Science in Context (2022), **35**, 426–437
doi:10.1017/S0269889724000127

RESEARCH ARTICLE

A dangerous preposition: The boundaries of mathematics

Alma Steingart

Department of History - Columbia University
Email: as2475@columbia.edu

Argument
This essay takes inspiration from Herbert Mehrtens' 1995 claim that the "history of mathematics is an integral part of intellectual history." It asks why, despite the tremendous transformation the historiography of mathematics has undergone since the field began to professionalize in the 1970s, historians of mathematics repeatedly complain that their field has been marginalized. The answer, I suggest, is not due to the fact that mathematics is less amenable to the types of social, cultural, or material analysis that came to dominate the history of science in recent decades. Rather, at issue is that most historians have adopted mathematicians' own definition of mathematics. The history of mathematics professionalized at a particular moment, one in which mathematicians were concerned about the limits and boundaries of their field. As such, they were invested in drawing boundaries around the "proper" confines of the field. Historians of mathematics have followed suit, and similarly restricted what did and did not belong in the history of mathematics. Following Mehrtens, however, and insisting that the history of mathematics is an integral part of broader intellectual history, a more capacious conception of the field is possible.

Keywords: Historiography; Herbert Mehrtens; professionalization; pure mathematics

In 1992, Herbert Mehrtens wrote an appendix to a paper he had published sixteen years earlier about the applicability of Kuhn's theory to the history of mathematics. Mehrtens' appendix was published as part of an edited volume entitled *Revolutions in Mathematics*. While his original article had been published when he was just making his way into the field, his appendix was written after the publication of his groundbreaking book, *Moderne—Sprache—Mathematik*. The historiography of mathematics had expanded since 1976, including a substantial literature on the social history of mathematics, but to some degree, Mehrtens reflected, it was still weighed down by old concerns. Whereas the "debate on the relative importance of 'internal' and 'external' factors in the development of science has faded away," Mehrtens wrote, "the history of mathematics … appears to be somewhat behind the trend"(Mehrtens 1995, 42). Despite these changes, he added, "I hear once in a while from historians of mathematics, that their field is too isolated from and too little recognized by colleagues in the history of natural sciences" (ibid.). Indeed, two years before he wrote his appendix, historian of mathematics Ivor Grattan-Guinness complained that the history of mathematics is "a classic example of a ghetto subject." Karen Hunger Parshall and David Rowe repeated the complaint two years after Mehrtens wrote his appendix in their 1994 book, *The Emergence of the American Mathematical Research Community*. "The history of mathematics," they wrote, has "been relatively neglected in the last several decades by American historians of science" (Parshall and Rowe 1997, x).

Such concerns over the marginalization of the history of mathematics within the history of science did not end in the 1990s. Almost two decades later, in 2011, *Isis* published a "focus" section on the history of science and the history of mathematics. In the introductory essay, Amir Alexander still complained about the dearth of work in the history of mathematics and its marginalization within the history of science. "Mathematics was hardly ever interrogated, historicized, or contextualized in the way that scientific theories and practices are. It was more or less accepted as a useful but ahistorical construct, of which not much more could be said" (Alexander 2011, 477).

What might explain the orphaned position of the history of mathematics? For Grattan-Guinness the answer was simple. The history of mathematics was "too mathematical for historians and too historical for mathematicians" (Grattan-Guinness 1990, 158). For him, it was the technical nature of the field that explained why it was practically neglected. Perhaps, but, considering that many other fields of science are highly technical, his explanation is unlikely. Parshall and Rowe, for their part, lay some of the blame at the feet of historians of mathematics who, they write, "have tended to pursue their work independently of the trends that have shaped and altered the historiography of the history of science"(Parshall and Rowe 1997, xii). In particular, the history of mathematics, they explain, was (wrongly) perceived to be less amenable to types of investigations popular in the history of science, which (following Alexandre Koyré and Thomas Kuhn) attended to ways philosophical and religious ideas impacted the development of scientific thought. This is a view with which Amir Alexander seems to agree, as he writes that the severing of the history of mathematics from the history of science is a "casualty of the social and cultural turn in the history of science"(Alexander 2011, 476). It is in consequence of this split, Alexander explains, that historians of mathematics turn to internalist studies.

The internalist-externalist debate that emerged in the 1970s was, of course, not unique to mathematics. Historians of medicine struggled with similar questions at the time (Reverby and Rosner 2004). However, whereas in other fields these early debates eventually quieted down and new historiographic approaches to the history of science emerged, as Parshall and Rowe noted, this was not the case for the history of mathematics. In her 1995 critical reappraisal of the state of the field, Joan Richards seemed to concur, noting that while the internalist-externalist debate has been declared dead in the history of science, "the division between the two camps is not only *a* but *the* critical problem in the history of mathematics" (Richards 1995, 123–124). While there is much merit in this view and the debates were undoubtedly lively within the history of mathematics, the explanation is peculiar considering that some of the most popular accounts of the history of mathematics that appeared in the preceding decades offered a deeply cultural and social analysis of the field.

