributed, and the second is its spectral class, or temperature.

The red component is well established to be filling its tidal, or Roche-lobe, and transfering material, both in the quiescent state and at outburst, toward the white dwarf companion. The stability of this star must be discussed in the appropriate potential, with the effect of the gravitational saddle-point at the inner Lagrangian point included. The turnover in potential leads to mass loss and the generation of a free-fall stream following expansion into contact with the Roche-surface. This condition of unbinding of matter at the inner $\mathscr{L}_1$ point is equivalent to the condition that the energy of escape of matter within the envelope (along the line of centes of the binary) is the energy required to lift it to the inner Lagrangian point and *not*, as in an isolated single star, the energy required to lift it to infinity. As Papaloizou and Bath (1975) have explicitly shown, the stability of the companion is severely affected by this major difference in the form of the potential surfaces as compared to an isolated star.

A star approaching filling it's Roche, or tidal, potential, is structurally affected by the presence of the companion. Perturbation of the configuration within the potential results in different behaviour of the eigenfunctions, as a consequence of weakening and eventual turnover of the gravitational field at the inner Lagrangian point. If the star is cool enough to possess, ionization zones, i.e., *G*, *K*, or *M* spectral class, then these can act to dynamically destabilize the envelope through the release of recombination energy. Destabilization is concentrated along the line of centres, affecting a region within at least a scale height of $\mathscr{L}_1$ (Bath, 1972, 1975).

A simply analogy with this problem is given by the following simple fluid model, shown in Figure 3. A cylindrical jar of water, area A, is suspended by elastic. A filled syphon, with an outlet bend as illustrated, is inserted until the water levels between the cylinder and the outlet are aligned. The water is analogous to the stellar envelope, with a 'leak' at a fixed point in the potential (analogous to the Roche potential and the inner Lagrangian point). The elastic, and its associated stored energy, is analogous to the influence of the ionization zones in the stellar envelope.

