Part II

Game Theoretic Semantics for the Language of Science

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On a Game-Theoretic Approach to a Scientific Language

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1. Introductory Remarks

This contribution gives an outline of a game theoretic foundation of the logical structure inherent to the language of a science. Game theoretic approaches to a language were considered and developed by several authors. In this volume Saarinen [10] examines the game theoretic semantics due to Hintikka and the dialog-game semantics due to Lorenzen. In the following I shall not re-examine the approaches by Hintikka [1] and Lorenzen [5]. However, some remarks about Lorenzen's semantics are necessary since the game theoretic approach considered here is essentially based on the idea of Lorenzen to use dialog-games for a foundation of logic.

1.1. The problem of the dialogic foundation of logic

A systematic game theoretic presentation of the rules of a dialog-game was given at first by Lorenz [3]. Starting with certain structural rules which constitute the general scheme of a dialog, a further dialog rule is necessary in order to guarantee a finite game. Three particular rules, each of which confines the possibilities of argumentation in a dialog, are distinguished: the 'strict' dialog rule, the 'effective' dialog rule and the 'classical' dialog rule. A formalisation of the dialog rules by means of calculi establishes the connection between the dialogic semantics and the usual formulation of logic by mathematical logic. While the 'strict' dialog rule leads to a new kind of logic, the 'effective' dialog rule leads to intuitionistic logic and the 'classical' dialog rule leads to classical logic. Within the dialogic approach of Lorenzen and Lorenz the problem of a foundation of logic precisely is the problem to distinguish particular dialog rules (i.e. the 'strict', 'effective', 'classical' dialog rule) among an infinite set of possibilities to confine a dialog-game. It depends on the choice of such a particular rule which kind of logic one obtains since, in the sense of the semantical completeness and consistency, the dialog-game and the logical

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calculus are equivalent. As a guiding principle in order to choose between the possibilities to confine a dialog, certain minimal and maximal properties of the game are considered to be decisive. But nonetheless it seems to be a matter of convention and not at all a necessity to select a particular dialog rule. It turns out that, in comparison with the 'strict' and the 'classical' dialog rule, the 'effective' dialog rule allows a maximal dialogical differentiation of compound propositions. The concept of truth, established by means of the 'effective' dialog-rule, is an extension of the concept of truth, established by means of the 'strict' dialog-rule, whereas the concept of truth, obtained by means of the 'classical' dialog-game, is a further extension which takes into account the particular hypothesis of the 'excluded middle'. It is argued by Lorenzen and Lorenz that because of the properties of maximal differentiation and the avoidance of the principle of 'excluded middle' (which can be introduced into the 'effective' dialog-game by means of appropriate hypotheses of the opponent) the 'effective' dialog rule is most appropriate as a basic dialog rule. In this way the intuitionistic logic is intended to be justified.

However, there is an argument that, following the very principle which distinguishes the intuitionistic logic, the priority of still another logic can be justified. In order to point out this argument I have to characterize the minimal and maximal conditions briefly which distinguish the 'strict', 'effective' and 'classical' dialog rule of Lorenzen and Lorenz. Whereas the 'strict' dialog rule only allows one attack by the proponent against the last argument of the opponent, which is a minimal possibility (and turns out to be too restrictive), the 'effective' dialog rule allows the proponent to choose a finite number of attacks against each argument, which is a maximal possibility. In both of the cases there is only one possibility to defend upon each attack. On the other hand, a maximal possibility of defence is granted to the proponent by the 'classical' dialog rule. One might think of still another possibility such that the possibility of attack is not maximal but restricted in a certain position of the game to particular arguments which are still 'available'. A similar restriction can be introduced with respect to the possibility of defence. A corresponding 'effective quantum' dialog rule and a 'quantum' dialog rule respectively are introduced in [11] and [12] in order to distinguish a logic for quantum mechanical propositions. The 'effective quantum' dialog rule allows a higher differentiation of propositions in a dialog-game than the three other dialog rules. The concept of truth, obtained by means of the 'effective quantum' dialog-game, is an extension of the concept of truth, obtained by means of the 'strict' dialog-game, whereas a second extension which takes into account the particular hypothesis of the 'unrestricted availability' leads to the concept of truth, established by means of the 'effective' dialog rule. The quantum mechanical propositional calculus which is a formalisation of the 'quantum' dialog-game is a well established structure of the language of quantum mechanics [2]. Its justification is usually based on the empirical conditions of quantum mechanical measurements as proof procedures for quantum mechanical propositions. By means of the conditions to measure quantum mechanical observables simultaneously, compound quantum mechanical propositions are defined and a quantum mechanical propositional system can be established. In this way the structure of the language of quantum mechanics is justifed as an appropriate possibility to describe the results of quantum mechanical measurements. In contrast to such an empirical justification of the quantum mechanical propositional calculus, the framework of the dialogic logic, as it was pointed out above and will be substantiated in the following, allows for a justification which is independent of the particular empirical conditions to prove propositions. The propositional calculus can be established as an autonomous logic, called quantum logic. The program of such a justification of quantum logic is due to Mittelstaedt [7].

Although a logic can be justified by means of a recourse to one or the other distinguished dialog rule, the problem of a foundation of logic exceeds such a justification. The further question concerning the foundation is how and in which sense is it possible to base a justification of the laws of logic on the rules of a dialog-game. In the framework of dialogic logic the universal validity of the laws of mathematical logic is lead back to the stipulation of certain rules within a dialog-game. But how can the necessity of such a stipulation of rules be understood?

In this contribution an approach towards a dialogic foundation of logic is presented which differs from Lorenzen's approach in some respect. The dialog-game which is distinguished by means of this approach is the 'effective quantum' dialog-game, established in [11] as a 'formal' dialog-game for quantum mechanical propositions. Its systematic location, as already pointed out, is "between" the 'strict' and the 'effective' dialog-game of Lorenzen and Lorenz. For the details of this game, its formal representation by means of logical calculi and its systematic relation to the usual caculi, I refer the reader to the articles ([11], [12], [8]). The main purpose of the following presentation of a game theoretic approach to a scientific language is to contribute to a better understanding of the stipulation of the dialog-game for a justification of logic.

