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The effectiveness of imperfect weighting in advice taking

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Abstract

We investigate decision-making in the Judge-Advisor-System where one person, the "judge", wants to estimate the number of a certain entity and is given advice by another person. The question is how to combine the judge's initial estimate and that of the advisor in order to get the optimal expected outcome. A previous approach compared two frequently applied strategies, taking the average or choosing the better estimate. In most situations, averaging produced the better estimates. However, this approach neglected a third strategy that judges frequently use, namely a weighted mean of the judges' initial estimate and the advice. We compare the performance of averaging and choosing to weighting in a theoretical analysis. If the judge can, without error, detect ability differences between judge and advisor, a straight-forward calculation shows that weighting outperforms both of these strategies. More interestingly, after introducing errors in the perception of the ability differences, we show that such *imperfect* weighting may or may not be the optimal strategy. The relative performance of imperfect weighting compared to averaging or choosing depends on the size of the actual ability differences as well as the magnitude of the error. However, for a sizeable range of ability differences and errors, weighting is preferable to averaging and more so to choosing. Our analysis expands previous research by showing that weighting, even when imperfect, is an appropriate advice taking strategy and under which circumstances judges benefit most from applying it.

Keywords: advice taking, judge-advisor-system, rational behavior, normative model.

1 Introduction

A famous saying holds that "two heads are better than one". Accordingly, when making important judgments we rarely do so on our own. Instead, we consult others for advice in the hope that our advisor will provide us with additional insights, expert knowledge or an outside perspective - in short, an independent second opinion. Previous research on advice taking has consistently shown that heeding advice does, in fact, increase the accuracy of judgments (e.g., Gino & Schweitzer, 2008; Minson, Liberman, & Ross, 2011; Sniezek, Schrah, & Dalal, 2004). However, a commonly observed phenomenon is the suboptimal utilization of advice, that is, judges do not heed the advice as much as they should according to its quality (e.g., Harvey & Fischer, 1997; Yaniv & Kleinberger, 2000); for reviews see Bonaccio and Dalal (2006); Yaniv (2004). As a consequence, the de facto improvement in judgment quality observed in many judge-advisor studies is inferior to the improvement that judges could have obtained if they had utilized the advice in the optimal way (Minson & Mueller, 2012). The critical question, however, is what constitutes the optimal advice taking strategy. Our main goal is to provide an answer to this question that goes beyond previous research. To this end, we will first discuss the existing approach on the optimal utilization of advice and, then, build on it to arrive at a normative model of advice taking.

Our analysis will build on the logic of the framework commonly used for studying advice taking, the judge-advisor-system (JAS, Sniezek & Buckley, 1995). In the JAS, one person (the "judge") first makes an initial estimate regarding a certain unknown quantity and then receives advice in the form of the estimate another person (the "advisor"), provided independently. The judge then makes a final, and possibly revised, estimate. Comparison of the initial and final estimates allows one to determine the degree to which the judge utilized the advice, and advice utilization is usually expressed as the percent weight of the advice when making the final estimate (e.g., Harvey & Fischer, 1997; Yaniv & Kleinberger, 2000). How strongly should the judge heed the advice in order to come up with the best possible final estimate? So far, our understanding of the optimal degree of advice utilization is lim-

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This paper is dedicated to Nicola Knight, whose untimely death saddened us all. Nicola contributed much inspiration and hard work during the design phase of this study.

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ited. In situations in which judge and advisor are known to be equally competent or in which comparable expertise is the best assumption—for example when judge and advisor are drawn from the same population and there is no valid information on their relative expertise—the normatively correct strategy is to average the initial estimate and the advice (e.g., Harvey & Fischer, 1997; Soll & Larrick, 2009; Yaniv & Kleinberger, 2000). Similarly, for multiple decision makers, the boundary condition for individual experts to be more accurate than the crowd average is very high ("wisdom of the crowd", Davis-Stober, Budescu, Dana, & Broomell, 2014). However, for situations in which there are ability differences between judge and advisor, determining the optimal advice taking strategy is more difficult.

One approach to answering the question is to employ more general models of judgmental aggregation that are concerned with tapping into the wisdom of the crowds (e.g., Davis-Stober et al., 2014; Einhorn, Hogarth, & Klempner, 1977; Mannes, Soll, & Larrick, 2014). These models aim at minimizing judgment errors by combining several judgments in the most sensible fashion. Despite differing in the underlying assumptions and/or the error measures applied, these models consistently reveal that averaging the individual judgments is a very effective strategy. In addition, simple averaging can usually be outperformed by choosing the supposedly best—or a small subset of particularly competent—judges if there are sufficient data to reliably identify the experts. One reason for the prevalence of averaging as the most robust strategy—particularly when compared to weighted averages—is the high number of individual judgments and the associated inflation of errors when trying to estimate their relative accuracy (Dawes, 1979).

