THE SEISMIC SUN

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Abstract. Helioseismic techniques allow us to probe the interior of the Sun with very high precision and in the process test the physical inputs to stellar models. The picture of the Sun that has been built in this manner may be termed "The Seismic Sun". After a brief discussion of some of the inversion techniques used in the process, our current view of the seismic Sun shall be reviewed. What we know so far suggests that the internal structure of the Sun can be represented by a standard model, however, one which has a smoother sound-speed and abundance variation than the solar models with the usual treatment of diffusion.

1. Introduction

Helioseismology is a powerful tool to study the internal structure of the Sun. The observed solar frequencies provide an unprecedented quantity of data than can be used to deduce the solar structure. This ability to determine the properties of solar interior is providing more stringent tests of stellar structure and evolution theory than those provided by the knowledge of just the global properties of the stars like luminosity, mass, radius etc. The value of solar oscillation frequencies as diagnostics of the interior of the Sun lies in the fact that they can be determined to very high accuracy, with the most precise observations having a relative error of less than 10^{-5}. Hence comparison of computed and observed frequencies provides a stringent test of the model. In fact, none of the solar models constructed so far are able to reproduce the observed frequency spectrum at the level of accuracy provided by the observations. Thus in order to exploit the full potential of the available frequency measurements, one has to resort to inversions. The model of the Sun built by helioseismic inversions may be termed the "Seismic Sun".

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In this review, I restrict myself to the spherically symmetric structure of the Sun. Solar rotation is therefore ignored. The presence of magnetic fields is also ignored. A discussion about solar rotation can be found in Sekii (*this volume*). Techniques for determining asphericities arising from other sources are discussed by Duvall et al. in this volume. The forward problem of building a solar model has been discussed by Provost (*this volume*). A review of how observed frequencies are related to solar structure can be found in Christensen-Dalsgaard (1996) and Christensen-Dalsgaard et al. (1996). The results quoted are based on different data sets – BBSO (Libbrecht et al. 1990), LOWL (Schou & Tomzcyk, 1996), BiSON (Elsworth et al. 1994), IRIS (cf., Gelly et al. 1996), GONG (cf., Harvey et al. 1996), etc.

2. Inversion Techniques

Solar oscillations eigenfunctions can be expressed in terms of spherical harmonics and described by three "quantum numbers" — the degree ℓ, the radial order n, and the azimuthal order m. In the absence of asphericities, all modes with the same value of ℓ and n have the same frequency. The main assumption involved while performing an inversion is that the mean frequency of an (ℓ, n) multiplet depends only on the spherically symmetric structure of the Sun. The modes that have been observed so far are acoustic modes and hence depend primarily on the sound speed c. They depend to a much lesser extent on the density ρ. The modes have small amplitudes and periods involved are much shorter than the thermal time scales in the Sun (except at the outermost layers) and hence the oscillations are linear and mostly adiabatic. The oscillations can be described as superposition of acoustic waves, each traveling in a resonant cavity. The upper boundary of the cavity is near the surface. The lower boundary of the cavity is determined by the frequency and degree of the wave — the higher the degree, the less deeply the waves penetrate (for details see Unno et al. 1989, Christensen-Dalsgaard & Berthomieu 1993). The fact that different waves travel to different depths enables us to determine the structure of that region of the Sun over which the waves travel (see Gough et al. 1996).

The first attempts at inversion were using the asymptotic dispersion relation of solar frequencies, the Duvall law (cf. Duvall 1982):

$$F(w) = \int_{r_t}^{R} \left(1 - \frac{c^2}{w^2 r^2}\right)^{-1/2} \frac{dr}{c} = \frac{(n + \alpha(\omega))\pi}{\omega}, \tag{1}$$

where $w = \omega/L$ and $L^2 = \ell(\ell + 1)$. In this approximation the mode frequencies depend only on the sound speed. Once $F(w)$ is determined by e.g., a least squares fit to the data, it can be inverted to determine the sound speed implicitly:

$$r = R_\odot \exp\left[-\frac{2}{\pi}\int_{a_s}^{a}(w^{-2} - a^{-2})^{-1/2}\frac{dF}{dw}dw\right] , \tag{2}$$

where $a = c/r$.