The question is whether a certain part of the formal structure of a scientific language can be founded to be necessary independently of the particular field of phenomena to which the language refers. There is probably only one possibility to understand the necessity of a certain structure within a language of a science, which consists in a demonstration that this structure can be founded on the necessary conditions which make a language of a science and thus science itself possible. Such a demonstration makes use of an argument which in the traditional philosophy (following Kant) is called a transcendental argument. This argument presupposes the fact that there are scientific languages which possess formal structures. Being in the possession of scientific languages their systematic reconstitution may be considered. The point of view adopted here is that a reconstitution of the (formalized) structure of a language is only meaningful if it is based on the acts which are performed in the praxis of a language. The transcendental argument formulates conditions which must be satisfied by linguistic acts such that a reconstitution of a scientific language is possible at all. The preconditions of a scientific language, the necessity of which can be understood in the sense of the transcendental argument, already determine certain structures of a scientific language irrespectively of the phenomena which are described by it.

1.2. Outline of the Approach

According to the above programmatic remarks a reconstitution of the formal structure (logic) of a scientific language is attempted which formulates the conditions which linguistic acts (argumentations) must satisfy for this aim. These conditions constitute certain possibilities of argumentation which can be represented by means of rules in an argumentation-game. The most general frame of an argumentation which characterizes a scientific argumentation is represented by the frame-rules of a dialog-game (Section 2). In the initial position of the game the proponent states the initial argument. This argument may be doubted by an opponent whereupon the proponent is obliged to justify the initial argument. An initial argument is defined to be a proposition. If and only if an unambiguous argumentation can be performed with respect to a proposition, the proposition is said to be 'dialog-definite'. In this case the particular possibilities of argumentation must be determined by the proposition as a formal structure. According to the particular possibilities of argumentation given by a proposition, the connective structure of propositions is defined (Section 3). An argumentation is considered which decides after a finite number of possible steps whether the initial argument is justified (i.e., the proponent wins the game) or refuted (i.e., the opponent wins the game). Since the possibilities of argumentation about a logically connected proposition are infinite, a finite game is only possible under the condition of the existence of 'availability' and 'elementary' propositions (Section 4). These propositions are not 'dialog-definite' but assumed to be 'proof-definite' "outside" of the dialog. By means of availability and elementary propositions a finite dialog-game is established which is called the material dialog-game. Material truth and falsity of propositions are defined by means of win and loss in the material dialog-game (Section 5). If availability and elementary propositions are not presupposed to exist, a finite dialog which decides between material truth and falsity of propositions is not guaranteed. However, because of the dialog-definiteness of propositions, which comprehends the infinity of the possibilities of argumentation about a proposition, win and loss are potential decisions of a dialog in the potential infinity of the dialog-game. This makes the following consideration meaningful. Propositions are not necessarily presupossed to be dialog-definite by a finite argumentation. Nevertheless there are propositions which are certainly won by the proponent in the potential infinity of a dialog-game. This is the case if and only if the proponent has a strategy of success against all potential arguments of the opponent within the infinite dialog-game. In order to examine strategies of success within a dialog-game, which depend only on the connective structure (formal structure) of propositions, a new dialog rule (the 'formal' dialog rule) is introduced into the game. The resulting 'formal' dialog-game leads (this can only be stated here)

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to the 'effective quantum' dialog-game under a particular hypothesis. It represents the structural (formal) possibilities of argumentation which are necessary in order to establish strategies of argumentation. Its formalization leads to a calculus which is called the calculus of the 'formal' logic. Finally (Section 6), the relation of the formal logic to the intuitionistic and classical logic is discussed.

2. The Frame-Rules of the Dialog-Game

The concept of a dialog-game represents the most general frame of a scientific argumentation. Hence it formulates a necessary condition which must be satisfied by any argumentation which can be characterized as a scientific argumentation. Considering the praxis of scientific languages, one can establish the following methodological distinction of a scientific argumentation with respect to other possible methods of argumentation: Any assertion put forward within the course of a scientific argumentation, is open to doubts or counterassertions and, upon a doubt, must be substantiated in a certain way by a justifying procedure. This means that it must be possible to examine each step of the procedure whether it is performed correctly or not and, in case the procedure is finished, it must be possible to examine whether a justification of the assertion is obtained by it or not. In this way a sequential order of argumentative acts is established which consists of an assertion, a doubt or an attack against the assertion and a justification or a defence upon the attack. Utterances which are used as assertions, attacks and defences respectively are called arguments. This most general stipulation of the possibilities of a scientific argumentation can be formulated by means of rules of a dialog-game:

F1: At the beginning of the dialog, the proponent (P) asserts the initial argument. In this way the initial position of the dialog-game is established.

F2: After the assertion of the initial argument, an opponent (O) may attack this argument. Thereupon, the proponent is obliged to defend the initial argument against the attack.

F3: The dialog consists in a sequence of arguments which are assertions, attacks or defences of the two participants.

F4: If one of the participants cannot continue to put forth an argument, he loses the dialog. In this case the other one wins and the final position of the dialog is established.

The rules F1-F4 are called the frame-rules of a dialog since they constitute the concept of a dialog. By means of the frame-rules a dialog is determined as a two-person zero-sum game. The restriction of a scientific argumentation to a dialog is not essential but only a simplification of the game. Every potential opponent to an argument may be embodied in the fictitious opponent of the dialog.

The frame-rules do not yet determine an exhaustive definition of a dialog. It is not made precise which arguments may be used as assertions, attacks and defences and in which combination they may be used. However, it is presupposed that one understands what it means to assert, to attack and to defend by an argument. Therefore we say that the concept of a dialog is introduced insofar as its definitional frame is determined. Further specifications of the arguments which are possible in a dialog fill out the definitional frame to an exhaustive definition of a dialog-game. The exhaustive definition formulates precise concepts such that a dialog can be performed unambiguously.