However, this error inflation might be less of a problem in classic judge-advisor systems with only two judgments. We, therefore, now turn to the more specific question of the optimal aggregation of opinions in judge-advisor dyads. To the best of our knowledge, the only formal model that addresses the question of optimal advice utilization in the face of ability differences between judge and advisor is the PAR model by Soll and Larrick (2009).

1.1 The PAR model of advice taking

The PAR model makes statements about the effectiveness of advice taking strategies based on the three parameters of the JAS, ability differences between judge and advisor (A), the probability of the judge detecting these differences (P), and the degree to which the two judgments contain redundant information (R). Based on these parameters, the PAR model compares two very specific weighting strategies, namely equal weighting (i.e., averaging) and choosing the supposedly more accurate estimate. Averaging is a powerful strategy because it is a statistical truth that the arithmetic mean of the judges' initial estimate and the advice is, on average, equally or more accurate than the initial estimate (Soll & Larrick, 2009). If the advisor's estimate is independent from the judge's initial estimate, averaging the initial estimate and the advice results in a reduction of unsystematic and—in some cases—systematic errors (Soll & Larrick, 2009; Yaniv, 2004).

The averaging strategy performs best if judge and advisor are equally competent. However, usually one judge is better. Averaging is unlikely to be optimal when the difference is large enough. The critical question, then, is how judges should utilize advice when they perceive it to be more or less accurate than their own initial estimates. The PAR model offers an alternative to averaging in the form of the choosing strategy, that is, the judge either maintains the initial estimate or fully adopts the advice, depending on which of the two estimates he or she thinks is more accurate.

The theoretical analysis of the performance of the two advice taking strategies suggests that judges should average their initial estimate and the advice in most of the cases. That is, even if judge and advisor differ in their ability, averaging often provides better results than choosing. The exceptions to this rule are situations in which there are strong and easily identifiable ability differences, and the advantage of choosing increases even more if judge and advisor share a systematic bias. In those cases, judges are usually better off simply choosing the supposedly more accurate estimate.

A possible downside of the PAR model is its focus on only two advice taking strategies. Soll and Larrick (2009) provide strong arguments for this restriction, namely that these strategies are simple to use and that these strategies, averaging and choosing, account for about two thirds of the strategy choices in advice taking. They back up this argument with data from four experiments showing that judges used a choosing strategy in close to 50% of the cases and relied on averaging in about 20% of the cases. However, these results imply that judges also may have adhered to a third strategy more than 30% of the time, namely weighting. In fact, while less frequent than choosing, judges seemed to prefer a weighting strategy to pure averaging. A study by Soll and Mannes (2011) showed a similar pattern; depending on the experimental conditions, judges utilized a weighting strategy in about 30 to 40% of the trials.

As previous studies (Soll & Larrick, 2009; Soll & Mannes, 2011) show, judges seem to engage in three rather than only two strategies when utilizing advice: choosing, averaging, and weighting. However, the PAR model allows us to compare only choosing and averaging. In order to make claims about the appropriateness of weighting, we require a different model that informs us about the optimal weight of advice. Ideally, we want to know, for any given constellation of a judge and an advisor who may differ with regards to their judgmental accuracy, how much weight the judge should assign to the advice in order to maximize the accuracy of the final estimates. Importantly, and comparable to the PAR model, these optimal weights need to be of normative character rather than being calculated post-hoc, that is, we need to state—a priori—which weighting scheme has the lowest expected judgmental error. In the following, we will describe a model that—similar to the PAR model—determines the effectiveness of weighted averaging based on ability differences between judge and advisor, as well as the ability of the judge to detect these differences. We will then compare the accuracy of the final estimates that would result from weighting to the expected accuracy of a pure averaging strategy as well as a choosing strategy and test under which conditions weighting is the more appropriate strategy.

2 Model and results

2.1 Weighted Mean

For the purpose of our model, and in accordance with the basic JAS, we assume that two people, a judge J and an advisor A, are tasked with estimating an unknown quantity (e.g., the distance between two cities). They first provide individual estimates, and then J wants to find the best possible final estimate after receiving A's estimate as advice. Let us denote J's a priori estimate by x_J and A's a priori estimate by x_A. The question is how to find an optimal method for combining the information from x_J and x_A. Most present models focus on comparing methods frequently observed in empirical studies (e.g., Soll & Larrick, 2009)[1]. In contrast, we seek to find the theoretically optimal method. Naturally, this comes at the price of making more bold assumptions. So, let us assume that the estimates of both judge and advisor are independent and drawn from a normal distribution centered on the true value x_T with variances σ_J^2 and σ_A^2. From this information, we can compute that the most likely estimation for the true value $\tilde{x}$ (using the most-likelihood method, see Appendix 4.1) is given by

$$\tilde{x} = \frac{x_J \sigma_A^2 + x_A \sigma_J^2}{\sigma_J^2 + \sigma_A^2} \tag{1}$$

which happens to be a weighted mean[2] x_w

$$x_w = w x_J + (1 - w) x_A \tag{2}$$

of x_J and x_A with the weight w.