The best two examples were Dirk Struik's *A Concise History of Mathematics*, which was published in 1948 and reprinted in 1967, and E. T. Bell's 1951 *Mathematics: Queen or Servant of Science*. An avowed Marxist, Struik offered a materialist reading of the history of mathematics. Compared to "oriental mathematics," Struik wrote, the Greeks' adaptation of the deductive method can partially be explained by the rise of a "politically conscious merchant class" (Struik, 1948, 40).[1] According to Struik, "the new social order created a new type of man," who as a result of slave labor could find time to "philosophize about this world of his." In this new environment, modern mathematics, he explains, "helped find order in chaos, to arrange ideas in logical chains, to find fundamental principles" (Struik 1948, 40–1).[2]

[1]For Struik, "Oriental mathematics," which preceded Greek mathematics, included Egyptian, Babylonian, Hindu, and Chinese mathematics. Such a division between "oriental" and "western" rational mathematics was common in the mid-twentieth century. These histories thus partake in what Dipesh Chakrabarty has described as "an honored tradition, both in Europe and elsewhere, of regarding 'rational outlook,' the 'spirit of science' and 'free enquiry' as constituting the 'progressive' aspects of modernity" (Chakrabarty 2000, 237).

[2]Struik's ideas were taken up by later mathematical historians. For example, Morris Kline explained that the highest social class in Greek society looked down on manual work, commerce, and trade and was therefore less likely to reason from experience or by analogy to the economy. "Since the Greeks were not idlers, they fell quite naturally into the modes of inquiry that suited their tastes and social attitudes" (Kline 1964, 30). This view was criticized as early as 1978 by mathematician Beatrice Lumpkin, who critiqued historians of mathematics for advancing an "imperialist view of the history of mathematics," according to which only classical Greece, the Renaissance, and modern Europe were included as important periods in the history of mathematics. Such views, she argued, cast as insignificant the work of the Babylonians and Egyptians, not to

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

Struik did not limit his Marxist analysis to ancient mathematics. In 1937, he wrote to George Sarton that he agrees that scientists make decisions that are not impacted by any other "force than what seems to them their own volition. . . . But this, I think, is not the whole story. The general cultural conditions demand certain scientific or artistic activity, or at any rate create the opportunity for it."[3] Struik's views that scientific and mathematical production is impelled by both personal freedom and social conditions fit neatly with the social turn in the history of science.[4]

E. T. Bell extended this type of analysis to modernity. Noting that the demands of World War II occasioned new mathematical discoveries, Bell explained that the Napoleonic Wars galvanized a similar moment of mathematical activity. "It would be interesting to debate which one—war or economic necessity—has been the more influential in the development of mathematics. . . . My own opinion is that war has outranked economics at least two to one. As this is written much research in pure mathematics is being financed by the military" (Bell 1951, 3–4). Far from offering an internalist account of mathematics, Bell understood both economic and military demands as foundational for the development of mathematics.

Struik and Bell are worth singling out because their books were widely read and translated to many languages. David Rowe suggested that "since its initial appearance," Struik's *Concise History* "has probably done more to promote interest in and appreciation for the rich diversity of mathematical ideas and cultures than any other single volume on the history of mathematics."[5] Their scholarship makes it readily apparent that mathematicians were accustomed to the notion that social and cultural conditions had a profound impact on the development of mathematics. This is not to suggest that mathematicians were steeped in social or cultural history. Rather, history of science's divorce from the history of mathematics in the last quarter of the twentieth century cannot be explained by claiming that the history of mathematics was not receptive to social and cultural analysis. In what follows, I want to suggest that this explanation is partial at best. The history of mathematics remained marginalized not because historians of mathematics could not agree to what degree social history impacted the development of mathematics, but because on the whole they adopted mathematicians' own definition of what mathematics *was.*

In his 1992 essay, Mehrtens similarly laid the blame at the feet of historians of mathematics. The problem, he wrote, was not the "inaccessibility of mathematics, but rather the inability of its historians to relate to the issues of interest in general history of science" (Mehrtens 1995, 42). Yet, it seems to me that what Mehrtens had in mind was slightly different than what his colleagues at the time were concerned about. He was not worried about the adoption of social and cultural analysis in the history of mathematics, which I believe he took as a given. Rather, what he was arguing for was the recognition that "history of mathematics is an integral part of intellectual history" (Mehrtens 1995, 48). It was up to the historian, he argued, to draw the necessary connections between the "epistemological ruptures" in mathematics and other intellectual concerns of the time. The problem, and the reason I believe the marginalization of the history of mathematics continues to be a concern two decades later, is that historians of mathematics have refrained from drawing such connections, for fear of no longer writing a history of "mathematics." Perhaps, stated somewhat differently, when historians did indeed consider the history of mathematics as part of intellectual history, their work no longer belonged squarely within the "history of mathematics."

mention Muslims, Jews, Hindus, and Chinese who "were credited only as copiers, adding nothing new for one thousand years. Thus, the history of mathematics was maintained as an exclusive preserve for white men and European civilization" (Lumpkin 1978, 178). Lumpkin took Kline and some of his contemporaries to task, arguing that their imperialist approach was rooted in histories of mathematics written at the height of colonialism in the late nineteenth century.

[3]"Dirk Struik to George Sarton," April 20, 1937, GSP, Series I bMS AM 1803, Folder "(1612) Struik, Dirk Jan, 1894-".