Any assertion within a language for which a justifying procedure is defined is said to be a proposition. In this sense propositions are said to be proof-definite. In particular we have:

D1: A proposition is dialog-definite if and only if it can be asserted as an initial argument in a dialog (i.e., its constitutive justifying procedure is a dialog-game).

3. Compound Propositions

In order to carry out a dialog about an argument unambiguously, the possibilities to attack and to defend an argument must be specified. This can be done by setting up all possible attacks against an argument and all possible defences of this argument upon each of these attacks. In general, according to the praxis of scientific languages, such a specification concerns what in the traditional philosophy is called the 'content' of a proposition. In particular, a proposition might be composed out of subpropositions. Then, the composition is a purely formal structure of the proposition (i.e., the composition is independent of the content of the subpropositions).

Considering the dialogic reconstitution of a language, the concept of composition of a proposition can be established by determining the 'argumentative' possibilities of attack and defence in a dialog. By an 'argumentative' possibility of attack I mean a possibility of attack where the attack is either a doubt, which is an unattackable challenge to defend, or a proposition, which is an initial argument in a new dialog. By an 'argumentative' possibiliy to defend I mean a possibility to defend where the defence is a proposition, which again is an initial argument in a new dialog. Argumentative possibilities make use of possibilities "within" the dialog; they reduce the dialog to a sequence of subdialogs, whereas non-argumentative possibilites make use of possibilities "outside" of the dialog. Each combination of argumentative possibilities of attack and argumentative possibilities of defence defines a particular connective. In the cases that one, two and infinitely many propositions are involved in the combinations, we have the following complete scheme of logical connectives:

D2:

logical connective	logically connected proposition	possibilities of attack	possibilities of defence
position	$\vdash A$	?	A
negation (not)	$\neg A$	A	

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conjunction (and)	$A \wedge B$	1? 2?	A B
disjunction (or)	$A \vee B$	?	A B
material implication (if-then)	$A \rightarrow B$	A	B
	$A \leftarrow B$	B	A
	$A \prec B$	A ?	B
(but not)	$A \succ B$	B ?	A
(neither-nor)	$A \veebar B$	A B	
universal quantifier (all)	$\bigwedge_\alpha A_\alpha$	n?	A_n
existential quantifier (some)	$\bigvee_\alpha A_\alpha$	?	A_n
(no)	$\veebar_\alpha A_\alpha$	A_n	

α ranges over a constructive index set of the minimal power χ_0

It can easily be shown that all combinations which involve finitely many propositions lead to connectives which are dialogically equivalent (i.e., the argumentative possibilities are the same) to iterations of the above unary and binary connectives. The unary and binary connectives are, furthermore, dialogically equivalent to iterations of the negation, the conjunction, the disjunction and the material implication which represent, together with the universal and the existential quantifiers, a minimal base for logical connectives.

The following argument-rule determines the argumentative possibilities in a dialog:

A1: a) Any assertion of a subproposition is an initial argument in a new dialog (a subdialog) about the subproposition. If an attack consists in the assertion of a subproposition, the corresponding obligation to defend is postponed until a final position of the subdialog is established.

b) A negation $\neg A$ may be attacked by the assertion of the subproposition A. Upon the attack no defence is possible (i.e., if the subdialog about A is lost by the participant who asserted $\neg A$,

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he loses the dialog about $\neg A$ also).

A conjunction $A \wedge B$ may be attacked by a challenge to defend by a subproposition which is chosen by the participant who attacks. The corresponding obligation of defence consists in the assertion of this subproposition.

A disjunction $A \vee B$ may be attacked by a challenge to defend by a subproposition. The corresponding obligation to defend consists in the assertion of a subproposition which is chosen by the participant who defends.

A material implication $A \rightarrow B$ may be attacked by the assertion of the subproposition A. According to a) the obligation to defend, which consists in the assertion of the subproposition B, is postponed until a final position of the subdialog about A is established. In case the participant who attacked loses the subdialog, he loses the dialog about $A \rightarrow B$ also, since he cannot continue the dialog. In case he wins the subdialog, the second participant has to continue the dialog by the assertion of B.

A universal quantifier may be attacked by the challenge to defend by a subproposition which is chosen by the participant who attacks.

An existential quantifier may be attacked by the challenge to defend by a subproposition which is chosen by the participant who defends.

c) Attacks may be repeated unrestrictedly (i.e., if a proposition is defended upon an attack, this proposition may be attacked again).

If a participant who is obliged to defend is allowed to choose between several possibilities to defend (cf., the disjunction and existential quantifier) he may repeat the defence-arguments unrestrictedly (i.e., in case he loses the subdialog about a defence-argument he may choose a defence-argument again).

The order of attacks and the order of defences is arbitrary.

If an order among the possibilities of attack and the possibilities of defence is introduced in addition to the fundamental succession of attack and defence, this leads to the definition of <u>sequential connectives</u>:

D3:

sequential connective	sequentially connected proposition	possibilities of attack	possibilities of defence
sequential conjunction (and then)	$A \sqcap B$	1. 1? 3. 2?	2. A 4. B

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sequential disjunction (or then)	$A \sqcup B$	1. ?	2. A 3. B
sequential material implication (if-then)	$A \dashv B$	1. A	2. B
sequential universal quantifier	$\sqcap_{\alpha} A_{\alpha}$	1. 1? ⋮ ⋮ 2n−1. n? ⋮ ⋮	2. A_1 ⋮ ⋮ 2n. A_n ⋮ ⋮
sequential existential quantifier	$\sqcup_{\alpha} A_{\alpha}$	1. ?	2. A_1 ⋮ ⋮ n+1. A_n ⋮ ⋮

α ranges over a constructive index set of the power $\aleph_0$

The succession of possible arguments is numbered in the above scheme. Each argument may be put forth at most once. It can be shown that the sequential connectives, defined in the above scheme, represent a minimal basis of sequential connectives (i.e., all sequential connectives are dialogically equivalent to iterations of the above connectives). In a dialog about a sequentially connected proposition the argument rule A1 a)-b) applies.