$$w = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_J^2} \tag{3}$$

Denoting the *ability ratio* by m

$$m = \frac{\sigma_A^2}{\sigma_J^2} \tag{4}$$

we can rewrite the weighted mean x_w as

$$x_w = \frac{m}{1+m} x_J + \frac{1}{1+m} x_A \tag{5}$$

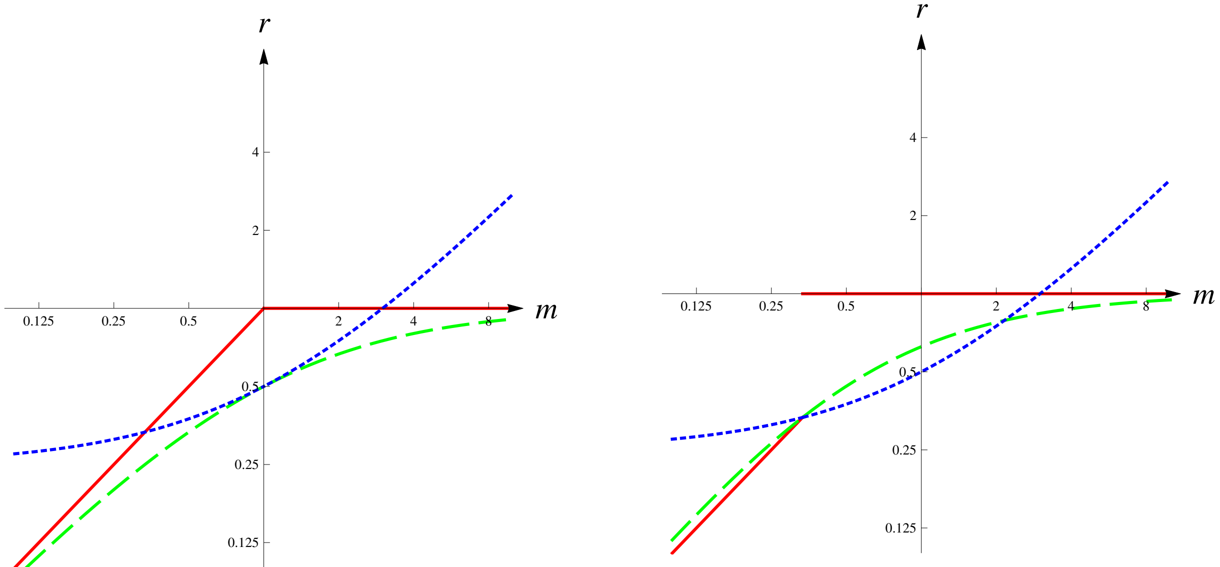
If $m > 1$, the judge is better than the advisor and, if $m < 1$, the advisor is better than the judge. In words, the judge needs to estimate "How much am I better at this task than my advisor?" or "How much is my advisor better than me?" For example, if the advisor's error variance is 1 arbitrary unit and the judge's error variance is 3 of those units, the weight that should be placed on the advice is 75%. If both error variances are equal, the optimal strategy is to weight the advice by 50%.

Essentially, the calculation yields two intuitive insights: first, as long as the error variance of both the judge and the advisor is nonzero and limited, their judgments should never be completely ignored. That is, weighting is bound to yield more accurate judgments than choosing the more accurate judgment. Second, the expected error of the weighted average is always smaller or equal to that of the arithmetic mean (they are equal if the optimal weight is 0.5, see Appendix 4.2). On a theoretical level, perfect weighting is therefore, by definition, superior to the PAR-models choosing and averaging strategies. In the next section we show that errors in the perception of the ability ratio imply that any of the three methods can be optimal, depending on the parameters.

[1] Our model differs from the PAR model in three aspects. First, whereas both the PAR and our model assume normally distributed estimates, our model makes the additional assumption of unbiased estimates for the sake of simplicity. Second, the error measures differ: while the PAR model measures judgment errors in terms of the mean absolute error, we chose the mean squared error due to its favorable mathematical properties. Note, that the choice of error measures can change the results only quantitatively, but not qualitatively. That is, if one aggregation strategy is superior to another it is so regardless of the error measure applied. Finally, our models differ in the way the recognition of ability differences is operationalized. Whereas the PAR model models it in terms of a correlation between two binary variables (which dyad member is more competent vs. which dyad member does the judge perceive to be more competent), our model treats the recognition of relative expertise as a continuous variable. This variable not only states which dyad member is more accurate but also quantifies the magnitude of the ability difference. The latter is necessary in order to determine the (perceived) optimal weight of advice.

[2] If, instead of deriving the optimal method theoretically, we would restrict ourselves on the method of assigning linear weights (*weighting*) to x_A and x_J, we could compute the optimal weight by simply optimizing the equation $\sigma_w^2 = (1-w)^2 \sigma_J^2 + w^2 \sigma_A^2$ with respect to σ_w^2.