[4]In 1942, Struik published "On the Sociology of Mathematics" in *Science & Society*. Here he explains that "the sociology of mathematics concerns itself with the influence of forms of social organization on the origin and growth of mathematical conceptions and methods, and the role of mathematics as part of the social and economic structure of a period" (Struik 1942, 58).

[5]Rowe notes that at last count the book has been translated to eighteen languages (Rowe 1994, 245).

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

The history of mathematics professionalized at a particularly fraught moment for American mathematicians, a time in which they were deeply concerned with the questions: what is mathematics? And what are its social and intellectual boundaries? The reason mathematicians were so invested in these questions was that, in the three decades after World War II, the mathematical landscape had completely changed. Europe's devastation coincided with the Golden Age of American academia, a convergence that placed American mathematicians in a singular position within the international scholarly community. Mathematics was growing in both scale and scope as new areas of research, from operations research to control theory, were pushing the bounds of mathematical theory further and further. Moreover, mathematical modes of thinking were becoming increasingly prevalent not only in the physical, but the biological and social sciences as well (Steingart 2023).

Faced with these transformations, mathematicians approached the question "what is mathematics" as both an epistemological and a political query. Concerns over the boundaries of the field and about the relation between "pure" and "applied" mathematics were, of course, not new. Amir Alexander has demonstrated how mathematicians' turn towards increased rigor during the early nineteenth century altered not only mathematical practice, but the persona of the mathematician as well (Alexander 2006, 2011).[6] What was unique about the post-World War II moment was that these debates were taking place within a new institutional terrain.[7] Propelled by national defense and government funds, the number of publications rose steeply, the number of mathematical departments grew, as did the number of students who needed training.[8] Yet a coherent national policy for mathematics could not be built upon the disciplinary matrix that defined the growth of the field before the war. It was to achieve that goal that American mathematicians turned to drawing borders around what constituted "proper" mathematics. The history of mathematics, coming along in these turbulent times, adopted this restricted conception of what mathematics is.

A turn to history

The clearest indication of mathematicians' concern with the question "what is mathematics" was their renewed interest in the postwar period in the history of mathematics. While interest in the history of mathematics had been alive and well in the nineteenth century, the postwar turn to history came on the heels of a profound transformation in the meaning of mathematics.[9] The growth of mathematics in the previous decades represented a series of epistemological problems for mathematicians: Were new areas of research such as game theory, control theory, and computing part of mathematics or were they allied fields? What united these disparate activities? What was the relation between pure and applied mathematics? To answer these questions and others, mathematicians turned to history.[10]

Historical writing by mathematicians took on different forms—from short historical surveys to more systematic studies by historically minded mathematicians such as Struik, Bell, Salomon

[6]One can go even further to question the use of the term mixed mathematics (Brown 1991).

[7]Up until the mid-twentieth century, the boundary between "pure" and "applied" mathematics was much more porous. Many mathematicians who contributed to "pure" mathematics also worked in "applied mathematics." In the United States, for example, both Oswald Veblen and George Birkhoff worked across this boundary. After World War II, the boundary solidified and there were simply fewer mathematicians who were able to work across it—not because of a lack on *intellectual* capacity, but because institutionally there was no space to do such work.

[8]I discuss these transformations in (Steingart 2013).

[9]For a broad and extensive overview of the historiography of mathematics see: (Dauben, Joseph W. and Scriba 2002; Remmert, Volker, Schneider, and Sørensen 2016).

[10]These works were not the first book-length histories of mathematics to appear in English. Earlier in the century, David Eugene Smith and Florian Cajori published similar historical accounts of mathematics from antiquity to modern times. Other mathematicians who turned to the history of the field before the war and continued to publish in its aftermath include J. L. Coolidge, Raymond Clare Archibald, and Orstein Ore.

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

Bochner, Max Dehn, Morris Kline, and Carl B. Boyer (Bochner 1966; Boyer 1968; Kline 1959, 1964).[11] These writings differed in their level of technicality and in expected audience, but all tried to offer an answer (if only a partial one) to the question of what mathematics "is" by identifying either a method, a way of thinking, or core ideas that connected the mathematics of the past with that of the present. In other words, while they offered different answers, all these works turned to history in order to identify an *essentialized* character of mathematics that connected the past to the present. A good example was a special issue entitled "Mathematics in the Modern World," which appeared in *Scientific American* in 1964. An essay by Richard Courant opened the magazine, setting the tone for the special issue. Aimed at the general public, contributors presented an illustrated guide to recent developments in the mathematical sciences, with the goal of teaching readers about modern mathematics. The image that accompanied the text, however, was not of a computer, a communications control room, or even a seminar room filled with young aspiring mathematicians. Rather, facing the text was a photograph of the Rhind Papyrus, one of the earliest extant sources of mathematical thinking from the second century BC. Discovered in the nineteenth century, the Rhind Papyrus compiles Egyptian problems on the measurement of land. It is, needless to say, a prime example of *ancient*, not *modern*, mathematics.

This was not just a stylistic choice on the part of *Scientific American* editors. Several of the articles included in the special issue offered a *longue durée* account explaining contemporary mathematics by reaching back to ancient mathematics. For example, in his article on "Number," mathematician Philip Davis began with Babylonian mathematics, then moved to negative numbers, complex numbers, and, finally Cantor's transfinite cardinals (Davis 1964). Morris Kline similarly covered a long period in his article on "Geometry." The history the authors narrate is one of constant change, in which the meaning of both number and geometry is continually under revision, while *at the same time* remaining remarkably singular by retaining an essential, yet unspecified, quality. History, for them, offered a way to make sense of the present.