Whereas sequential connectives are defined by a succession of possible arguments the definition of logical connectives depends on the attack-defence succession only, but not on an order among attacks or among defences. However, each dialog about a logically connected proposition consists in a particular application of the possibilities to attack and to defend and, thus, in a particular sequence of arguments. For a representation of a dialog it is convenient to write down the arguments of O and P in a pair of two columns. Each row gets a number. Arguments which are used as attacks are associated with an index (i), where i indicates the row in which the attacked argument stands. Arguments which are used as defences are associated with an index ⟨i⟩ where i indicates the row in which the defended argument stands.

An example of a dialog about a conjunction $A \wedge B$ is represented by the scheme:

	O			P	
0				$A \wedge B$	
1	2?	(0)	⟨0⟩	B	the subdialog about B is won by P
⋮	⋮			⋮	
m	1?	(0)	⟨0⟩	A	the subdialog about A is won by P
⋮	⋮			⋮	
n	2?	(0)	⟨0⟩	B	the subdialog about B is lost by P
⋮	⋮			⋮	
r					

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The subdialogs are assumed to be finite. In row r, P loses the dialog about $A \wedge B$ since he has no argument to continue the dialog. In case P wins the last dialog about B, O is allowed to continue by an attack against $A \wedge B$.

Consider a dialog about a disjunction $A \vee B$ which is for example:

	O			P	
0				$A \vee B$	
1	?	(0)	⟨0⟩	A	the subdialog about A is lost by P
⋮	⋮			⋮	
m			⟨0⟩	B	the subdialog about B is lost by P
⋮	⋮			⋮	
n			⟨0⟩	A	the subdialog about A is won by P
⋮	⋮			⋮	
r					

Since P wins the last subdialog about A he wins the dialog about $A \vee B$. In case he loses the last subdialog, P is allowed to continue the dialog by defending against the attack in row 1.

A dialog about a material implication $A \rightarrow B$ is for example:

	O			P	
0				$A \rightarrow B$	
1	A	(0)			the subdialog about A is lost by P
⋮	⋮			⋮	
m			⟨0⟩	B	the subdialog about B is won by P
⋮	⋮			⋮	
n	A	(0)			the subdialog about A is lost by P
⋮	⋮			⋮	
r			⟨0⟩	B	the subdialog about B is lost by P
⋮	⋮			⋮	
s					

P loses the dialog about $A \rightarrow B$ in row s since he loses the last subdialog about B. In case P wins the last subdialog, O is allowed to continue the dialog by a new attack against $A \rightarrow B$.

In order to use the dialog-game as a justifying procedure for compound propositions a criterion for the <u>truth</u> of a proposition must be given. The truth of a logically connected proposition should not depend on the particular choice of attacks by O in a dialog which is won by P. Considering the definition of logical connectives, it should be certain that the dialog about a logically connected proposition is won irrespectively of the particular sequences of attacks by the opponent. Thus we have the following definitions:

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D4: a) A proposition A is <u>true</u> if and only if P wins the dialog-game about A against all possible sequences of attacks by O.

b) A proposition A is <u>not true</u> if and only if P wins the dialog-game about A, which is stated by O as a subproposition in a dialog, against all possible sequences of attacks by O.

From the above examples of dialogs about logically connected propositions it is clear that the conditions which define the truth of propositions can be satisfied at all only in dialog-games about propositions which are composed by particular connectives. In a dialog-game about a disjunction or an existential quantifier, the conditions of truth are satisfied if one of the subpropositions is established to be true. In a dialog-game about a material implication $A \rightarrow B$ the truth conditions are satisfied if the truth of the subproposition A cannot be established. However, in all other cases of dialogs, as it can be seen most clearly in a dialog about a conjunction, the conditions of truth cannot be established by means of the purely argumentative possibilities of a dialog-game which we considered until now.

One might ask why we do not restrict the argumentative possibilities to sequential connectives only. In this case, the possible attacks of the opponent are determined by a particular sequence in a dialog such that a sequentially connected proposition is true if P wins the dialog about this proposition. But it can be seen that sequential connectives do not establish what is usually meant by a logical connective in the language of a science. If, for instance, a conjunction is used in a scientific language, its truth is not interpreted as any sequential truth of its subproposition but as a simultaneous truth of its subpropositions. Let us consider physical propositions of the kind $a(S,t)$: "The physical system S has the property a at the time t." Let us assume that the proof of such a proposition consists in a measurement of a with respect to the system S at the time t. A time metric is presupposed. The conjunction $a(S,t) \wedge b(S,t)$ cannot be established by a proof of $a(S,t_1)$ and a proof of $b(S,t_2)$ where the time interval $[t_1,t_2]$ (which includes t) is infinitesimally small, if the system S is a quantum mechanical system and a and b are not commensurable ([6],[7] p. 23). In the limiting case $t_2-t_1 \rightarrow +0$, the sequential proposition $a(S,t_1) \sqcap b(S,t_2)$ cannot be interpreted to be independent of the succession of the propositions $a(S,t)$ and $b(S,t)$ without leading to contradictions within the language of quantum mechanics. A conjunction $a \wedge b$ of quantum mechanical propositions can be established by a proof of a and a proof of b only if, in addition, a and b are commensurable, i.e., it is guaranteed that the proofs of a and b are reproduced in any sequence of measurements of a and b. This example shows that in scientific languages compound propositions are used, the proof conditions of which indeed cannot be reduced to sequential proofs of the subpropositions.

The truth conditions of the logically connected propositions, defined by means of D2, can be considered to be most general argumentative truth conditions with respect to compound propositions. However, as it was pointed out above, these truth conditions cannot be decided by purely argu-

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mentative means in the dialog-game. Yet, such logically connected propositions occur and are proven in scientific languages. Considering a dialogic reconstitution of the language of a science we are lead to the question how the truth of a logically connected proposition can be established in a dialog.