Figure 1: Plots of relative improvement r of accuracy (i.e., reduction of variance) depending on the ability ratio m after considering the advisor's advice using three different methods: Choosing the better estimate (red plain), averaging both estimates equally (blue dotted), and weighting the estimates according to ability ratio (green dashed). Since r is measuring the change of variance compared to the initial estimate, $r < 0$ means an improvement while $r > 1$ means worsening of the initial estimate. Both axes are in logarithmic scale.



(a) Here, weighting uses the precise ability ratio m and choosing identifies the correct expert at 100%.

(b) The judge overestimates her ability relative to that of the advisor by 200% (i.e., $p = 3$), resulting in imperfect weighting and, for some values of m, choosing the wrong estimate.

2.2 Imperfect weighting: The effect of errors in assessing the ability differences

As we have demonstrated in the last subsection, perfect weighting is superior to choosing and averaging. However, perfect weighting requires that the ability ratio between judge and advisor is known to the judge. Despite judges' ability to differentiate between good and bad advice beyond chance level (e.g., Harvey & Fischer, 1997; Harvey, Harries, & Fischer, 2000; Yaniv, 2004; Yaniv & Kleinberger, 2000) exact knowledge of m is unlikely. Let us, accordingly, assume that m must be estimated by the judge and is, therefore, subject to errors or biases. In essence, regardless of whether such a mistake is systematic or not, the judge can either under- or over-estimate the true value of m, and we denote the degree to which the judge does so by the factor p. If p equals 1, the judge has a perfect representation of the ability ratio. In contrast, values greater than 1 indicate that the judge's perception of the ability erroneously shift in his or her favor, whereas values smaller than 1 mean that the judge overestimates the ability of the advisor. Technically speaking, p varies misconception by either magnifying or dampening the ratio m. Thus, instead of (5) the judge's final result reads as

$$\tilde{x}(p) = \frac{pm}{1+pm} x_J + \frac{1}{1+pm} x_A \tag{6}$$

and the variance of $\tilde{x}(p)$ is given by

$$\sigma_p^2 = \frac{m^2 p^2 \sigma_J^2 + \sigma_A^2}{(1+pm)^2} \tag{7}$$

In this case, the final estimate by weighting the two initial estimates differently might end up being worse than taking the simple average. This would happen if the ability ratio is (i) not very large and (ii) poorly estimated. The weighted mean might also end up being worse than choosing the better guess. This would happen if the competence ratio is actually large, but is perceived as small. To see the full picture we need to compare the relative improvements

$$r = \frac{\text{variance of final guess}}{\text{variance of initial guess}} \tag{8}$$

of the judge. Values smaller than 1 indicate that the error variance of the final estimates is smaller than that of the initial estimate, that is, the final estimates are more accurate. In contrast, if the final estimates are less accurate than the initial estimates, r will assume values greater than 1. We determine the expected values of r for the three

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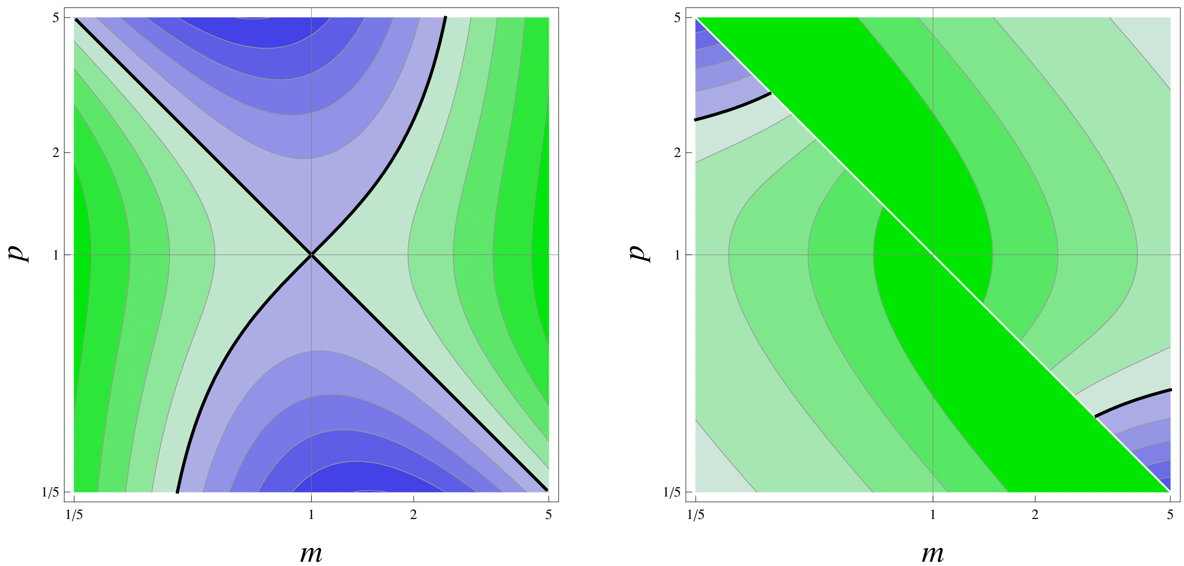
Figure 2: Contour plot of the relative difference k of averaging/weighting (a) and choosing/weighting (b). The two methods are equally efficient at the thick black lines. In the green region weighting is more efficient while in the blue region averaging (a) / choosing (b) are more efficient. Again, efficiency is measured in the reduction of variance compared to the initial estimate: if weighting reduces more variance than averaging/choosing, it is more efficient. At the thick black line, $k = 1$. Contour lines represent steps of 10%, i.e., $k = 0.6, 0.7, ..., 1.4, 1.5$