In his 1966 *The Role of Mathematics in the Rise of Science*, Bochner tackled the question head on. The volume begins by posing the question: "what is mathematics?" "What indeed is mathematics?," Bochner continues, "This question, if asked in earnest, has no answer, not a satisfactory one; only part answers and observations can be attempted" (Bochner 1966, 13). In lieu of an answer, Bochner offered history—starting with the Greeks and moving in fits and starts to the present. Bochner identified abstraction as the dominant feature of mathematical knowledge, noting that while it had its inception with the Greeks, it only took off in the nineteenth century. Modern mathematics, for him, was defined by "untrammeled escalation of abstraction, that is, abstraction from abstraction, abstraction from abstraction from abstraction, and so forth" (Bochner 1966, 18).

Even the French group Bourbaki included historical surveys, at times going all the way back to Babylonian mathematics in their monographs. Moreover, they too used history to answer the question, "what is mathematics." The answer, not surprisingly, was that mathematics was the study of abstract structures. As Leo Corry has showed, Bourbaki's historical writing "has been strongly connected with their overall conception of mathematics" (Corry 2004, 331). Corry quotes Dieudonné as stating that "today when we look at the evolution of mathematics for the last two centuries, we cannot help seeing that since about 1840 the study of specific mathematical objects has been replaced more and more by the study of mathematical *structures*" (Corry 2004, 332). Such an assertion, Corry demonstrates, is doubtful at best, but the point is not whether Dieudonné's analysis is accurate or not, but rather the fact that history served as justification for the present.

Mathematicians' search for some inherent characteristic of mathematics was not just an intellectual quandary. The postwar institutional and funding regime that supported the growth of mathematics at the time put additional pressure on mathematicians to define the boundaries of their field. During a 1966 conference on the education of applied mathematicians, mathematician

[11]In addition, Jean Dieudonné, Herman Goldstein, Raymond Wilder, André Weil, Herman Weyl, and many other mathematicians developed a strong interest in the history of mathematics during this period.

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

Lipman Bers, for example, suggested that more historical research is necessary to better direct the future of the field:

> Let us stimulate research in the history of modern mathematics . . . let us try to find "what did really happen." I think if we do we will find that the traditional picture of problems coming from the outside into mathematics, being solved there and then going back, is exceedingly oversimplified. . . . The true history of the interplay between applications and pure mathematics is highly interesting and should be studied and taught. (Greenberg 1967, 315)

Bers' appeal was not hypothetical. Having recently moved to Columbia University after spending fifteen years as a professor of mathematics at the Courant Institute at NYU, Bers was put in charge of a national effort by the mathematical community to study the state of American mathematics and put forward policy recommendations for federal and military agencies. The final report, which was published in 1968, further elucidated how mathematicians sought to draw boundaries around the "proper" domain of the field.

Bounding mathematics

On the face of it, the Committee on Support of Research in the Mathematical Sciences (COSRIMS) presented the most expansive view of mathematics. The committee included representatives from the various mathematical fields—statistics, probability, computing, applied and pure mathematics—and represented a massive undertaking by the mathematical community. It produced surveys on the state of undergraduate education, graduate education, and employment opportunities, as well as a detailed analysis of national changes in the mathematical curriculum. In addition to these national surveys, which were published independently, the committee established several individual panels to study and produce reports on specific topics. Besides the eleven members of the standing committee, approximately forty-five additional, highly respected mathematicians took part in the work of these panels.[12]

As a national policy document, the report unsurprisingly emphasized the ubiquity of mathematics. Mathematics, the report begins, had long played a role in scientific and technological developments. Yet such an assertion "hardly begins to convey or account for the current explosive penetration of mathematical methods into other disciplines, amounting to a virtual 'mathematization of culture'" (National Research Council 1968, 3). The use of mathematical techniques is not limited to academic domains. Government, business, and industry, the report continues, are becoming progressively dependent on mathematics and computers to solve problems of resource and time allocation. The message was clear: mathematics was everywhere. The authors proclaimed, "it is no exaggeration to say, therefore, that the fundamental problems of national life depend now, more than ever before, upon the existence and the further growth of the mathematical sciences and upon the continuing activities of able people skilled in their use" (National Research Council 1968, 47).

Almost every aspect of daily life was affected by mathematics. As examples, the authors noted that mathematics was an "absolute necessary condition" for developments in electronics, and that the growth of information theory, network synthesis, and feedback theory was "unthinkable" without mathematics.[13] They added that telephone and radio communication also depend on mathematics, as well as the transmission of pictures, and the "collection, classification, and transmission of data in general" requires mathematics. If that was not enough, transportation both

[12]Among others, the mathematicians who took part in the work of the Committee included Mina Rees, William Prager, Allen Newell, Harold Grad, Joseph LaSalle, Andrew Gleason and Adrian Albert.

[13]Ibid, 46.