4. Availability and Elementary Propositions

From the discussion concerning the above dialogs about the logical connectives it can be seen that an additional argument is necessary in order to decide on the truth of propositions. If the proponent wins a dialog about a logically connected proposition the truth of this proposition can be established only by means of an argument which confirms that the dialog is won irrespectively of the particular sequence of attacks which are chosen by the opponent. This argument leads to the concept of availability of propositions. If, for instance, in the above dialog about the conjunction $A \wedge B$ it is guaranteed by an additional argument (an availability argument) that the win of the subdialogs about A and B is inherited by arbitrary sequences of the subdialogs, the dialog can be confined to only one attack by 1? and 2? each. In this case, the truth conditions of the conjunction are reduced to the truth of the two subpropositions and the truth of the additional availability argument which asserts that, once the dialogs about A and B are won, the win is available throughout any sequence of dialogs about A and B. Consider the above dialog about a disjunction $A \vee B$ where the first two subdialogs are lost by P. The truth of $A \vee B$ is established if in a finite, but unbounded, continuation of trials to defend by A and B the truth of a subproposition can be obtained. However, if this is known by an additional argument (an inavailability argument) the continuation of the dialog is redundant and the dialog is bounded.

The assertions of the two availability arguments, denoted by k(A,B) and $\bar{k}$(A,B) in the following, are made precise by the definition:

D5: a) k(A,B) states that each sequence of dialogs about A and B satisfies that all dialogs about A are won by the same participant and all dialogs about B are won by the same participant.

b) $\bar{k}$(A,B) states that a sequence of dialogs about A and B satisfies that a win or loss of a dialog about A or B by a participant at the beginning of the sequence is changed into a loss or win respectively of the repeated dialog at the end of the sequence.

The mere introduction of availability arguments does not lead to an extension of the dialog-game by means of which the truth of logically connected propositions can be established, unless we are in the possession of justifying-procedures for these arguments. The existence of proof-procedures, which determine k(A,B) and $\bar{k}$(A,B) to be proof-definite propositions, is a necessary condition in order to decide on the truth of logically connected propositions. Proofs of availability propositions cannot be performed by means of the structural possibilities of argumentation within a dialog, but only with respect to some special knowledge about the material 'content' of the propositions. In this sense we say that proofs of availability propositions are performed "outside" of the dialog. The justifying procedures for availability propositions can only be defined within the particular set-up of a scientific language. In a language for mathematics or in a language for classical physics availability propositions k(A,B) and $\bar{k}$(A,B) are formally true and formally false propositions respectively such that they do not occur in the languages explicitely. A scientific language in which the availability of propositions is not trivially satisfied is a language for quantum mechanics. Here, the availability propositions state the commensurability of quantum mechanical properties. Within the framework of quantum theory the commensurability is precisely defined by means of commutation relations between observables.

Considering the dialogic reconstitution of a scientific language, the necessity of the existence of availability propositions is understood without reference to the particular material set-up of a scientific language.

The proof of $\bar{k}$(A,B) is interpreted as a disproof of k(A,B) and a proof of k(A,B) is interpreted as a disproof of $\bar{k}$(A,B). If a disproof is defined for a proposition, this proposition is said to be <u>disproof-definite</u>. A disproof establishes the <u>falsity</u> of a proposition. k(A,B) and $\bar{k}$(A,B) are <u>counter-propositions</u>; this means:

k(A,B) true $\leftrightsquigarrow$ $\bar{k}$(A,B) false, $\bar{k}$(A,B) true $\leftrightsquigarrow$ k(A,B) false,

whereas

k(A,B) true $\rightsquigarrow$ $\bar{k}$(A,B) not true, $\bar{k}$(A,B) true $\rightsquigarrow$ k(A,B) not true.

If availability propositions exist they can be incorporated into the dialog-game by means of the additional argument-rule:

A_m2: a) If an availability proposition k(A,B) is asserted in a dialog it may be attacked by the argument k(A,B)?. The obligation of defence consists in a proof of k(A,B) which is performed outside of the dialog. If a proof of k(A,B) is established, this is indicated by the argument k(A,B)! in the dialog.

b) analogously for $\bar{k}$(A,B).

With the help of availability propositions and the argument-rule A_m2 the argumentative possibilities in a dialog about logically connected propositions can be confined to only one attack by each argument and to only one defence by each subproposition. In a dialog about a conjunction A ∧ B, the availability attack k(A,B)? may be used as a third attack:

	O		P		
0				A ∧ B	
1	1?	(0)	⟨0⟩	A	the subdialog about A is won by P
⋮	⋮			⋮	
m	2?	(0)	⟨0⟩	B	the subdialog about B is won by P
⋮	⋮			⋮	
n	k(A,B)?	(0)	⟨0⟩	k(A,B)!	

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In case P succeeds in defending against the three attacks he wins the dialog about $A \wedge B$, and the truth of $A \wedge B$ is established if the win of the two subdialogs leads to the truth of A and B.

A dialog about a disjunction $A \vee B$ obtains a third possibility to defend $A \vee B$ by the proposition $\bar{k}(A,B)$:

	O		P		
0				$A \vee B$	
1	?	(0)	$\langle 0 \rangle$	A	the subdialog about A is lost by P
⋮	⋮			⋮	
m			$\langle 0 \rangle$	B	the subdialog about B is lost by P
⋮	⋮			⋮	
n			$\langle 0 \rangle$	$\bar{k}(A,B)$	
n+1	$\bar{k}(A,B)$?	(n)	$\langle n \rangle$	$\bar{k}(A,B)$!	

In case P wins one of the dialogs about A,B and $\bar{k}(A,B)$, he wins the dialog about the disjunction $A \vee B$. $A \vee B$ is true if one of the propositions A,B, $\bar{k}(A,B)$ can be established to be true.

