Proceedings of the Edinburgh Mathematical Society (1992) 35, 167–168 ©

BOOK REVIEW

SHARP, R. Y., *Steps in commutative algebra* (London Mathematical Society Student Texts 19, Cambridge University Press, Cambridge 1990) pp. xii + 321, cloth 0 521 39338 8, £30, paper 0 521 39732 4, £10.95.

Commutative Algebra is a delightful and important area of Mathematics, drawing its inspiration from Geometry and Number Theory and having interwoven into its fabric connections with Topology, K-theory, Field Theory, Linear Algebra, and so on. It is a fruitful meeting place of many different ideas, it offers a range of beautiful results, some of them very deep, and, of course, it is still a buoyantly active area of research. The author writes in the Preface, "This book has been written to persuade more young people to study Commutative Algebra by providing 'stepping stones' to help them into the subject." He goes on to assert that many of the existing books on the subject are too sophisticated for present-day students, and his hope is that this book will provide a means of proceeding to more advanced texts.

The reader is expected to have done a basic course on Linear Algebra and to be familiar with the material covered in *Rings and fractions* by D. Sharpe (Cambridge University Press 1987). The pace is extremely leisurely and the presentation crystal clear. The author goes to great pains to explain details, sometimes giving (in different parts of the text) different explanations for the same point. Whenever there is the slightest possibility of confusion, a clarifying comment is made. The ordering of items is interesting, and I surmise that the introduction of some material is delayed because it is considered by the author to be more demanding.

The contents of the chapters are as follows: 1. Commutative rings and subrings, 2. Ideals, 3. Prime ideals and maximal ideals, 4. Primary decomposition, 5. Rings of fractions, 6. Modules, 7. Chain conditions on modules, 8. Commutative Noetherian rings, 9. More module theory, 10. Modules over principal ideal domains, 11. Canonical forms for square matrices, 12. Some applications to field theory, 13. Integral dependence on subrings, 14. Affine algebras over fields, 15. Dimension theory.

Some more details will show how the book proceeds. The first three chapters are very basic indeed, the most complicated theorem being the Prime Avoidance Theorem (page 56). Primary decomposition is worked through in a traditional way, and then Chapters 5 and 6 are also very basic. Chapter 7 has an introduction to Artinian and Noetherian rings and modules, with the Jordan–Hölder Theorem (page 140) for modules over a commutative ring. Hilbert's Basis Theorem (page 148) is in Chapter 8, together with Cohen's Theorem, Nakayama's Lemma and Krull's Intersection Theorem, the latter proved via Primary Decomposition rather than by the Artin–Rees Lemma which is not mentioned. Chapter 9 concerns modules over rings of fractions, but tensor products are not used, and primary decomposition for modules, associated primes of modules, etc. Chapters 10 and 11 are what you would expect, and Chapter 12 is an introduction to field extensions and transcendence degree. Standard results (Lying over, Going up, etc.) are given in Chapter 13. Hilbert's Nullstellensatz is in Chapter 14 on page 267 and Noether's Normalization Theorem on page 274, with the standard results about primes in polynomial rings deduced. Chapter 15 has Krull's Principal Ideal Theorem (page 289) and a discussion about systems of parameters in quasi-local rings and regular local rings. Various ideas play no part in the development, in particular, tensor products, projective, injective and flat modules, completions, Hom, Ext and Tor.

Does the book achieve its stated objective? Yes and no! I have already mentioned the clarity. On the positive side, I should also mention the clear type, the welcome lack of misprints (I can only remember one!), the good references in the text, the high degree of organisation of the material and the low price. On the other hand, I find the lack of any details about the origins of the ideas a miss. For example, Dedekind domains appear as an exercise on page 306, and the geometrical motivation behind Hilbert's theorems is not explained. "Why are local rings called 'local'?" is a perfectly fair question. The better student will find the repetition of detail somewhat tedious, I suspect, and really has to wait a long time to get his teeth into more meaty material. Given the level of sophistication he (or she) is expected to be able to exercise in other branches of Mathematics, he may conclude too early that there is not much depth in Commutative Algebra, and that may be a prejudice which is hard to dispel later! The moderate student, on the other hand, will find this book most helpful and a very gentle introduction to the subject. But is such a student ever going to read Matsumura?

PATRICK F. SMITH