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

on the ground and in the air is demonstrated to require mathematics in consideration of traffic control problems. Mathematics, the authors thus proved, was implicated in the life of the nation.[14]

This sweeping conception of mathematics, however, is misleading, for it came with a subtle but crucial change in terminology. The report considered the state of the "mathematical sciences"—not mathematics. This was a new term of art that emerged in the 1960s as a response to the growing use of mathematics across myriad domains. "Mathematics" was no longer an inclusive enough category to describe all the new areas of research that had emerged over the preceding two decades. The "mathematical sciences" was a means by which to reflect the plurality of new areas of research that now laid claim to the mathematical terrain. Rejecting the common division between pure and applied mathematics, the report divided the field into "core mathematics" and the "applied mathematical sciences," which consisted of four major areas of research: computer science, operation research, statistics, and physical mathematics (classical applied mathematics). The goal of the authors was to offer a unified vision of the field, but in effect the growth of the mathematical sciences only served to cordon off the domain of pure mathematics. Put somewhat differently, as the "mathematical sciences" turned increasingly outward, "mathematics" could safely turn inwards.

This inward move is exemplified by the changes instituted throughout the 1960s, in *Mathematical Reviews* (MR), the leading reviewing journal of the mathematical community. As the number of mathematical publications ballooned, an ad hoc committee was established in 1962 to study the coverage of the *Mathematical Reviews*. Their final report stated that "the increased rate of mathematical production causes headaches for MR." The problem, however, was not just the increase in size. "Of greater impact," the committee continued, "is the change of character of various fields."[15] Symbolic logic, computing, and operation research, the committee noted, had expanded the scope of the *Mathematical Reviews*. Whereas in the past, the tendency was to be as inclusive as possible, the committee suggested that in the future "only mathematical papers should be reviewed and reviewing should be from a mathematical point of view. The importance of a paper of applications or teaching, and the fame of an author, are irrelevant for MR."[16] To ensure prompt coverage and to better serve the mathematical community, coverage, the committee suggested, had to be more selective.

Four years later, as the production of mathematical research continued to grow, the selection for inclusion became even stricter. Starting in 1967, the *Mathematical Review* followed a coverage policy that specifically sought to exclude "routine applications of known mathematics."[17] The effects of this new policy were immediately noticeable. Whereas in 1965 and 1966 the percentage of reviews in the "applied" sections was about 33%, in the last volume of 1967 it dropped to 14%.[18] Noting the effect of this new policy, the editorial committee for *Mathematical Review* admitted that, in some cases, making the distinction between pure and applied was quite difficult. Section 93 (Control Theory), they noted, "always contained quite a few papers in 'pure' mathematics." In conclusion, they observed that "none of the dividing lines are sharp, but the net effect is discernible in the general trend of the figures: a larger total number of reviews, a smaller proportion of applied mathematics, very little of what could be called physics."[19] Acknowledging that any attempt to distinguish pure from applied was by definition artificial, mathematicians were nonetheless

[14]The most pressing message the report aims to convey is the need to strengthen the mathematical literacy of the nation. The demand, according to its authors, is for professional mathematicians, more mathematically trained scientists, and a mathematically informed public. This of course was the era of the "New Math" movement that saw, for the first time, the involvement of research mathematicians in elementary and high-school mathematical education. Chris Phillips has demonstrated how debates about the New Math curriculum were debates about the "necessary" intellectual discipline of American citizenship (Phillips 2015).

[15]"Report on Mathematical Reviews Coverage," 22 March 1962, AMSR, Box 16, Folder 65.

[16]Ibid.

[17]"Editorial Committee For Mathematical Reviews," AMSR, Box 16, Folder 128.

[18]In the *Mathematical Reviews*, the "pure" sections are those numbered from 00–65, while the "applied" sections are those from 68–94.

[19]Ibid.

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

drawing new boundaries around their discipline. In the case of the *Mathematical Reviews* this was done in part due to the unprecedented expansion in the production of research papers, but these boundaries were also being marked into the field's institutional formation, with the rise of new departments in statistics, computer science, and operation research, independent from the traditional mathematical departments.

In 1970, mathematician Saunders MacLane was ready to dispense altogether with the new terminology. In a memorandum, which he circulated among a small number of mathematicians, MacLane suggested that a new organization representing mathematicians' demands was necessary. "I submit that there is a real need for a strong mathematical posture on questions of national science policy and that this need will grow greater with the shortage of funds and the inevitable wider application of mathematical methods." MacLane held that existing organizations representing mathematics on the national level were broken, and urged his colleagues to form a new national organization. One of the "essential" conditions for such an organization, he added, was that "it is concerned with mathematics (*which has existed for millennia*) and not with that novelty 'mathematical sciences.'"[20] It is against this background that historians of mathematics and historically inclined mathematicians found themselves debating one another in the 1970s. Their disagreement was less about the applicability of social and cultural history to mathematics, but about what mathematics *was*. It is in reaction to the proliferation of mathematical applications that many American mathematicians sought to draw borders around what constituted "proper" mathematics.