(a) Averaging vs. weighting

(b) Choosing vs. weighting

advice-taking strategies as a function of the parameters m and p (except, for averaging, which does not depend on p). For averaging, we get

$$r_{averaging}(m) = \frac{\sigma_a^2}{\sigma_J^2} = \frac{1}{4}\frac{\sigma_J^2 + \sigma_A^2}{\sigma_J^2} = \frac{1+m}{4} \tag{9}$$

with the expected variance of averaging $\sigma_a^2 = \frac{1}{4}(\sigma_J^2 + \sigma_A^2)$. For weighting, we get

$$r_{weighting}(m,p) = \frac{\sigma_p^2}{\sigma_J^2} = \frac{m^2p^2\sigma_J^2 + \sigma_A^2}{(1+pm)^2\sigma_J^2} \tag{10}$$

$$= \frac{m^2p^2}{(1+pm)^2} + \frac{m}{(1+pm)^2} = \frac{m(1+p^2m)}{(1+pm)^2} \tag{11}$$

For choosing, we first observe that $r_{choosing}$ can only be either 1, or m. In the first case, the judge chooses her own estimate and therefore can neither improve nor worsen. In the latter case, the accuracy changes exactly by the competence ratio m. Essentially, the judge must guess whether $m > 1$ or $m < 1$. However, she knows only pm instead of m which gives

$$r_{choosing}(m,p) = \begin{cases} m, & \text{if } pm < 1 \\ 1, & \text{else} \end{cases} \tag{12}$$

Obviously, the judge does not always identify the correct expert. This happens if either m is chosen despite $m > 1$ (because $pm < 1$) or of 1 is chosen despite $m < 1$ (because $pm > 1$). Essentially, these three r-functions tell us how much the judge improves or worsens her initial estimate by using either averaging, weighting or choosing.

In Figure 1, we show LogLog Plots[1] with fixed p, $p = 1$ (left panel) and $p = 3$ (right panel) varying the ability ratio m. In line with the reasoning above, Figure 1(a) shows that if the judge can correctly assess the ability differences, weighting outperforms both averaging and choosing. However, as we can see in Figure 1(b), the relative performance of the three strategies differs for specific parameter regions. In our example, the judge overestimates her ability relative to that of the advisor by 200% (i.e., $p = 3$). In this case, averaging outperforms weighting for small ability ratios, and choosing outperforms weighting if the advisor is substantially more accurate than the judge.

Next, we want to explore the full parameter space of m and p. To this end, we need to compare the relative im-

[1] A brief remark for readers unfamiliar with LogLog plots: Since the variables m and r that we wish to plot are relations, we need to scale the axes accordingly. A value of $m = 0.5$ means that the judge is twice as good as the advisor while $m = 2$ means that the advisor is twice as good as the judge. Similarly for $m = 0.1$ and $m = 10$. This means that we need to treat the two intervals $(0; 1)$ and $(1; \infty)$ equally. Further, we must center the plot around 1 instead of 0 because a value of $m = 1$ indictaes equal accuracy of judge and advisor. This is accomplished by Log(-arithmic) scaling. Double logarithmic scaling (i.e., LogLog Plots) scales both axes logarithmically.

provement in accuracy obtained by the different strategies as a function of the model parameters p and m. Specifically, we are interested in the relative performance of weighting on one hand and either choosing or averaging on the other (for an in-depth comparison of choosing and averaging, see Soll & Larrick, 2009), which we denote as

$$k_{averaging} = \frac{r_{weighting}}{r_{averaging}} \tag{13}$$

and

$$k_{choosing} = \frac{r_{weighting}}{r_{choosing}} \tag{14}$$

respectively. A value of $k = 1$ indicates that weighting and the comparison strategy (averaging or choosing) perform equally well whereas values of $k > 1$ indicate superior performance of weighting, and values of $k < 1$ indicate that the respective comparison strategy performs better. The target value k is represented by the shade in the contour plot spanned by the parameters m and p (see Figure 2). The bold line separating the blue and green regions is the iso-accuracy curve which indicates that the accuracy of the weighting strategy equals that of the comparison strategy (i.e., $k = 1$). For each subsequent line in the green area, k increases by 0.1, that is, the weighting-method performs 10% better than averaging/choosing, while in the blue area the opposite is true.