This inversion method is however not very accurate, and, when applied to artificial data, the inverted sound-speed deviates substantially from the exact sound-speed, particularly near the centre. The accuracy of the inversion can be improved by taking higher order terms in Eq. (1) (cf., Vorontsov & Shibahashi 1991). The next logical step therefore was to assume that the Sun is not very different from a standard solar model (SSM) and linearize Eq. (1) around the model, so that the difference in sound speed between the model and Sun is related to the difference in frequency (cf., Christensen-Dalsgaard et al. 1988). Thus

$$S(w)\frac{\delta\omega}{\omega} = H_1(w) + H_2(\omega), \tag{3}$$

where the function $S(w)$ is a known function of the reference model alone, while $H_1(w)$ can be determined along with $H_2(\omega)$ by a fit to the data. $H_1(w)$ can be expressed in terms of the sound-speed difference and can be inverted to find the sound-speed difference,

$$\frac{\delta c}{c} = -\frac{2r}{\pi}\frac{da}{dr}\int_{a_s}^{a}(a^2 - w^2)^{-1/2}\frac{dH_1}{dw}w\,dw\,. \tag{4}$$

The function $H_2(\omega)$ contains information about the surface layers of the Sun, where our assumption of adiabaticity breaks down. The functions $H_1(w)$ and $H_2(\omega)$ have also been used on their own to study various aspects of solar structure (e.g., Pérez Hernández & Christensen-Dalsgaard 1994, Basu & Antia 1995).

Although the differential asymptotic relation (Eq. 3) gives much better sound-speed results than those obtained with Eq. (1), there are still substantial inaccuracies in the core. Thus for more detailed work, one often uses a complete numerical inversion. Detailed descriptions of these asymptotic techniques and an in-depth analysis of the errors in the different methods can be found in Gough (1985) and Gough & Thompson (1991). Better results can also be obtained by alternative higher order asymptotic approximations (cf., Roxburgh & Vorontsov 1996, Marchenkov et al. 1996).

For numerical inversion for solar structure (e.g., Gough & Kosovichev 1990, Dziembowski et al. 1990; Däppen et al. 1991, Antia & Basu 1994, Basu et al. 1996a) the variational principle for the frequencies of adiabatic oscillations (cf., Chandrasekhar 1964) is used to express the frequency differences between the Sun and a model in terms of corresponding differences in structure. To this are added the effects of near surface errors. Thus, we can write

$$\frac{\delta\omega_i}{\omega_i} = \int K^i_{c^2,\rho}\frac{\delta c^2}{c^2}dr + \int K^i_{\rho,c^2}\frac{\delta\rho}{\rho}dr + \frac{F_s(\omega_i)}{E_i} \tag{5}$$

(cf. Dziembowski et al. 1990). Here $\delta\omega_i$ is the difference in the frequency ω_i of the ith mode between the solar data and a reference model. The functions c and ρ are the sound speed and density respectively. The kernels $K^i_{c^2,\rho}$ and K^i_{ρ,c^2} are known functions of the reference model which relate the changes in frequency to the changes in c^2 and ρ respectively; and E_i is the inertia of the mode, normalized by the photospheric amplitude of the displacement. The term F_s results from the near-surface differences, not taken into account by the adiabatic oscillation equations. The kernels for the (c^2, ρ) combination can be easily converted to kernels for others pairs of variables like (Γ_1, ρ), (u, Γ_1) with no extra assumptions (cf., Gough 1993), where $u \equiv p/\rho$.

There are two complementary methods of using Eq. (5) to determine $\delta c^2/c^2$ or $\delta\rho/\rho$: the regularized least squares (RLS) and the optimally localized averages (OLA). In the former, one tries to fit the given data under the constraint that the solution is smooth. The latter involves finding a linear combination of the kernels localized in spatial coordinates. The complementary nature of the two techniques is discussed by Sekii (*this volume*). Details of the RLS method can be found in Dziembowski et al. (1990), Antia & Basu (1994), Antia (1996). The details on how different versions of OLA are implemented can be found in Kosovichev et al. (1992), Christensen-Dalsgaard & Thompson (1995), Basu et al. (1996b) etc. A combination of RLS and OLA has also been used (cf., Dziembowski et al. 1994).

3. Inversion results — sound speed and density

Fig. 1 shows the relative squared sound-speed and density differences between the Sun and a SSM — model S of Christensen-Dalsgaard et al. (1996). The model was constructed with OPAL equation of state (EOS) (Rogers, Swenson & Iglesias, 1996), OPAL opacities (Iglesias, Rogers & Wilson 1992), observed surface Z/X (from Grevesse & Noels 1993) and incorporates diffusion of helium and heavy elements. The inversion was performed using a combination of modes from the BiSON and LOWL groups, and was done using a subtractive OLA technique (cf., Pijpers & Thompson 1992,1994; Basu et al. 1996a,b). Note that the difference between the Sun and the model is extremely small – fractions of a percent in the case of sound speed. However, the differences are still significant. The most no-