Also a dialog about a material implication $A \rightarrow B$ can be confined by means of the availability proposition k(A,B):

	O		P		
0				$A \rightarrow B$	
1	A	(0)			the subdialog about A is lost by P
⋮	⋮			⋮	
m			$\langle 0 \rangle$	B	the subdialog about B is won by P
⋮	⋮			⋮	
n	k(A,B)?	(0)	$\langle 0 \rangle$	k(A,B)!	

Whenever P wins the subdialog about B the opponent may attack $A \rightarrow B$ again by asserting A. However, if the availability of B is guaranteed, i.e., k(A,B) is true, P always can win the subdialog about B. If the truth of B can be established, the truth conditions are satisfied for $A \rightarrow B$.

It is obvious that the above possibilities in dialogs about logically connected propositions are equivalent to the possibilities in dialogs about particular sequentially connected propositions. The following argument-rule, which replaces the argument-rule A1, formulates this correspondence:

$A_m 1$:

logically connected proposition	sequentially connected proposition
$A \wedge B$	$(A \sqcap B) \sqcap k(A,B)$
$A \vee B$	$(A \sqcup B) \sqcup \bar{k}(A,B)$
$A \rightarrow B$	$A \dashv (B \sqcap k(A,B))$

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A dialog about a compound proposition is a nesting of subdialogs about the subpropositions. The concept of availability must recursively be applied with respect to the subpropositions in order to confine the subdialogs.

However, an exhaustive definition of a dialog-game can be given only if the nesting of the subdialogs is based on a set of elementary dialogs. This means that there must exist propositions which are not proven by means of a further dialogic argumentation, and that each compound proposition must be composed of these elementary propositions. It needs no further explication that scientific languages use elementary propositions as a basis for compound propositions and formulate justifying procedures for these propositions. Examples for elementary propositions in a language for mathematics are propositions of the kind $\vdash_K f$: "The figure f is deducible in the calculus K". The proofs consist in demonstrations of the deductions. The language for physics uses elementary propositions of the kind a(S,t), as formulated already above. The proofs of such propositions are performed by means of measuring processes, which are interpreted by means of the particular physical theory, and by means of deductions within the mathematical formalism of the theory.

For the dialogic reconstitution of a scientific language, the necessity of the existence of elementary propositions is understood, irrespectively of the particular material set-up of a scientific language.

The introduction of elementary propositions into the dialog-game is analogous to the introduction of availability propositions. The game is extended by means of the argument-rule:

$A_m 3$: If an elementary proposition a is asserted in a dialog, it may be attacked by the argument a?. The obligation to defend consists in a proof of a which is performed outside of the dialog. If a proof of a is established, the argument a! is put forth in the dialog.

5. The Material and the Formal Dialog-Game

Being in the possession of availability and elementary propositions, the dialog-game, established by the argument-rules $A_m 1$ to $A_m 3$, exhaustively defines a justifying procedure for logically connected propositions. This dialog-game is called the material dialog-game, and the concept of truth, which is constituted by it, is called the material truth:

$D_m 4$: a) A proposition A is materially true if and only if P wins the material dialog about A.
b) A proposition A is materially not true if and only if P loses the material dialog-game about A.

As an example we consider a dialog about the proposition $(a \rightarrow b) \rightarrow (c \vee d)$:

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	O			P
0				$(a \rightarrow b) \rightarrow (c \vee d)$
1	$a \rightarrow b$	(0)		
2			(1)	a
3	a?	(2)	⟨2⟩	a!
4	b	⟨1⟩		
5	b!	⟨4⟩	(4)	b?
6	k(a,b)!	⟨1⟩	(1)	k(a,b)?
7			⟨0⟩	$c \vee d$
8	?	(7)	⟨7⟩	d
9	d?	(8)	⟨7⟩	c
10	c?	(9)	⟨9⟩	c!
11	$k(a \rightarrow b, c \vee d)$?	(0)	⟨0⟩	$k(a \rightarrow b, c \vee d)$!

It is assumed that in row 9 P cannot prove the elementary proposition d. Hence, he cannot place the argument d! but has to continue the subdialog about $c \vee d$ by choosing c or $\bar{k}(c,d)$ as another defence. The win of the above dialog establishes the truth of the proposition.

In addition to the proof-procedures for elementary propositions we consider the possibility of disproof-procedures which define the <u>falsity</u> of elementary propositions. If elementary propositions a are disproof-definite, their counter-propositions $\bar{a}$ can be defined such that

$$\bar{a} \text{ true} \leftrightsquigarrow a \text{ false},$$

and

$$a \text{ true} \leftrightsquigarrow \bar{a} \text{ not true}.$$

The argument-rule $A_m 3$ applies also for counter-propositions $\bar{a}$.

The dialog-game, as a proof-procedure, decides on the truth of propositions; but it does not decide on their falsity, since the falsity of a proposition is defined by means of a disproof-procedure. The negation $\neg A$ is true if and only if the participant who attacks loses the subdialog about A (i.e., A is not true). If A is not true, this does not imply the falsity of A in general. In the following, however, the dialog-game with respect to a negation $\neg A$ will be extended such that the win of the dialog about $\neg A$ (by the participant who asserted $\neg A$) establishes the falsity of A. If a disproof-procedure exists for A, a counter-proposition $\bar{A}$ can be defined which is proved if and only if A is disproved. In this case we stipulate the additional argument-rule:

$A_m 4$: A participant who asserted a negation $\neg A$, may give up the subdialog about the attack A. In case he gives up or wins the subdialog about the attack A, he is obliged to prove the counter-proposition $\bar{A}$ outside of the dialog.

By means of the material dialog-game and $A_m 4$ we define:

$D_m 4$ c) A proposition A is <u>materially false</u> if and only if P wins the material dialog-game about $\neg A$.