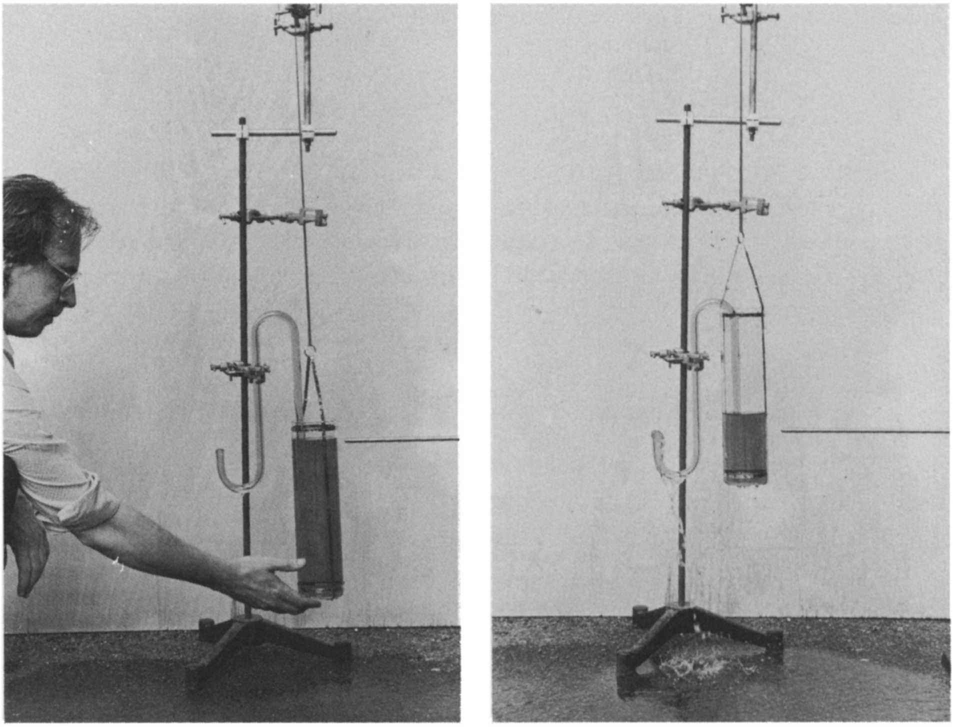

Fig. 3. Simple experimental demonstration of dynamical instability in a system with close analogy to a cool ($T_e < 10\,000$ K) star filling its Roche-lobe. The glass cylinder is suspended by elastic from the clamp at the top edge of the picture. Perturbation of the initial equilibrium configuration (left) leads to an exponentially growing instability (right) with water loss driven through the syphon by release of stored elastic energy. If the elastic is instead held rigidly by the central clamp, the elastic energy is insufficient to drive dynamical instability. This is analogous to the stability of hotter stars. In that case the cylinder oscillates until following viscous damping of the oscillations a new equilibrium with water level below the syphon outlet is achieved.

If the cylinder is perturbed upwards water will leak out of the syphon. The subsequent behaviour then depends on the stored elastic energy. If this is insufficient to lift the water over the gravitational potential the cylinder oscillates, and eventually settles down to a new equilibrium level with the water level below the end of the outlet.

However, if the stored elastic energy is sufficient, then the cylinder starts to rise, rising exponentially against gravity. Water is expelled with increasing force through the syphon outlet. The condition for instability is that if the associated elastic stretch length, x, is initially (in the equilibrium state) longer than the height of the water column, h, then the system is dynamically unstable to water loss. In practice a finite perturbation is required, but only in order to overcome surface tension effects.

The cause of instability is easily seen. If, following water loss from the syphon, and hence, the cylinder, the new equilibrium level of the water is higher in the Earth's gravitational potential than it was before water removal, then it will be above the syphon outlet, and further water loss will take place.

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The important factor is whether the water level rises or falls as water is poured into the cylinder. If the level falls as it is filled, the cylinder will be dynamically unstable to water loss. The process is closely analogous, both physically and mathematically, to the stability problem of semi-detached binaries. However, in stellar instabilities of the type experienced by cataclysmic variables the instability is eventually damped by the changing thermal conditions in the envelope. These are only restored to their initial state following energy transfer from the interior and a subsequent eruption may then take place. The energy transfer must be sufficient to refill depleted energy resources in the envelope, as described in the previous section.

In the stellar context the elastic property of the ionisation zones is an intrinsic property of the fluid. Their effect is also apparent in the context of Cepheid pulsations. Stellar pulsations are the equivalent of oscillations of a free cylinder with no syphon, or with the syphon continuing sufficiently above the outlet bend that there is no water loss following perturbation. Cataclysmic variables are a consequence of a leak through a 'hole' in the gravitational potential, equivalent to instability of the cylinder when there is a cut-off in the syphon.

5. Disc Structure and Evolution

The response of the accretion disc to sudden fluctuations in the mass transfer rate requires the development of time-dependent disc evolution models. The general evolution equation of a time-dependent, thin, viscous, axially-symmetric, Keplerian disc, is given by

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left(R^{1/2}\frac{\partial}{\partial R}\left(\nu\Sigma R^{1/2}\right)\right) + \frac{1}{2\pi R}\frac{\partial \dot{m}}{\partial R} + $$
$$+ \frac{1}{\pi R}\frac{\partial}{\partial R}\left(R\left(1 - \frac{R_k^{1/2}}{R^{1/2}}\right)\frac{\partial \dot{m}}{\partial R}\right). \tag{7}$$

The associated vertical structure may be described by one zone, vertically averaged, structure equations (Lightman, 1974; Bath and Pringle, 1981). In this expression ν is the vertically averaged kinematic viscosity; Σ, the surface density; $\partial\dot{m}/\partial r$, the rate of matter supply as a function of radius due to stream/disc collision; and R_k, the circular Keplerian radius at which the stream would orbit if there were no viscous angular momentum transfer within the disc.