As can be seen in Figure 2a, if there are ability differences between judge and advisor and the judge has a rough representation of these differences, weighting is superior to simple averaging. In contrast, whenever the ability differences are small and/or difficult to detect, judges will benefit more from averaging. The accuracy differences between weighting and choosing are more pronounced (see Figure 2b). Obviously, the judge must make extreme errors when assessing m in order for choosing to be the better advice taking strategy. In addition, choosing can outperform weighting only if correctly identifying the better estimate. This is the case above the white diagonal in Figure 2b for $m > 1$, and below the diagonal for $m < 1$. Note that the second prerequisite creates an asymmetry in the results. This asymmetry is rooted in the fact that choosing is heavily penalized if the judge erroneously chooses the wrong estimate while weighting is much less prone to such extreme errors because it still assigns some weight to the more accurate judgment.

Our analysis so far revealed that weighting is quite a powerful strategy when comparing it to either averaging or choosing. However, one rationale that we can derive from Soll and Larrick's (2009) PAR model is that judges should switch between averaging and choosing in order to maximize the accuracy of their final estimates. Specifically, they should average when ability differences are small and/or difficult to detect and choose when the opposite is true. An interesting vantage point, then, is to compare weighting to a combination of choosing and averaging.

2.3 Combining averaging and choosing

Let us assume that judges know when they should switch from averaging to choosing based on their (potentially biased) perception of m. We can easily compute this threshold by equating $r_{choosing}$ and $r_{averaging}$

$$\frac{1+m}{4} = 1 \tag{15}$$

$$\Leftrightarrow m = 3 \tag{16}$$

if, choosing one self, and

$$\frac{1+m}{4} = m \tag{17}$$

$$\Leftrightarrow m = 1/3 \tag{18}$$

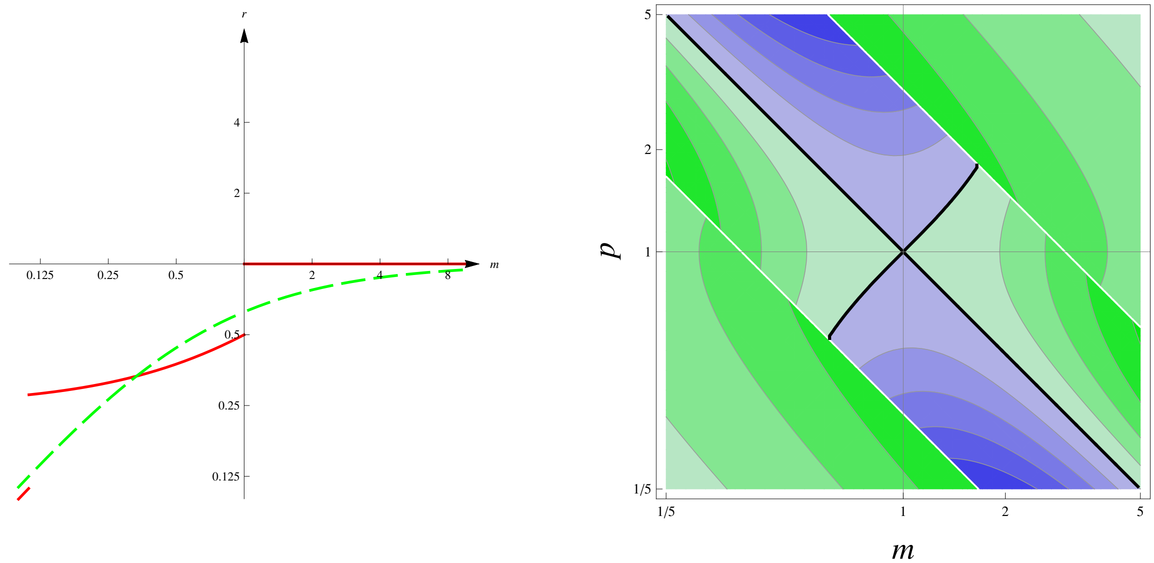
if choosing the advisor. Since the judge estimates m as pm, she will change whenever $pm = 3$ or $pm = \frac{1}{3}$. In other words, a perfect application of the combined strategy implies that judges average their initial estimates and the advice until they perceive the initial estimates to be three times as accurate as the advice or vice versa; if this threshold is passed, they choose the more accurate estimate. If m is estimated without error (i.e., $p = 1$), dynamically switching between choosing and averaging is a powerful strategy. However, we have to take into account that if $p \neq 1$, choosing will not always be correct, since the judge may erroneously choose the less accurate judgment. This flaw drastically reduces the performance of the combined strategy, because choosing the wrong expert has highly negative consequences.