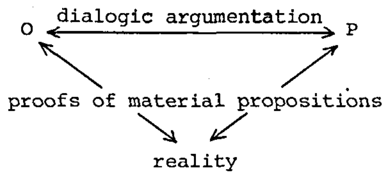
For example, a material dialog about the negation $\neg a$ of an elementary proposition a reads:

	O		P	
0				$\neg a$
1	a	(0)		
2			(1)	a?
3			⟨0⟩	$\bar{a}$
4	$\bar{a}$?	(3)	⟨3⟩	$\bar{a}$!

With the help of $A_m 4$ the negation $\neg A$ satisfies:

$$\neg A \text{ true} \leftrightsquigarrow \bar{A} \text{ true} \leftrightsquigarrow A \text{ false}.$$

The material dialog-game is founded by means of the conditions under which the truth of compound propositions can be established in a dialogic procedure. Availability propositions, elementary propositions and counter-propositions confine the argumentation insofar as they may be stated in a dialog but their proof procedures are performed outside of the dialog. These material propositions are not dialog-definite (i.e., justified by means of the argumentative possibilities in a dialog) but they are justified by means of their particular material 'content' which refers to the conception of reality formulated by a scientific language.



If we restrict our consideration to the dialogic argumentation between P and O (not including the proofs of material propositions), the truth and the falsity of propositions cannot be decided. However, even under the conditions of the purely structural rules of the dialog, one can distinguish propositions which would certainly be true if a material dialog-game were performed and if the opponent were able to decide on the truth and on the falsity of material propositions. The truth of such propositions does not depend on the particular material set-up of the propositions; it is guaranteed only by the formal structure of the propositions.

Consider for instance the dialog-game about the proposition $A \rightarrow A$. Upon the opponent's attack by the assertion of A, a subdialog is performed about A. The proposition A is dialog-definite (i.e., an unambiguous dialog can be performed about A) only if A can be decomposed into elementary propositions and if availability propositions exist according to the material argument-rule $A_m 1$. In case A is not true, the proponent wins the dialog about $A \rightarrow A$. In case A is true, the proponent defends by

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the assertion of the same subproposition A. Within the subdialog about A P is allowed to make use of the arguments of O within the preceding subdialog. As an example for A let us consider an elementary proposition a. Then we have the dialog:

	O		P	
0				$a \rightarrow a$
1	a	(0)		
2	a!	⟨1⟩	(1)	a?
3			⟨0⟩	a
4	a?	(3)		

It is assumed that O can prove the elementary proposition a in row 2. For instance, if a is a mathematical proposition of the kind $\vdash_K f$: "The figure f is deducible in the calculus K", and K is undecidable, it is assumed that O knows a deduction of f. If it is guaranteed that the proof of a can be reestablished again in row 4, P certainly can defend by a!. This condition implies that the availability proposition k(a,a) is true. Hence, P wins the dialog about $a \rightarrow a$.

Another example is a dialog about the material implication $(\bigwedge_\alpha A_\alpha) \rightarrow (\bigwedge_\alpha A_\alpha)$

	O		P	
0				$(\bigwedge_\alpha A_\alpha) \rightarrow (\bigwedge_\alpha A_\alpha)$
1	$\bigwedge_\alpha A_\alpha$	(0)		
2	A_i	⟨1⟩	(1)	i?
⋮	⋮			⋮
m	A_k	⟨1⟩	(1)	k?
⋮	⋮			⋮

In case proposition $\bigwedge_\alpha A_\alpha$ can be proved by O, i.e., all subpropositions A_k are true, P is obliged to defend by the assertion of the same proposition $\bigwedge_\alpha A_\alpha$. Upon any attack k?, P can take over the opponent's proof of A_k, again under the condition that A_k is still available. P then possesses a strategy of success in the continuation of the dialog.

Propositions which can be justified to be true only because of their formal structure are called <u>formally true</u> propositions. In order to obtain a complete survey about formally true propositions, a new dialog-game is introduced which takes into account the above conditions for the formal truth of propositions. In this way the <u>formal dialog-game</u> [11] can be established. The argument-rules of the <u>formal dialog-game</u>, which cannot be justified in detail here, may be characterized in the following way:

(1) Elementary propositions must not be attacked. O is allowed to state elementary propositions unrestrictedly. P is allowed to state elementary propositions only if they have been asserted by O previously and if they are still available.

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(2) Availability propositions k(A,B), which state the availability of the proposition A after the succeeding proposition B, are treated like elementary propositions. However, there are availability propositions which can be shown to be formally true and which may be asserted by P unrestrictedly. Availability propositions can be eliminated in the formal dialog-game. The formal truth of k(A,B) is equivalent to the win of the formal dialog-game about the compound proposition $A \rightarrow (B \rightarrow A)$.

(3) Counter-propositions $\bar{A}$, which state the falsity of the proposition A, are considered like elementary propositions. There are counter-propositions $\bar{A}$ which can be shown to be formally true since they prove A to be a formal contradiction, and which may be asserted by P unrestrictedly. Counter-propositions too can be eliminated in the formal dialog-game. The formal truth of $\bar{A}$ is equivalent to the win of the formal dialog-game against the opponent's proposition A. This equivalence is established by extending the argument-rules to the rules D3 and D4, used by Lorenz ([3], [4]); also cf., the rule F4 in [11]).

(4) Formally true propositions exist only if the following condition (which is already taken into account in the above examples and in (1)-(3)) is satisfied: Win and loss of a dialog about a material proposition is inherited by arbitrary repetitions of the dialog.

By means of the formal dialog-game we obtain:

D_f4 a) A proposition A is <u>formally true</u> if and only if P has a strategy of success for A in the formal dialog-game, i.e., P wins the dialog-game about A irrespective of the arguments of O.

b) A proposition A is <u>formally false</u> if and only if P has a strategy of success against A in the formal dialog-game, i.e., if O asserts A, P wins the subdialog about A irrespective of the arguments of O.