A history of mathematics

By the early 1970s, mathematicians were no longer the only ones interested in this history of mathematics. As the history of science began professionalizing as an academic discipline, the history of mathematics emerged as a distinct subfield. No longer the sole purview of elder mathematicians, papers and dissertations on the history of mathematics began to be published by young scholars trained in the history of science and historical methods more broadly. First, however, they had to contend with the mathematicians. In 1974, mathematicians and historians of mathematics convened at a workshop on the evolution of modern mathematics, hosted under the auspices of the American Academy of Arts and Sciences. The workshop was organized by a committee that included Garrett Birkhoff (chair), I. B. Cohen, Thomas Hawkins, Kenneth May, and Felix Browder. This membership alone demonstrates the changing nature of the history of mathematics as a recognizable academic field in its own right. Birkhoff and Browder were celebrated mathematicians, May was a mathematician turned historian, and Hawkins was a younger historian of mathematics. The workshop brought together more than forty scholars interested in the history of modern mathematics. It included, among others, the celebrated mathematicians Alan Baker, Jean Dieudonné, Churchill Eisenhart, and Antoni Zygmund, as well as mathematically trained historians Morris Kline and Carl Boyer, and historians William Aspray, Michael Crowe, Ivor Grattan-Guinness, Judith Grabiner, and Frederick Gregory.

Garret Birkhoff explained that its goal was "to direct into constructive channels *the rising tide of interest* on the part of mathematicians in the history and philosophy of their subject" (Birkhoff

[20]Saunders MacLane, "Global Organization for American Mathematics: An Informal Examination of a Possible Development," 18 February 1970, AMSR, Box 62, Folder 83. Emphasis added. Thirteen years earlier, Marston Morse similarly insisted that the independence of mathematics (understood as pure mathematics) must remain in light of all the new mathematical sciences. He explained, in a letter to fellow mathematician G. Bailey Price, that there were three fundamental groups in the country: "those concerned with basic mathematics," "those concerned with teaching," and "various societies which are concerned with applications." Morse predicted that with time the AMS, which was still the largest professional organization for mathematicians, would actually become the smallest one. The other societies dedicated to the applications of mathematics, he explained, would eventually represent the majority of American mathematicians. "Unless the historic development of mathematics is to be inverted and the tail wag the dog, the first group [AMS], having great intelligence and ideals, must at least remain *independent*." Marston Morse to G. Baley Price, 18 March 1957, MMP, Box 1, Folder A.

and Cohen 1975). Indeed, the mathematicians outnumbered the historians by nearly three to one. The conference marks an important juncture point in the professionalization of the history of mathematics in the United States, not because it gave rise to a new research tradition, but because it symbolizes the difficult disciplinary position in which American historians of mathematics found themselves in the early 1970s. By the mid-1970s, the history of science was expanding as an academic profession, and Thomas Kuhn's *The Structure of Scientific Revolutions* was more than a decade old. Historians of mathematics had to find their place among mathematicians, who were *deeply* invested in their own history.

This tension between the two groups was palpable throughout the conference. In his opening speech, I. B. Cohen reported that the historians had met the previous night to discuss their objectives for the workshop.[21] Consequently, the historians' two motivating questions were: first, to what degree should the presentation of historical material depend on the intended audience? and second, what is the relation of history of mathematics "not only to mathematics but to general history and the history of science?"[22] The latter of the two was the perennial question occupying historians of mathematics throughout the decade and, as the workshop makes evident, historians were unwilling to break ties with mathematicians. In a nod of deference to the mathematicians in the audience, Cohen noted, for example, that all of the historians in attendance agreed that active mathematicians brought to the study of history "a special kind of insight that goes beyond the power of the historians."[23] Collaboration with mathematicians was especially necessary for studying recent history, for which the historian's mathematical knowledge might be lacking. The problem was that aligning themselves with mathematicians implied alienating themselves from historians of science. Trying to speak to both mathematicians and historians of science partially explains the separation of the history of mathematics from the history of science.

The publication of *Historia Mathematica* in 1974 similarly points to the growing interest in the history of mathematics at the time and the difficult disciplinary position it presented. In the second issue of the journal, the editor of the journal Kenneth May published an article titled, "Should we be Mathematicians, Historians of Science, Historians, or Generalists?" (May 1974). The answer, according to May, was all of the above. May insisted that every center for the history of science (of which there were by then a few) must include a historian of mathematics, but recognized that the most common and obvious place for historians of mathematics was within the mathematical community. "From the beginning, history of mathematics has been a part of mathematical activity, and its practitioners a part of the mathematical community" (May 1974). Most historians of mathematics, May added, earn their living as part of the mathematical community, which is also their main audience. "The mathematicians have a real need for the historians of mathematics, especially in periods like the present, when there is considerable disarray over matter of science policy" (May 1974).

May's article came as a response to the academic job crisis of the early 1970s. The history of mathematics, he believed, could at the very least help direct educational programs. By arguing that historians of mathematics should be all of the above (historians, generalists, etc.), May was hoping to point towards additional academic positions that young PhDs might assume. However, such a multifaceted definition of what the history of mathematics entailed came with its own difficulties.

May, who was one of the participants in the conference, clearly recognized the tension between the groups. He began his talk by chastising mathematicians for being willing to forgive the mistakes of their colleagues but having zero tolerance for those of non-mathematicians:

[21]The historians of mathematics included William Aspray, Stephen Brush, Michael Crowe, Judith Grabiner, I. Grattan-Guinness, Frederick Gregory, Thomas Hawkins, Erwin Hiebert, and Kenneth Manning. Mathematicians turned historians were Carl Boyer, Hans Freudenthal, Morris Kline, and Kenneth May. Philosophers Burton Dreben and Hilary Putnam were also present.