The first term is the diffusion term describing matter and angular momentum redistribution through viscous stress. This term causes a disc configuration to be formed on a viscous evolution time-scale, in which matter spirals inward, losing angular momentum to more slowly rotating outer regions, and radiating the energy dissipated as a result of the viscous stress through the top and bottom of the disc. In α-viscosity discs ν is parametrized as $\nu = \alpha c H$. $\alpha \simeq 1$ is the largest value of the viscosity parameter compatible with subsonic turbulence, c is the sound speed, and H the disc thickness.

The second term describes the way in which matter is inserted into disc material already in Keplerian motion. $\partial\dot{m}/\partial R$ measures the rate of input of matter into the disc and the degree of stream penetration. This may be parametrized in terms of the momentum available in disc material for acceleration of stream material into disc orbits.

$$\frac{\partial\dot{m}}{\partial R} = \beta\Sigma\left(\frac{GM_1}{R}\right)^{1/2}. \tag{8}$$

If $\beta = 1$ all the momentum of disc material is absorbed by the stream and used in deflecting the maximum flux of matter out of the stream into the disc. $\beta = 1$ is an absolute upper limit on the stream injection rate. A more realistic upper limit, corresponding to equipartition of energy, is $\beta = 0.5$.

The third term describes the tendency of new material with angular momentum appropriate to a Keplerian orbit at radius R_k, to squeeze the disc into an annulus orbiting at that radius. R_k is less than the outer disc radius, R_{out}. In typical cataclysmic models $1 \times 10^{10} \lesssim R_k \lesssim 2 \times 10^{10}$ cm and $2 \times 10^{10} \lesssim R_{out} \lesssim 8 \times 10^{10}$ cm.

The inclusion of Cox–Stewart opacities leads to multiple solutions for disc structure at ionization temperature of $\simeq 10000$ K. In this region the kinematic viscosity is no longer a single valued function of the surface density, but shows a characteristic S-band with triple solutions. This leads to the possibility of limit cycle behaviour, with the structure oscillating between the upper and lower stable branches. The middle branch is unstable (Bath and Pringle, 1982).

The model described below allows such behaviour. In addition the ionization state of hydrogen and helium is computed assuming the validity of Saha's equation. The optically thin regions of the disc are computed using a time-dependent structural procedure similar to the approach of Tylenda (1981). No treatment of vertical convection is included, since the convective turnover times are typically longer than the viscous radial transport time, and convection is therefore inefficient.

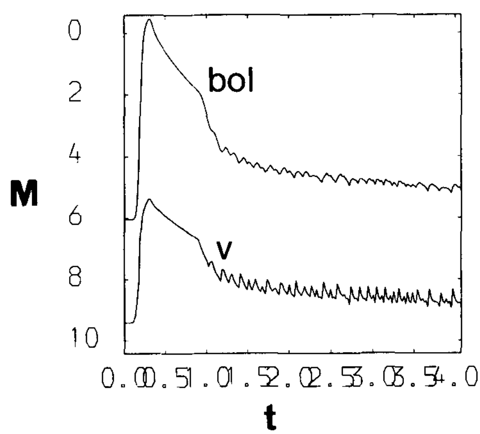

Fig. 4. Outburst bolometric and visual light curves of disc models with multiple solution regions giving disc instabilities (small oscillations) together with the normal outburst due to a mass transfer burst. t is the time in units of 10^6 s.

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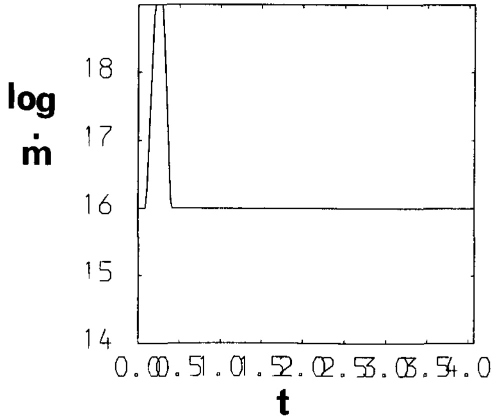

Fig. 5. Mass transfer burst assumed as initial input.