The formal dialog-game can be replaced by the following calculus L of <u>formal logic</u> such that:

$$\vdash_L A \leq B \quad \leftrightsquigarrow \quad A \rightarrow B \text{ formally true.}$$

(L1.1) $A \leq A$
(L1.2) $A \leq B \;\&\; B \leq C \Rightarrow A \leq C$
(L2.1) $A \wedge B \leq A$
(L2.2) $A \wedge B \leq B$
(L2.3) $C \leq A \;\&\; C \leq B \Rightarrow C \leq A \wedge B$
(L2.4) $\bigwedge_\alpha A_\alpha \leq A_n$
(L2.5) $C \leq A_n \Rightarrow C \leq \bigwedge_\alpha A_\alpha$ (n does not appear in the conclusion)
(L3.1) $A \leq A \vee B$
(L3.2) $B \leq A \vee B$
(L3.3) $A \leq C \;\&\; B \leq C \Rightarrow A \vee B \leq C$
(L3.4) $A_n \leq \bigvee_\alpha A_\alpha$
(L3.5) $A_n \leq C \Rightarrow \bigvee_\alpha A_\alpha \leq C$ (n does not appear in the conclusion)

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(L4.1) $A \wedge (A \rightarrow B) \leq B$
(L4.2) $A \wedge C \leq B \Rightarrow A \rightarrow C \leq A \rightarrow B$
(L4.3) $A \leq B \rightarrow A \Rightarrow B \leq A \rightarrow B$
(L4.4) $A \leq B \rightarrow A \;\&\; A \leq C \rightarrow A \Rightarrow A \leq (B \wedge C) \rightarrow A$
(L4.5) $B \leq A_n \rightarrow B \Rightarrow B \leq (\bigwedge_\alpha A_\alpha) \rightarrow B$ (n does not appear in the conclusion)

(L5.0) $\Lambda \leq A$
(L5.1) $A \wedge \neg A \leq \Lambda$
(L5.2) $A \wedge C \leq \Lambda \Rightarrow A \rightarrow C \leq \neg A$
(L5.3) $A \leq \neg A \rightarrow A$
(L5.4) $\left.\begin{array}{l} A \leq B \rightarrow A \;\&\; A \leq \neg B \rightarrow A \\ A \leq C \rightarrow A \;\&\; A \leq \neg C \rightarrow A \end{array}\right\} \Rightarrow A \leq (B \vee C) \rightarrow A$
(L5.5) $\left.\begin{array}{l} A \leq B \rightarrow A \;\&\; A \leq \neg B \rightarrow A \\ A \leq C \rightarrow A \end{array}\right\} \Rightarrow A \leq (B \rightarrow C) \rightarrow A$
(L5.6) $B \leq A_n \rightarrow B \;\&\; B \leq \neg A_n \rightarrow B \Rightarrow B \leq (\bigvee_\alpha A_\alpha) \rightarrow B$
(n does not appear in the conclusion)

This result cannot be demonstrated here. The calculus L is obtained analogously to the procedure in ([11], [12]). However, L differs from the calculus of effective quantum logic Q_{eff} cf., [12], p. 363) with respect to some rules concerning propositions $A \rightarrow (B \rightarrow A)$, since the dia--logic semantics of Q_{eff} involves an additional rule for availability propositions which is satisfied for quantum physical propositions.

6. Connection to Logical Calculi

The calculus of effective quantum logic Q_{eff} can be established if the rule:

$$A \leq B \rightarrow A \Rightarrow A \leq \neg B \rightarrow A$$

is added to the calculus L. This additional rule is justified by means of the dialogic semantics if the following particular condition with respect to availability propositions is satisfied: If the availability proposition k(A,B) is true under the condition that A and B are true, then the availability proposition k(A,B) is true also under the condition that A is true and B is false.

For quantum mechanical propositions A,B and their commensurability propositions k(A,B) the above condition means: If a measurement of the property B with the result "true" is commensurable with a measurement of the property A with the result "true", then also a measurement of B with the result "false" is commensurable with a measurement of A with the result "true". This condition, which corresponds to the possibility of 'perfect measurements' [8], is necessary in order to interpret the property B by means of a four-element Boolean algebra $\{0, B, \neg B, 1\}$.

The calculus of (full) quantum logic Q (cf., [12], p.374) can be obtained from Q_{eff} by the additional axiom of the 'excluded middle':

$$V \leq A \vee \neg A \;.$$

This axiom can be justified if the condition of the value-definiteness

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(i.e., the decidability of the proof procedure) is satisfied. In the case of quantum mechanical propositions this condition corresponds to the possibility of 'yes-no measurements' which decide between the truth and falsity of the propositions.

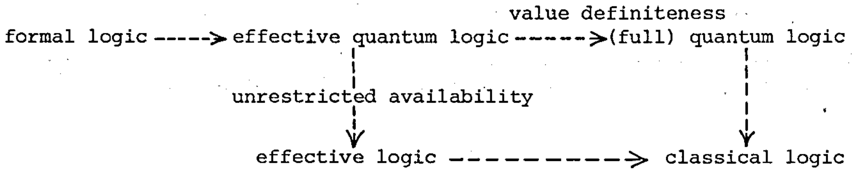
The algebraic representation of the calculus of (full) quantum logic by means of the Lindenbaum-Tarski algebra is an orthocomplemented quasimodular (= orthomodular) lattice. This lattice is well-known for an axiomatic formulation of abstract quantum theory [2]. A detailed presentation of the dialogic quantum logic is found in [7].

The dialogic logic of Lorenzen and Lorenz can be reestablished if the additional axiom of the 'unrestricted availability':

$$A \lesssim B \to A$$

is added to the calculi Q_{eff} and Q respectively. This axiom is justified in the framework of the dialogic semantics if availability propositions k(A,B) can be established to be always true.

The connection between the calculi is summarized in the scheme:



The formal logic seems to be a universal structure of a formal scientific language which can be founded by the argumentative preconditions of a scientific language independently of the empirical content of propositions. Particular empirical conditions like the unrestricted availability and the value-definiteness of propositions, in case they are confirmed within a scientific praxis, lead to distinguished logical calculi. However, their necessity is not understood as a structure of argumentation but is confirmed as a structure of reality which is concerned by a scientific language.

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