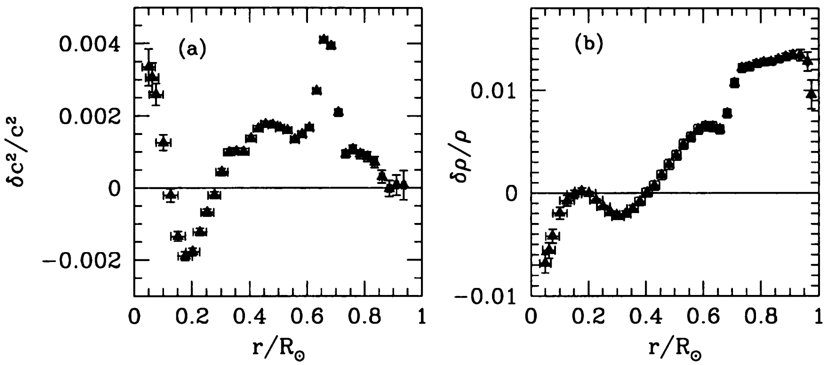

Figure 1. The relative squared sound-speed [panel (a)] and density [panel (b)] differences between the Sun and solar model S. The vertical error bars are 1σ propagated errors, while the horizontal bars are the distance between the quartile points of the averaging kernel and are a measure of the resolution.

ticeable difference is the larger sound-speed in the Sun just below the base of the convection zone. This could be due to the accumulation of excess helium below the convection zone and is a signature of mixing below the convection zone (cf., Gough et al. 1996). Model S does not incorporate any mixing below the convection zone (CZ). If there were mixing, the helium abundance locally would be reduced, decreasing the mean molecular weight, and hence increasing the sound-speed, thereby reducing the difference between the Sun and the model. The difference at the CZ base can also be removed by selective changes in the opacity (Tripathy et al. 1996).

The other region of large sound-speed difference is the core. The structure of the core is, however, still quite uncertain. Gough & Kosovichev (1993) and Gough et al. (1995), showed that the different sets of low-degree modes that are available give different results for the core, while Basu et al. (1996a) demonstrated that an indiscriminate combination of modes from different data sets could result in an erroneous interpretation of the core structure. The reason that the solar core is so uncertain is that only a few modes — those with very low degrees — penetrate to the core, and even those sample the core for a comparatively short time because of the large sound speeds there. It is hoped that more precise data from the SOHO instruments and the GONG network can improve the situation.

Model S is accepted as a SSM. But it does include diffusion of helium and heavy elements. Not so long ago, the term SSM was restricted to models without diffusion. There is evidence that diffusion of helium and heavy elements is important in the context of solar structure (cf., Cox et al. 1989, Christensen-Dalsgaard et al. 1993, etc.) For illustration we have shown the squared sound-speed difference between the Sun and a model without diffusion in Fig. 2(a). Also shown for comparison is the sound-speed difference

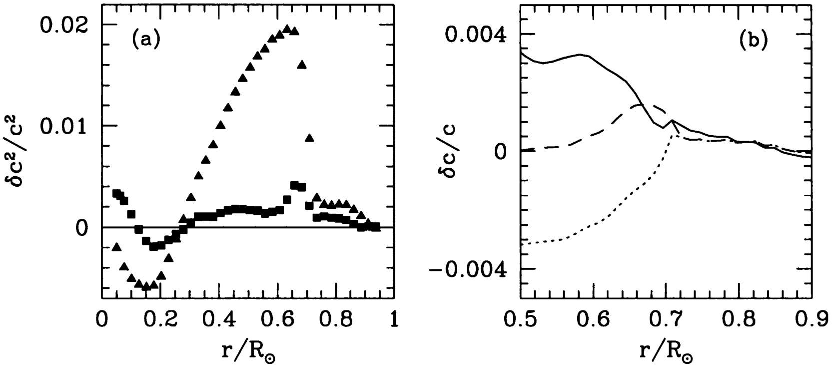

Figure 2. (a) The relative squared sound-speed difference between the Sun and a model without diffusion (triangles) and model S (squares). (b) The relative sound-speed difference between the Sun and solar envelope models without diffusion constructed with the Cox-Tabor opacities (solid line), OPAL opacities (dotted line), and a model with OPAL opacities which incorporates diffusion (dashed line). These models have a CZ depth of $0.287R_\odot$

with model S. The improvement on addition of diffusion is quite obvious.

Since the main cause of improvement in models with diffusion is a change in the convection zone depth – the model without diffusion has a shallower convection zone than the Sun, it can be argued that a change in the depth of CZ by any means will improve the results. This is not quite correct, as shown by Basu & Antia (1994a). As an illustration, in Fig. 2(b) the sound-speed difference between three solar-envelope models is shown. All models have identical CZ depths of $0.287R_\odot$. One model without diffusion has been constructed with the Cox-Tabor opacities (Cox & Tabor, 1976), the other two models were constructed with OPAL opacities, one with diffusion and one without. Thus we see that unless solar opacities are substantially lower than OPAL opacities, models need diffusion to achieve a better match to the solar sound-speed profile. Of course, it could still be argued that opacity can be changed and changing opacity gradient sufficiently could still be a way to match the solar sound-speed profile. Baturin & Ajukov (1996a) have found that substantial opacity changes are needed to construct a no-diffusion model with a sound-speed profile which agrees with the seismically determined profile. The indirect evidence that still goes against models without diffusion is the helium abundance in the solar envelope.

