THE CRITICAL FREQUENCY IN THE STELLAR PULSATION THEORY

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Abstract: The standing oscillation in an isothermal semi-infinite plane-parallel atmosphere has a critical frequency. The significance of critical frequency in the stellar pulsation is discussed briefly. The critical period is calculated for the cepheid instability strip. A star having relatively large radius compared with the mass should be examined by the criterion. The pulsational instability is reduced by the linear running wave.

1. INTRODUCTION

The resonance theory of stellar pulsation is still hopeful to explain the bumps and multi-periodicity, even if the linear analysis has not succeeded in the agreement with the observed periods of double-mode cepheids (Petersen 1979, Takeuti and Aikawa 1980) and the non-linear dynamical calculation has not shown the double-periodicity yet (Simon, Cox, and Hodson 1980). We have to wait for the result of much more study on the response integral and the non-linear effect and/or the effect of damping forces on the resonance period. It seems important to study the all resonances expected theoretically comparing with the all observational features likely suggesting the resonances. For this reason the study on the RV Tauri stars is interesting. We should also study much more carefully higher overtones which have the possibility of interacting with lower-mode oscillations. It is necessary to study the response of atmospheric layers with the high frequency oscillations. A critical frequency appearing the oscillation in a semi-infinite plane-parallel atmosphere should be also studied. In the present note, I shall examine the critical frequency on the cepheid instability strip and the correct boundary condition in the case that the linear running wave exists. It seems interesting to take into account the existence of linear running wave for studying the variable stars having low value of the mass to radius ratio.

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2. CRITICAL FREQUENCY

Few of theoretical works on the critical frequency have been published. The standing wave of an isothermal semi-infinite atmosphere is permitted with the frequency less than a critical value. Periodic motion is possible only in the form of progressive waves with higher frequency. Rosseland (1949) pointed out the significance of the critical frequency in the study of pulsation theory. R.L.Simon (1957) obtained again the critical value for the periodic motion of plane-parallel atmosphere. Kamijo (1962) argued its role in the pulsation of long-period variable stars. The standing oscillation of an infinite spherical system has been also investigated by Simon (1957) and no critical frequency was obtained. Takeuti (1979) has tried the problem in the infinite spherical atmosphere and found that the case expressed analytically is restricted in Simon's case.

Simon (1957) has demonstrated that the total kinetic energy of oscillation becomes infinite for high frequency. Strictly speaking, the standing oscillation is never permitted with a frequency larger than the critical value, because an infinitely large energy is required to maintain the oscillation. Unno (1965) and Takeuti (1969) have studied the case including the non-adiabatic effect. The critical frequency is also obtained. The surface boundary condition studied by Baker and Kippenhahn (1962, 1965) is but a limiting case of atmospheric oscillations. Another limiting case corresponds to the oscillation including the progressive wave.

3. CRITICAL FREQUENCY ON THE CEPHEID INSTABILITY STRIP

We calculate here the critical frequency of atmospheres on the cepheid instability strip using the expression of atmospheric oscillation by Schatzman (1956). We put C and S as follows:

$$C = H_0/R, \tag{1}$$

$$S = \omega^2 R/g + 4 = 0.013/Q^2 + 4, \tag{2}$$

where H_0 is the scale height at the outermost atmosphere (the optical depth = 0 in the theory), R the stellar radius, ω the frequency of pulsation, g the surface gravity, and Q the pulsation constant in days. Then we have

$$1 - 4\alpha CS = 0, \tag{3}$$

for the critical case, where α is $(\partial \ell n\rho/\partial \ell nP)$. And we have

$$C = (0.000253/\mu)(T_0 R_s/M_s), \tag{4}$$

where T_0 is the temperature at the outermost layer in the units of solar value, μ the molecular weight, R_s and M_s the stellar radius and mass in solar units, respectively.

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We put here α, μ, and T_0 at solar values. The critical normalized period Q_c is a function of (M_s/R_s). Figure 1 shows the variation of Q_c with (M_s/R_s). The critical pulsation constant decreases with increasing (M_s/R_s) because H_0/R decreases for dwarf-like objects. To calculate the pulsation constant of model cepheids, periods calculated by Dr. Jørgen Otzen Petersen (unpublished) were used. Models were constructed by using the Cox-Stewart opacities (X = 0.7, Z = 0.02) with chemically homogeneous envelopes and the evolutionary masses. Pulsation constants for the fundamental mode, the first overtone, the second and third overtones are demonstrated on Figure 1. We must not concerned about the critical frequency for these four modes of classical cepheids. Even if the running wave does not appear in almost all lower-mode oscillations for classical cepheids, it seems useful to take into account the linear running wave in the study on the lower-mode of RV Tauri stars. Takeuti and Petersen (1980) have shown that the first harmonic oscillation of these variables fulfills the running wave criterion while their masses are nearly one solar mass which is suggested by the spectroscopic and photometric observations. In their paper, they have proposed that the strong progressive wave which appears with the time interval of a half period might be the linear running wave studied here.