[22]"Opening Remarks for the Conference by Garrett Birkhoff and I. B. Cohen," 434.

[23]Ibid.

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

"mathematicians forgive a good mathematician for his blunders, but they tend to be outraged by the clumsy intrusions of outsiders" (May 1975, 453). The historians, however, were not blameless. "The historian finds hilarious such naïve historical mistakes as assuming that words have fixed meanings or that a brilliant mathematician of past centuries must have understood a concept or had a proof because these would be evident to lesser lights today" (ibid.). The best history, May insisted, was one that tried to be sensitive to both the mathematics and the history. This, however, was easier said than done.

After Thomas Hawkins' talk, Morris Kline felt the need to defend the historians in the audience. "They came today, not to tell us the history of mathematics, but to discuss the problems of history" (Hawkins 1975, 565). Commenting that the mathematicians' talks were very detailed, he added, "the purpose of this meeting was not so much to get the history down from the mathematicians, who would be the best source, but rather to discuss more the problems of how we can all co-operate" (ibid., 565–66). Birkhoff reiterated Kline's point, but with a caveat, "discussion between mathematicians and historians of mathematics should be related to specific examples, with a minimum of philosophical superstructure" (ibid., 566). Birkhoff's advice came as response to the constant references throughout the workshop to the work of Kuhn. Despite mathematicians' general aversion to the topic, the historians in the group questioned again and again the appropriateness of Kuhn's ideas to the history of mathematics.

In particular, they questioned whether revolutions happened in mathematics. M. J. Crowe, one of the young historians who presented at the conference, took the most decisive position, stating as a "law" that "revolutions never occur in mathematics" (Crowe 1975). Crowe argued that for an event to count as a revolution something needed to be discarded. However, a mathematical theorem, once proven, stands forever. A year after the conference, Crowe's paper, along with the rest of the proceedings, was published in *Historia Mathematica*. After reading the paper, Herbert Mehrtens submitted "T.S. Kuhn's Theories and Mathematics: A Discussion Paper on the 'New Historiography' of Mathematics" to the journal. In his paper, Crowe explained that in formulating his "law," the preposition "in" was crucial. Revolution, he explained, may occur in mathematical nomenclature, symbolism, metamathematics, and methodology, but not *in* mathematics. In addition to his analysis of Kuhn's ideas, Mehrtens' response also homed in on Crowe's use of the preposition "in". As Mehrtens explained, only by cordoning off an aspect of mathematics that is distinct from nomenclature, symbolism, metamathematics, methodology, and historiography was Crowe able to argue that revolutions never occurred in mathematics. But what exactly, he asked, was this part of mathematics? What did it consist of? What was the "content" or "substance" of mathematics which supposedly remained the same despite great transformations? "There is a danger," he added, "for the historian of mathematics in this preposition *in*" (Mehrtens 1976, 301–2). By adopting the distinction between that which is *in* mathematics and that which is not, historians of mathematics inadvertently adopt mathematicians' definition of what should and should not belong in the history of mathematics. I would add that that they are also in danger of following mathematicians' own definitions of its boundaries.

Wonderful, insightful, and rich books in the history of mathematics have been written since the 1970s, but these works always feel like an exception rather than the rule. A few authors have managed to please both the mathematicians and the historians of science, but their work stands out. This is why, despite all of these works, historians of mathematics periodically feel compelled to bemoan that history of mathematics is tuned to a minor key compared to the history of science. At stake, I believe, is not the old internalist-externalist debate, but the fact that historians of mathematics have followed mathematicians too closely in deciding what counts as the history of mathematics. Acknowledging that the image of mathematics as an abstract and insular field of study is one which mathematicians constructed not only because of intellectual convictions, but also due to social and political concerns, should give historians of mathematics freedom to write against the grain. Accepting the notion that "proper" mathematics is independent of its applications risks accepting the essentialized conception of the field that mathematicians have advanced. What if historians of

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

mathematics followed the COSRIMS report, not in cordoning off mathematics, but in recognizing that such divisions are up to us to make? The history of invariant theory is obviously part of the history of mathematics, but what about the history of information theory or axiomatics? Postwar mathematicians spent much time and energy demarcating mathematics. Historians, like mathematicians, must decide what mathematics is before they can agree on what comprises the history of mathematics. Perhaps the field has been flourishing all along.[24]

Bibliography

Alexander, Amir. 2006. "Tragic Mathematics: Romantic Narratives and the Refounding of Mathematics in the Early Nineteenth Century." *Isis* **97** (4): 714–26.

Alexander, Amir. 2010. *Duel at Dawn: Heroes, Martyrs, and the Rise of Modern Mathematics*. Cambridge, MA: Harvard University Press.

Alexander, Amir. 2011. "The Skeleton in The Closet: Should Historians of Science Care about the History of Mathematics?" *Isis* **102** (3): 475–80.

Bell, Eric Temple. 1951. *Mathematics: Queen and Servant of Science*. Washington: Mathematical Association of America.

Birkhof, Garrett and I. B. Cohen. 1975. "Opening Remarks." *Historia Mathematica* **2** (4): 433–436.

Bochner, Salomon. 1966. *The Role of Mathematics in the Rise of Science*. Princeton: Princeton University Press.