In order to compare weighting to the combined strategy of choosing and averaging, we first determine the accuracy gains relative to the initial estimates that would result from a combination of choosing and averaging, $r_{combined}$. Figure 3 (left panel) compares the accuracy ratios of the combined strategy as well as that of weighting as a function of m and assuming that the judge is strongly overestimating his or her own accuracy ($p = 3$). We next calculated the ratio of the accuracy gain obtained by weighting and that obtained by the combined strategy:

$$k_{combined} = \frac{r_{weighting}}{r_{combined}} \tag{19}$$

The right panel of Figure 3 shows $k_{combined}$ as a function of m and p. The white lines denote the threshold at which judges switch from averaging to choosing based on their perception of the relative accuracy of judge and advisor (i.e., when the product pm is greater than 3 or smaller than 1/3). The bold lines, again, denote the iso-accuracy-curves. The analysis reveals some interesting findings.

Figure 3: Comparing weighting to the combination of choosing and averaging.



(a) Relative improvement of accuracy (as in Fig.1) of weighting (green dashed) and the combined method (red plain), both for $p = 3$. Note that imperfect estimation of m leads to choosing the wrong judgment in a certain parameter regions.

(b) Generalization of (a) by allowing for varying p (as in Fig. 2). In the green area, weighting is the better strategy, while in the blue area the combined method performs better. The contour lines denote increases or decreases in steps of 10%.

First, weighting is superior to the combined strategy in a wide range of situations. Second, the superiority of the weighting strategy is mostly due to the relatively weak performance of choosing. The problem is that the application of the combined strategy sometimes leads to choosing in situations in which averaging would outperform weighting but choosing does not. This happens when ability differences are small and difficult to assess (i.e., m close to 1 and p either very small or very large). Instances where the choosing part of the combined strategy performs better than the weighting strategy occur only for extreme competence differences outside of the parameter range of Figure 3.

3 Discussion

The aim of our theoretical analysis was to answer the question which advice-taking strategy judges in a judge-advisor system should utilize in order to maximize the accuracy of their revised estimates. Previous research has suggested that judges should average their initial estimates and the advice unless the difference in accuracy between the two estimates is large and easily identifiable; in such cases they should simply choose the more accurate estimate (Soll & Larrick, 2009). It is a mathematical fact that averaging two independent and unbiased estimates leads to, on average, more accurate judgments (e.g., Larrick & Soll, 2006; Yaniv, 2004). However, if the error variance of the two judgments is unequal, there is an optimal weight of advice that produces combined estimates that are always equal or better than simple averaging with regards to accuracy. As a consequence, judges in a judge-advisor system would benefit the most from weighting the advice according to its accuracy relative to that of the judges' initial estimate (D. Budescu, 2006; D. V. Budescu & Yu, 2006). Similar to choosing the better estimate, the potential superiority of the weighting strategy compared to pure averaging comes at the cost of additional information, namely knowledge of the ability difference between judge and advisor.

If this ability difference is known, a weighting strategy is bound to be superior to both, averaging and choosing. Yet, it is rather unlikely that judges will be able to correctly recognize differences between their own and their advisor's ability with perfect accuracy. Instead, previous research suggests that while judges have some ability to assess the relative quality of advice they frequently underestimate it (e.g., Harvey & Fischer, 1997; Harvey et al., 2000; Yaniv & Kleinberger, 2000). In other situations, for example, when judges perceive the task as very difficult (Gino & Moore, 2007) or when they are very anxious, they are prone to overestimate the quality of the advice relative to that of their own initial estimates (Gino, Brooks, & Schweitzer, 2012). If judges' assessment of the ability differences are subject to errors the resulting weighting strategy will result in less accurate judgments, and if these errors become too large, simple averaging turns out to be the better strategy. The fact that the averaging strategy can outperform weighting strategies that are based on erroneous weights has been previously documented in multi-cue judgments (Dawes, 1979), and the advantage of averaging increases as the number of cues grows. Hence, the first question we aimed to answer was under which conditions imperfect weighting outperforms averaging. To this end, we compared the expected performance of both strategies as a function of ability differences between judge and advisor as well as the accuracy of the judge when estimating these differences.

Our analysis revealed that imperfect weighting outperforms averaging as long as there are at least moderate ability differences. This performance advantage of the weighting strategy is rather robust against moderate misperceptions of the ability differences. For example, if the judge's error was 50% larger than that of the advisor, weighting is superior to averaging even if the judge under- or overestimates the ability difference by 50%. Additionally, the larger the ability differences become the more robust the weighting strategy becomes against erroneous assessment of these differences. In other words, averaging is likely to produce better estimates than imperfect weighting only when ability differences are small and/or difficult to detect.

We also compared an imperfect weighting strategy to imperfect choosing, finding that the former outperformed the latter with very few exceptions. Specifically, choosing was superior to weighting only when there were large differences in accuracy which the judge recognized but severely underestimated. The reason for this finding is that the choosing strategy is insensitive to the magnitude of the ability differences whereas the weighting strategy is not. Consider the case where the advisor is much more accurate than the judge but the judge erroneously perceives the advisor to be only slightly better than him- or herself. In this case the judge will still correctly identify the advisor as the expert, and because the actual difference in expertise is large, choosing the advice will produce a rather good result. In contrast, weighting will produce a final estimate that is not too different from (but slightly superior to) the one obtained by averaging because the difference in weights is bound to be small. Based on the misperception of the ability differences, the judge does not assign enough weight to the advice.