4. The solar helium abundance

Spectroscopic measurements of the abundance of helium in the Sun are very uncertain, hence, helioseismology plays a major role in determining the helium abundance in the solar envelope. The abundance is obtained

TABLE 1. Recent Estimations of the Solar Helium Abundance

Authors	MHD EOS	OPAL EOS	Method
Pérez Hernández, Christensen-Dalsgaard (1994)	0.242	-	Asymptotic
Basu, Antia (1995)	0.246	0.249	Asymptotic
Richard et al. (1996)	0.244	0.2505	Variational
Kosovichev (1996)	0.232	0.248	Variational
Baturin, Ajukov (1996b)	0.25	0.23	Asymptotic

from the variation of the adiabatic index of the solar material in the second helium ionization zone. This variation causes a change in the sound speed of that region. The changes in sound speed leave their signature on the function $H_1(w)$ and since the helium and hydrogen ionization zones are close to the surface, the function $H_2(\omega)$ is affected too.

Helioseismic techniques to determine the solar helium abundance can be roughly divided into two categories: the asymptotic and the variational. In the asymptotic case either the function $H_1(w)$ or $H_2(\omega)$, or the sound-speed as derived from $H_1(w)$ are used to determine the helium abundance, (cf., Gough 1984, Pérez Hernández & Christensen-Dalsgaard 1994, Vorontsov et al. 1992, Basu & Antia 1995). The variational method involved converting the kernels for c^2 and ρ (cf., Eq. (5)) to kernels for u and Y using the derivatives of the equation of state, and then using either RLS or OLA techniques to determine the helium abundance in the solar CZ (cf., Däppen et al. 1991, Kosovichev et al. 1992)

However, the results obtained by different workers have not been fully consistent with each other. It is found that the helioseismic measurement of helium abundance is sensitive to the equation of state of stellar material, which is used in translating the variation of Γ_1 to the difference in Y, and most of the difference in the older determinations can be attributed to this fact. However, some discrepancy still remains. Table 1 shows some of the more recent determination of the solar helium abundance. Results for the MHD (Hummer & Mihalas 1988; Mihalas et al. 1988, Däppen et al. 1988) and OPAL EOS (Rogers 1994, Rogers, Swenson & Iglesias, 1996) are shown, these being the two detailed EOS available for solar applications. The errors in the determination due to errors in the observed frequencies is much smaller than the differences due to EOS effects, and hence have not been included. The main point to note is that the helium abundance is quite low, between 0.24–0.25. This is compatible with solar evolution theories only if helium settles out of the envelope into the radiative zone. In absence of settling, the present day helium abundance in the solar envelope has to be about 0.27–0.28 to satisfy solar luminosity constraints.

TABLE 2. Position of the base of the solar convection zone

Authors	Position of CZ base ($R_\odot$)
Vorontsov (1988)	0.70 ± 0.01
Christensen-Dalsgaard, Gough, Thompson (1991)	0.713 ± 0.003
Kosovichev, Fedorova (1991)	0.713 ± 0.003
Guzik, Cox (1993)	0.712 ± 0.001
Basu, Antia (1996)	
Calibration models without diffusion	0.7141 ± 0.0002
Models with usual treatment of diffusion	0.7105 ± 0.0002
Models with X-profile from inversions	0.7133 ± 0.0002
Final value (including systematic errors)	0.713 ± 0.001

5. Depth of the solar convection zone

The transition of the temperature gradient from the adiabatic to radiative values at the base of the solar convection zone (CZ) leaves its signature on the sound speed. Thus if there are two otherwise similar solar models with different depths of the convection zone, then the model with a deeper convection zone will have an excess of sound speed over the other just below the base of the convection zone. Thus helioseismic measurement of the sound speed enables a determination of the position of the base of the convection zone. This has been used by a number of authors to estimate the position of the base of the convection zone. The results of various determinations of the CZ depth are shown in Table 2. All results roughly agree with each other within the error bars. All authors, except Guzik & Cox (1993), use the inverted sound-speed to determine the CZ depth. Guzik & Cox (1993) use a direct comparison of frequencies to estimate the CZ depth.

Unfortunately, the change in temperature gradient is not the only factor which leaves its imprint on the sound speed near the base of the convection zone. The abundance profiles also affect the sound-speed profile, and can confuse the signal due to change in temperature gradients. The excess helium just below the CZ due to settling causes an increase in the mean molecular weight below the base of the convection zone, and this reduces the sound speed. Thus a model with helium diffusion will appear to have a shallower convection zone, i.e., in regions just below the base of CZ, it will have sound speed similar to a no settling model with shallower CZ. Most authors have not taken this into account, since most CZ-depth determinations were made before the importance of settling was realized.