Boyer, Carl B. 1968. *A History of Mathematics*. New York: John Wiley & Sons, Inc.

Brown, Gary I. 1991. "The Evolution of the Term 'Mixed Mathematics'." *Journal of the History of Ideas* **52** (1): 81–102.

Chakrabarty, Dipesh. 2000. *Provincializing Europe: Postcolonial Thought and Historical Difference*. Princeton: Princeton University Press.

Corry, Leo. 2004. *Modern Algebra and the Rise of Mathematical Structures*. New York, NY: Springer.

Crowe, Michael. 1975. "Ten 'Laws' Concerning Conceptual Change in Mathematics." *Historia Mathematica* **2** (4): 469–70.

Dauben, Joseph W., and Christoph J. Scriba, eds. 2002. *Writing the History of Mathematics: Its Historical Development*. Boston, MA: Birkhäuser-Verlag.

Davis, Philip J. 1964. "Number." *Scientific American* **211** (3): 51–59.

Grattan-Guinness. 1990. "Does History of Science Treat of the History of Science? The Case of Mathematics." *History of Science* **28** (2): 149–173.

Greenberg, H. J., ed. 1967. "Proceedings of a Conference on Education in Applied Mathematics." *SIAM Review* **9** (2): 289–415.

Hawkins, Thomas. 1975. "Mathematical Progress Without Fusion." *Historia Mathematica* **2** (4): 563–66.

Kline, Morris. 1959. *Mathematics and the Physical World*. London: John Murray.

Kline, Morris. 1964. *Mathematics in Western Culture*. New York: Oxford University Press.

Lumpkin, Beatrice. 1978. "History of Mathematics in the Age of Imperialism." *Science & Society* **42** (2): 178–184.

May, Kenneth O. 1974. "Should We Be Mathematicians, Historians of Science, Historians, or Generalists?" *Historia Mathematica* **1** (2): 127–28.

May, Kenneth O. 1975. "What Is Good History and Who Should Do It?" *Historia Mathematica* **2** (4): 449–55.

Mehrtens, Herbert.1976. "T.S. Kuhn's Theories and Mathematics: A Discussion Paper on the 'New Historiography' of Mathematics" *Historia Mathematica* **3** (3): 297–320.

Mehrtens, Herbert. 1995. "Appendix (1992): Revolution Reconsidered." In *Revolutions in Mathematics*, edited by Donald Gillies, 42–48. Oxford: Clarendon Press.

National Research Council. 1968. *The Mathematical Sciences: A Report*. Washington: The National Academies Press.

Parshall, Karen Hunger, and David E. Rowe. 1997. *The Emergence of the American Mathematical Research Community, 1876–1900*. Providence: American Mathematical Society.

Phillips, Christopher. 2015. *The New Math: A Political History*. Chicago: University of Chicago Press.

Remmert, Volker, Martina R. Schneider, and Henrik Kragh Sørensen, eds. 2016. *Historiography of Mathematics in the 19th and 20th Centuries*. Switzerland: Birkhäuser.

Reverby, Susan M., and David Rosner. 2004. "'Beyond the Great Doctors' Revisited: A Generation of the 'New' Social History of Medicine." In *Locating Medical History: The Stories and Their Meanings*, edited by John Harley Warner and Frank Huisman, 167–93. Baltimore: John Hopkins University Press.

Richards, Joan L. 1995. "The History of Mathematics and L'esprit Humain: A Critical Reappraisal." *Osiris* **10** (Constructing Knowledge in the History of Science): 122–35.

[24]As Stephanie Dick notes, the history of Artificial Intelligence serves as a good parallel. Claiming that the history of AI received little scholarly attention makes sense only if you ignore the fact that the history of AI is actually the history of management, communication, computing, etc. From that perspective, the history of AI is rich and substantive (private communication with author).

https://doi.org/10.1017/S0269889724000127 Published online by Cambridge University Press

Rowe, David E. 1994. “Dirk Jan Struik and His Contributions to the History of Mathematics.” *Historia Mathematica* **21** (3): 245–73.

Steingart, Alma. 2013. “Conditional Inequalities: American Pure and Applied Mathematics, 1940-1975.” Dissertation, Massachusetts Institute of Technology.

Steingart, Alma. 2023. *Axiomatics: Mathematical Thought and High Modernism*. Chicago: University of Chicago Press.

Struik, Dirk. 1942. “On the Sociology of Mathematics.” *Science & Society* **6** (1): 58–70.

Struik, Dirk. 1948. *A Concise History of Mathematics*. New York: Dover Publications.

Alma Steingart, an assistant professor in the Department of History at Columbia University, researches the interplay between American politics and mathematical rationalities. Professor Steingart’s current project, *Accountable Democracy*, examines how mathematical thought and computing technologies impacted electoral politics in the United States in the twentieth century. It follows on her first book, *Axiomatics: Mathematical Thought and High Modernism* (2023). Steingart’s work has appeared in *Social Studies of Science, Grey Room,* and the *Los Angeles Review of Books.* Her work is supported by a CAREER Award from the National Science Foundation.

Cite this article: Steingart, Alma. 2022. “A dangerous preposition: The boundaries of mathematics,” *Science in Context* **35** (4): 426–437. doi:10.1017/S0269889724000127