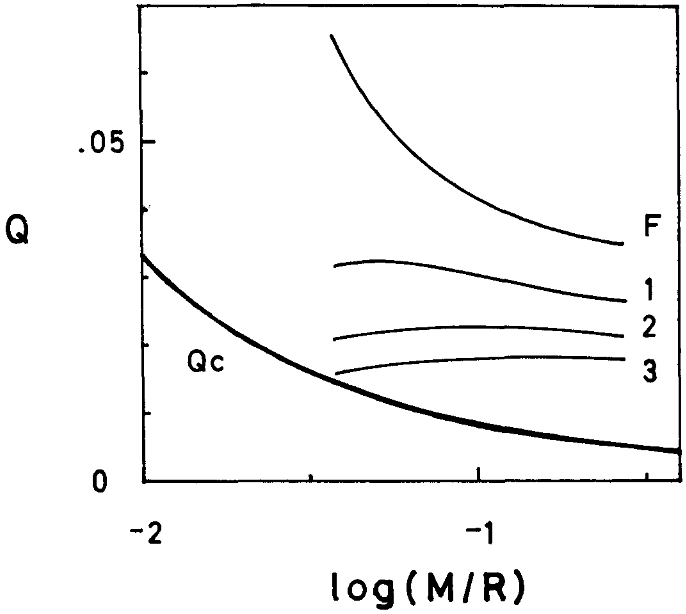

Figure 1. Pulsation constants for models on the cepheid instability strip and the normalized critical period. The heavy line indicates the normalized critical period Q_c and thin lines show pulsation constants for several modes.

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4. SURFACE BOUNDARY CONDITION

In case the linear running wave exists, the surface mechanical boundary condition is replaced by the following equation:

$$p = - 2S(1 - iE)x/(1 + E^2), \tag{5}$$

where

$$E = (4\alpha CS - 1)^{1/2} \tag{6}$$

for the case the linear running wave exists. This expression means that the phase difference appears between the displacement and the pressure variation. The work integral W at the stellar surface is not zero in this case. So, we estimate here the energy dissipation from the photosphere by the running wave using the formula,

$$W = 4\pi^2 P_0 r_0{}^3 \mathrm{Im}(p^*x). \tag{7}$$

Because we have

$$\mathrm{Im}(p^*x) = - 2SE|x|^2/(1 + E^2) \tag{8}$$

and $P_0|x|^2$ seems nearly constant throughout the upper atmosphere, the dissipation caused from the running wave may reach a considerable amount compared to the excitation under the photosphere while $\mathrm{Im}(p^*x) \gtrsim 10$. S is usually very large with small Q, therefore we can estimate that Im(p*x) is larger than 10 for many cases.

The effect of running wave on the pulsational instability has been studied carefully by Unno (1965) for the cepheid with unrealistic dense corona. The variable star with low value of (M_S/R_S) was not examined however in his paper. It is likely important to investigate the dissipation by the running wave in the star having low (M_S/R_S) value.

5. CONCLUSION

In conclusion, we remark first the fundamental mode, the first overtone, the second overtone, and the third overtone satisfy the criterion for the standing wave, on the cepheid instability strip. The fourth overtone and higher modes have to examined by the criterion for running waves even if for usual classical cepheids. The boundary condition for the standing wave is not adequate to apply on the variable stars with relatively small masses. The BL Herculis stars are also candidates which show the linear running wave studied here as well as the RV Tauri stars.

The solution of pulsation equation near the stellar surface is generally very sensitive to the change of frequency, therefore we may expect that the period is unchanged with the boundary condition adapted to the running wave. On the other hand, the damping of oscillation may increases in this case. It is possible that the excitation of higher overtone is restricted efficiently by this type of damping.

The strongly damped higher overtone may work as a sink of the energy of oscillation. It can make lower-mode oscillation more stabilize by the coupling with them.

It seems useful and important to take into account the effect of linear running wave on the pulsation theory.

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DISCUSSION

A. COX: I don't completely understand this. May I ask if one does the linear nonadiabatic solution with the Castor technique, do you actually get this effect of the damping?

TAKEUTI: I think almost all of the published linear results are safe with the standing wave boundary condition.

J. COX: What effect do these running waves have on the period?

TAKEUTI: I think that the period is not effected, just the stability. The period change is another problem.