TABLE 3. Extent of overshoot below the solar convective zone

Authors	Overshoot
Gough, Sekii (1993)	" No convincing evidence"
Roxburgh, Vorontsov (1994)	$< 0.25\ H_p$
Monteiro et al. (1994)	$< 0.07\ H_p$
Basu et al. (1994)	$< 0.10\ H_p$
Basu, Antia (1994b)	$0.05^{+0.08}_{-0.05}\ H_p$
Christensen-Dalsgaard et al. (1995)	$< 0.10 H_p$
Basu (1996)	$< 0.05 H_p$

6. Overshoot below the solar convection zone

Theoretical estimates of the extent of overshoot are uncertain since that requires a non-local theory of turbulence. Recently, such theories have become available (e.g. Canuto & Dubovikov 1996) which naturally lead to overshoot. Work currently in progress (Canuto et al. 1996) seems to indicate that the extent of penetration is small.

The discontinuity in the derivatives of the sound speed at the base of the overshoot layer below the solar convection zone (CZ) introduces a characteristic oscillatory component in the frequencies of solar p-modes as a function of the radial order n (Gough 1990). The amplitude of these oscillations depends on the 'severity' of the discontinuity, which in turn depends on the extent of overshoot, while the period of the signal gives an estimate of the position of the discontinuity. This signal can be extracted and calibrated to find the depth of overshoot, as has been done by a number of groups and the results are shown in Table 3. The consensus seems to be that any overshoot below the convection zone is small. These studies however, assume that the overshoot layer is adiabatically stratified. This is probably true in regions where the convective velocity is large enough to transmit significant convective flux, but if convective velocity becomes too small then the temperature gradient is likely to approach the radiative value and the resulting structure will not be different from the radiative layers and it may not be possible to detect such layers helioseismically.

Like the case of the position of the CZ base, the abundance gradients caused by diffusion at the CZ base confuse the signal from overshoot. In this case any gradient in the helium abundance increases the signal due to the discontinuity since the helium abundance gradient causes a sharp change in the sound-speed. Thus models with a sharp abundance profile have a larger signal than models with smooth or no abundance gradients. This fact was used by Basu & Antia (1994b), Basu (1996) to determine the validity of the abundance profiles produced by different formalisms of diffusion. Only models which have a smooth abundance profile at the CZ base are consistent with observations. These include models constructed with the abundance profiles obtained from models which have mixing below the CZ (e.g., Richard et al. 1996), or from secondary inversions. Model S does not satisfy the observational constraint and this is consistent with the sharp feature found in the sound-speed difference with the Sun (cf. Fig. 1). Models which appear to satisfy the sound-speed constraint, but have steep abundance gradients (e.g., the gradual mass-losing model 3b of Guzik & Cox 1995) do not satisfy the solar constraint either. Thus this appears to be a fairly sensitive test of the abundance gradient.

7. The solar equation of state

Tests for the equation of state so far have only been indirect – either through a comparison of the frequency differences between the Sun and models constructed with different EOS, or through a comparison of the sound-speed differences. Reviews of the solar EOS can be found in Christensen-Dalsgaard & Däppen (1992), and Däppen (1996).

For reasonably simple EOS's, like the EFF (cf., Eggleton et al. 1973), a simple comparison of frequencies is enough to know that the equation of state is not good enough to satisfy solar constraints (Christensen-Dalsgaard et al. 1988). With more sophisticated equations of state, like MHD and OPAL, the frequency differences are dominated by the signature of the improperly modeled solar surface and hence one needs to look at the inversion results.

Sound-speed inversion results first showed that the MHD equation of state was deficient in the CZ, just below the HeII ionisation zone (cf., Dziembowski et al. 1992, Antia & Basu 1994). The OPAL EOS does not show this deficiency (Basu & Antia 1995). In fact, there is now evidence that models constructed with the OPAL EOS give a better fit to the solar data in the lower convection zone and below the CZ too (Basu et al. 1996c). Recently Basu & Christensen-Dalsgaard (1996) have shown how one can invert for the intrinsic difference in Γ_1 (i.e., difference at fixed pressure, density and composition) between the Sun and a model. Unfortunately, the propagated errors are still quite large, so although the EFF EOS can be ruled out, one cannot make a significant distinction between the MHD and OPAL equations of state. The expected increase in data precision should enable us to use this inversion in the future.