In Figure 4, the light curve of a typical model following a mass-transfer burst is shown, with burst form as in Figure 5. At outburst the whole disc is optically thick, hotter than 10 000 K, and the limit cycle instabilities disappear. They return after outburst, as the accretion flow dies down to the quiescent level, and the disc cools. A new unstable region then migrates back into the disc from the outside edge.

This change in structure is illustrated in Figure 6. Oscillations only appear with the reappearance of the optically thin exterior. Also shown in this figure are the positions (at increasing radius) of points with ionization fractions = 0.5 of He II, He I, and H. The depth of penetration of the stream, and associated shrinkage of the disc during the mass transfer burst is also shown.

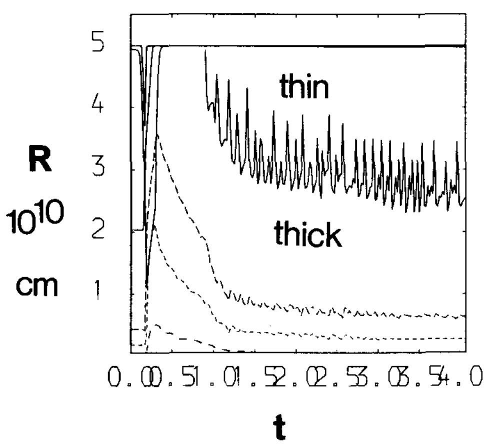

Fig. 6. Change in disc structure associated with the burst input. In the burst the disk shrinks to 3.8×10^{10} cm and the stream penetrates to 1.1×10^{10} cm. Following the burst the disc becomes optically thick out to the outermost radius at 5×10^{10} cm, and the unstable disc region is expelled. Later, following viscous evolution the optically thin outer region is re-established, and associated disc instabilities return. The positions of partial ionization of hydrogen, He I, and He II are shown by broken lines at successively smaller radii. Note that the disc instabilities lie on top of the global outburst behaviour introduced by the mass transfer burst.

We find that with our treatment disc instabilities do not produce eruptive behaviour but rather irregular oscillations with period of hours to days. The properties of the oscillations are determined by the assumption of axial symmetry. In realistic conditions in which large azimuthal changes in the shear, and hence, in the viscous stress, are produced due to deviations from circular motion, then the instabilities are likely to be confined to discrete local regions of the disc. It is tempting to speculate that the final effect may be produce rapid flickering of exactly the type observed in the quiescent state in cataclysmic variables.

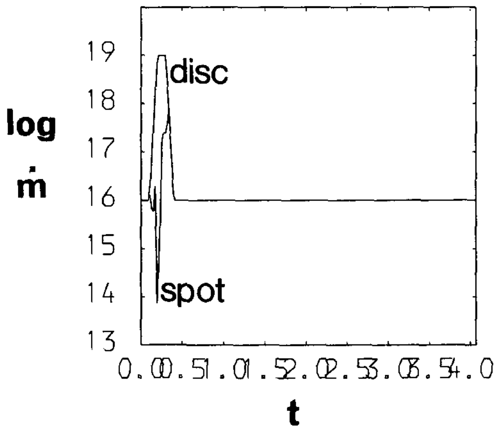

Fig. 7. The fraction of stream material, $\dot{m}$, which is stripped in the outer spot region, and that stripped deeper within the disc. Note the delay in the spot flux, similar to the observed luminosity delay.

Finally, in Figure 7, the fraction of the impact stream which is stripped at the hot-spot, on the disc edge, and the fraction stripped deeper within the disc is shown. The mass flux in the spot is responsible for the spot luminosity. It is clear that stream penetration effects cause a delay in the response of the hot-spot. It is not surprising, therefore, that the change in the hump in cataclysmic variables at outburst is not simultaneous with the outburst itself.

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