8. Secondary Inversions

Helioseismology gives direct constraints on only the mechanical properties of the Sun. If the equation of state is assumed, or where solar plasma is fully ionized (e.g. the core), the sound-speed constraint gives a constraint on T/μ. To be able to extract information of the thermal structure or the composition, additional input, such as equation of state, nuclear reaction rates, and opacities are required. Thus inversions for T or μ are termed "secondary inversions". Secondary inversions are particularly important in the context of the solar neutrino problem, since neutrino flux predictions require a knowledge of the temperature and chemical composition profiles in the solar core. Three techniques have been used for secondary inversions so far: (1) Use the additional input to convert sound-speed and density kernels to kernels for X and Z (cf., Gough & Kosovichev 1990, Kosovichev 1996). This method assumes that the additional inputs are exact and have no errors. (2) Assume that inputs such as opacity may have uncertainties and determine the temperature and composition which satisfies the sound-speed (i.e., T/μ) and ρ constraints, but which requires a minimum change in opacities to satisfy the thermal balance equations (cf., Antia & Chitre 1995, 1996). (3) Solve the stellar structure equations, but instead of specifying composition profiles, as is usual, specify the sound-speed profile. The sound-speed profile is obtained by primary inversions. (cf., Shibahashi & Takata 1996).

Inversions for the helium abundance profile (Shibahashi et al. 1995, Kosovichev 1996, Antia & Chitre 1996) show clear evidence of helium settling, even though the results do not completely agree with one another. Kosovichev's results show the change in the helium abundance earlier than the accepted position of the solar CZ base. That is probably a reflection of the fact that the reference model used was one without diffusion and hence had a shallow convection zone depth. Antia & Chitre (1996) find that the profile is very close to that of a SSM like model S, however, the profile is much smoother below the CZ base, and the change in the helium abundance is not as sharp as in the model, which supports the scenario of turbulent diffusion. The inverted helium profile is similar to that of model 5 of Richard et al. (1996).

One of the aims behind investigating the thermal structure and abundance gradients in the Sun is to be able to predict neutrino fluxes. The earliest results on neutrino fluxes were quite uncertain. Whereas Gough & Kosovichev (1990) said that seismic constraints led to a lower neutrino flux than SSMs (which invariably have higher neutrino fluxes than observed values, see Bahcall & Pinsonneault 1992, 1995), Dziembowski et al. (1991) claimed that seismic constraints increased the neutrino fluxes. The later inversions still show that with the current input physics the neutrino fluxes are larger than the observed values. Shibahashi & Takata (1996) claim that ^{8}B and ^{7}Be neutrino fluxes can be reduced, but that model fails to satisfy the solar luminosity constraint. Antia & Chitre (1995, 1996) show that assuming the nuclear reaction rates are not uncertain, large changes in opacity are required to lower the neutrino fluxes, however, they say that even arbitrary changes in just opacity are not sufficient to satisfy any two of the three solar neutrino constraints (i.e., Chlorine, Gallium and Boron) simultaneously. Roxburgh (1996) is of the view that neutrino fluxes may be lowered in models that have a structure consistent with helioseismic results if there is slow diffusive mixing, the opacities are changed, and there are some other contributions to energy transport within the Sun. But it is fair to say that no solar model exists thus far which satisfies both helioseismic and neutrino constraints.

9. Conclusions

So what is the current seismic model of the Sun? Helioseismic results show that the structure of the Sun is remarkably close to that of a standard solar model. The structure of the solar core is still somewhat uncertain and it is hoped that the new data will help towards reducing this uncertainty. The sound-speed profile of the Sun is, however, smoother than that of a standard solar model. The most visible difference lies just below the solar convection zone and is most probably a consequence of mixing below the solar convection zone base.

Helioseismic estimations of the abundance of helium in the solar convection zone yields a value of between 0.24 and 0.25 in most cases. There is some uncertainty caused by the uncertainty in the equation of state. The depth of the solar convection zone is $0.287 \pm 0.001\mathrm{R}_\odot$, and it appears that there is very little overshoot ($< 0.05H_p$, i.e., < 2800 Km) below the solar convection zone. Although direct inversions for equation of state are not very precise yet, indirect evidence shows that of the equations of state available today, OPAL gives the best results, though discrepancies still remain.

Since helioseismology puts constraints only on T/μ, additional inputs are required to estimate the solar temperature and neutrino fluxes. From the results available so far, it does appear that solar neutrino constraints cannot be satisfied without changes in the opacity, nuclear reaction rates, or unless other contributions to the energy transport are present.

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References

Antia, H.M. (1996) *A&A*, 307, 609

Antia, H.M., Basu, S. (1994) *A&AS*, **107**, 421

Antia, H.M., Chitre, S.M. (1995) *ApJ*, **442**, 434

Antia, H.M., Chitre, S.M. (1996) *ApJ*, submitted

Bahcall, J.N., Pinsonneault, M.H. (1992) *Rev. Mod. Phys.*, **60**, 297

Bahcall, J.N., Pinsonneault, M.H. (1995) *Rev. Mod. Phys.*, **67**, 781

Basu, S. (1996) in preparation

Basu, S., Antia, H.M. (1994a) *JAA*, **15**, 143

Basu, S., Antia, H.M. (1994b) *MNRAS*, **269**, 1137

Basu, S., Antia, H.M. (1995) *MNRAS*, **276**, 1402

Basu, S., Antia, H.M. (1996) *MNRAS*, submitted

Basu, S., Christensen-Dalsgaard, J. (1996) in *proc: IAU Symp. 181, Posters Volume*, eds. J. Provost, F.-X. Schmider (Nice Observatory), in press

Basu, S., Antia, H.M., Narasimha, D. (1994) *MNRAS*, **267**, 209

Basu, S., Christensen-Dalsgaard, J., Schou, J., Thompson, M.J., Tomczyk, S. (1996a) *ApJ*, **460**, 1064

Basu, S., Christensen-Dalsgaard, J., Pérez Hernández, F., Thompson, M.J. (1996b) *MNRAS*, **280**, 651

Basu, S., Christensen-Dalsgaard, J., Schou, J., Thompson, M.J., Tomczyk, S. (1996c) *Bull. Astron. Soc. India*, **24**, 147

Baturin, V.A., Ajukov, S.V (1996a) *Astron. Reports*, **40**, 259

Baturin, V.A., Ajukov, S.V (1996b) in *Solar Convection and Oscillations and their Relationship*, eds., F.P. Pijpers, J. Christensen-Dalsgaard, C. Rosenthal, (Dordrecht: Kluwer), in press

Canuto, V.M., Dubovikov. M.S. (1996) *Phys. Fluids*, **8**, 12

Canuto, V. M. et al (1996) (in preparation)

Chandrasekhar, S. (1964) *ApJ*, **139**, 664

Christensen-Dalsgaard, J. (1996) *Bull. Astron Soc. India*, **24**, 103

Christensen-Dalsgaard, J., Berthomieu, G. (1991) in *Solar Interior and Atmosphere*, eds. A.N. Cox et al. Space Science Series, (Tucson: University of Arizona Press) p401

Christensen-Dalsgaard, J., Däppen, W. (1992) *A&AR*, **4**, 267

Christensen-Dalsgaard J., Thompson M.J. (1995), in *Proc. GONG'94*, eds., R.K. Ulrich, E.J. Rhodes Jr., W. Däppen, PASPC, **76**, p 144

Christensen-Dalsgaard, J., Däppen, W., Lebreton, Y. (1988) *Nature*, **336**, 634

Christensen-Dalsgaard, J., Gough, D.O., Pérez Hernández, F. (1988) *MNRAS*, **235**, 875

Christensen-Dalsgaard, J., Gough, D.O., Thompson, M.J. (1991) *ApJ*, **378**, 413

Christensen-Dalsgaard, J., Proffitt, C.R., Thompson, M.J. (1993) *ApJ*, **403**, L75

Christensen-Dalsgaard, J., Monteiro, M.J.P.F.G., Thompson, M.J., (1995) *MNRAS*, **276**, 283

Christensen-Dalsgaard, J., Däppen, W., Ajukov, S.V., et al. (1996) *Science*, **272**, 1286

Cox, A.N., Tabor, J.E. (1976) *ApJS*, **31**, 271

Cox, A.N., Guzik, J.A., Kidman, R.B. (1989) *ApJ*, **342**, 1187

Däppen, W. (1996) *Bull. Astron. Soc. India*, **24**, 151

Däppen, W., Mihalas, D., Hummer, D.G., Mihalas, B.W. (1988), *ApJ*, **332**, 261

Däppen, W., Gough, D.O., Kosovichev, A.G., Thompson, M.J. (1991) in *Lecture Notes in Physics*, eds., D.O. Gough, J. Toomre, **388** (Heidelberg: Springer), p 111

Duvall, T. L. (1982) *Nature*, **300**, 242

Dziembowski, W.A., Pamyatnykh, A.A., Sienkiewicz, R. (1990) *MNRAS*, **244**, 542

Dziembowski, W.A., Pamyatnykh, A.A., Sienkiewicz, R. (1991) *MNRAS*, **249**, 602

Dziembowski, W.A., Pamyatnykh, A.A., Sienkiewicz, R. (1992) *Acta Astron.*, **42**, 5

Dziembowski, W.A., Goode, P.R., Pamyatnykh, A.A., Sienkiewich, R., (1994) *ApJ*, **432**, 417

Eggleton, P.P., Faulkner, J., Flannery, B.P. (1973) *A&A*, **23**, 325

Elsworth, Y., Howe, R., Isaak, G.R., et al. (1994) *ApJ*, **434**, 801
Gelly, B., Fierry-Fraillon, D., Fossat, E., et al. (1996) *A& A*, submitted
Gough, D.O. (1984) *Mem. Soc. Astron. Ital.*, **55**, 13
Gough, D.O. (1985) *Sol. Phys.*, **100**, 65
Gough, D.O. (1990) in *Lecture Notes in Physics*, **367**, eds., Y. Osaki, H. Shibahashi, (Berlin: Springer) p 283
Gough D.O. (1993) in *Astrophysical Fluid Dynamics: Les Houches, Session XLVII*, eds. J.-P. Zahn, J. Zinn-Justin (Elsevier)
Gough, D.O., Kosovichev, A.G. (1990) in *Proc. IAU Colloq. 121: Inside the Sun*, eds., G. Berthomieu & M. Cribier, (Dordrecht: Kluwer), p 327
Gough, D.O., Kosovichev, A.G. (1993), *MNRAS*, **264**, 522
Gough, D.O., Sekii, T., in *GONG 1992:* , ed., T.M. Brown, ASPCS **42**, p 177
Gough., D. O., Thompson, M. J. (1991) in *Solar Interior and Atmosphere*, eds. A.N. Cox et al. Space Science Series, (Tucson: University of Arizona Press) 519
Gough, D.O., Kosovichev, A.G., Toutain, T. (1995) *Solar Phys.*, **157**, 1
Gough, D.O., Kosovichev, A.G., Toomre, J., et al. (1996) *Science*, **272**, 1296
Grevesse, N., Noels, A. (1993) in *Origin and evolution of the Elements*, eds., N. Prantzos, E. Vangioni-Flam, M. Cassé, (Cambridge: Cambridge Univ. Press), p 15
Guzik, J. A., Cox, A. N. (1993) *ApJ*, **411**, 394
Guzik, J. A., Cox, A. N. (1995) *ApJ*, **448**, 905
Hummer, D.G., Mihalas, D. (1988) *ApJ*, **331**, 794
Harvey, J.W., Hill, F., Hubbard, R.P., et al. (1996) *Science*, **272**, 1284
Iglesias, C.A., Rogers, F.J., Wilson, B.G. (1992) *ApJ*, **397**, 717
Kosovichev, A.G. (1996) *Bull. Astron. Soc. India*, **24**, 355
Kosovichev, A.G., Fedorova, A.V. (1991), *Sov. Astron.*, **35**, 507
Kosovichev, A.G., Christensen-Dalsgaard, J., Däppen, W., Dziembowski, W.A., Gough, D.O., Thompson, M.J. (1992) *MNRAS*, **259**, 536
Libbrecht, K.G., Woodard, M.F., Kaufman, J. M. (1990) *ApJS*, **74**, 1129
Marchenkov., K, Roxburgh, I.W., Vorontsov, S.V., (1996) in *proc: IAU Symp. 181, Posters Volume*, eds. J. Provost, F.-X. Schmider, (Nice Observatory), in press
Mihalas, D., Däppen, W., Hummer, D.G. (1988) *ApJ*, **331**, 815
Monteiro, M.J.P.F.G., Christensen-Dalsgaard, J., Thompson, M. J. (1994), *A&A*, **283**, 247
Pérez Hernández, F., Christensen-Dalsgaard, J. (1994) *MNRAS*, **267**, 111
Pijpers, F.P., Thompson, M.J., (1992) *A&A*, **262**, L33
Pijpers, F.P., Thompson, M.J., (1994) *A&A*, **281**, 231
Richard, O., Vauclair, S., Charbonnel, C., Dziembowski, W.A. (1996) *A&A*, **312**, 1000
Rogers, F.J. (1994) in *The Equation of State in Astrophysics, IAU Colloq. 147*, eds., G. Chabrier & E. Schatzman (Cambridge: Cambridge University Press), p 16
Rogers, F.J., Swenson, F.J., Iglesias, C.A. (1996) *ApJ*, **456**, 902
Roxburgh, I.W. (1996) *Bull. Astron. Soc. India*, **24**, 89
Roxburgh, I.W., Vorontsov, S.V. (1994) *MNRAS*, **268**, 880
Roxburgh, I.W., Vorontsov, S.V. (1996) *MNRAS*, **278**, 940
Schou, J., Tomczyk, S. (1996) in preparation
Shibahashi, H., Takata, M. (1996) *Bull. Astron. Soc. India*, **24**, 301
Shibahashi, H., Takata, M., Tanuma, S. (1995) in *Proc. Fourth SOHO Workshop, Vol. 2*, eds. J.T. Hoeksema, V. Domingo, B. Fleck, B. Battrick, (ESA SP-376), p 9
Tripathy, S.C., Basu, S., Christensen-Dalsgaard, J. (1996) in *proc: IAU Symp. 181, Posters Volume*, eds. J. Provost, F.-X. Schmider, (Nice Observatory), in press
Unno, W., Osaki, Y., Shibahashi, H. (1989) *Non-radial Oscillations of Stars, 2nd ed.*, (Tokyo: Univ. of Tokyo Press)
Vorontsov, S.V. (1988) in *Seismology of the Sun & Sun-like Stars*, eds., V. Domingo, E.J. Rolfe, ESA SP-286, p 475
Vorontsov, S.V., Shibahashi, H. (1991) *PASJ*, **43**, 739
Vorontsov, S.V., Baturin, V.A., Pamyatnykh, A.A. (1992) *MNRAS*, **257**, 32