Finally, we compared imperfect weighting to a strategy that dynamically switches from averaging to choosing when the (potentially biased) perceived ability differences between judge and advisor become large (Soll & Larrick, 2009). Our analysis revealed that weighting is superior to the combined strategy in a wide range of situations. Interestingly, weighting is better than the combined strategy mainly because the application of the combined strategy leads judges to choose between estimates in situations where averaging would outperform weighting. These situations are characterized by the judge correctly recognizing whether the advisor is more competent than him- or herself or vice versa, but at the same time greatly overestimating the ability differences. The interesting thing about those situations is that simple averaging would have performed better than weighting, but since the ability differences are perceived as too high, the combined strategy must use choosing instead.

3.1 Implications and directions for future research

An important implication of our analysis is that weighting is a highly effective strategy in advice taking. This finding extends previous research on judgmental aggregation. So far, the respective literature has unanimously supported averaging as the most robust strategy when it comes to utilizing the wisdom of the crowds (e.g., Clemen, 1989; Davis-Stober et al., 2014; Smith & Wallis, 2009). In addition, some recent studies showed that a combination of choosing and averaging can outperform mere averaging. In these studies, the average of all individuals judgments were compared to the average of a subset comprised of the most accurate judgments (Davis-Stober et al., 2014) or those judgments supposedly more accurate based on incomplete historic data (Mannes et al., 2014). In contrast, differential weighting of the individual judgments usually performs worse than simple averaging (e.g., Dawes, 1979; Genre, Kenny, Meyler, & Timmermann, 2013). The reason for this is the inflation of errors when estimating the optimal weights of a large set of individual judgments (Smith & Wallis, 2009). However, in the context of the judge-advisor dyad, the judge needs only estimate one parameter when estimating the optimal weight of advice. Therefore, the risk of error inflation is minimal and, as a consequence, weighting becomes a powerful strategy.

Furthermore, the fact that participants in previous studies adhered to a weighting strategy in a substantial number of trials (Soll & Larrick, 2009; Soll & Mannes, 2011) as well as its potential superiority to averaging highlight its importance when studying advice taking. Whereas the PAR model suggests that judges should engage in averaging in case of small or difficult to detect ability difference and rely on choosing otherwise, our analysis makes a partially different statement. In case of small and difficult to detect ability differences, averaging is still the best option. However, in case the ability differences become larger and easier to detect, judges should attempt to weight the two judgments by perceived accuracy instead of choosing between the two. Interestingly, weighting the two estimates by their perceived accuracy allows judges to mimic an aggregation strategy that has proven to be very effective if three or more judgments are involved, namely taking the median. Research on group judgment (Bonner & Baumann, 2008; Bonner, Gonzalez, & Sommer, 2004; Bonner, Sillito, & Baumann, 2007) suggests that the way in which groups or judges combine the individual estimates is best described by the median or similar models that discount outliers. The same is true when judges combine several independent judgments (Yaniv, 1997) or receive advice from multiple advisors (Yaniv & Milyavsky, 2007). Importantly, the median strategy outperforms the average because it discounts extreme judgments which are usually less accurate. Naturally, in the JAS with only one advisor, the median is per definition, equal to the mean, but assigning more weight to the more accurate judgment, even if the weight is not optimal due to misperceptions of the ability differences, also leads to discounting the less accurate judgments.

Our theoretical analysis does not only provide a normative framework to compare the expected performance of different advice taking strategies. It also allows to evaluate the effectiveness of judges' advice taking strategies. Similar to Soll and Larrick's (2009) empirical analysis, our model provides performance baselines against which to compare the de facto improvements in accuracy between judges' initial and final estimates. Soll and Larrick's analyses already showed that in the majority of the cases frequent averagers outperformed frequent choosers. An interesting question would, then, be whether or under which conditions frequent weighting can outperform frequent averaging.

Finally, a potential venue for further developing our model would be to include biased judgments. In our theoretical analysis, we made the simplifying assumption that there is no systematic bias in the judge's and advisor's estimates. Incorporating systematic biases of judge and advisor will necessarily make the model more complex, but it may be worthwhile if it allows us to draw conclusions about the relative performance of weighting, choosing and averaging in a wider range of decision situations.

3.2 Conclusion

Advice taking is not only an integral part of our daily social reality but also one of the most effective ways to increase the quality of our judgments and decisions. In order to make the best use of the wisdom of others, we need a thorough understanding of how well we utilize advice depending on its quality. An elegant way to provide answers to this question is provided by normative models of advice taking. We built on and extended the most prominent normative model of advice taking and, by doing so, furthered our understanding of how effective different advice taking strategies are in different situations. More importantly, however, normative modeling allows us to detect and, ultimately intervene against, deviations from optimal strategies, that is, they can help us utilize the benefits of advice to its full effect.

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4 Appendix

4.1 Deriving the most likely final estimate

Let us assume that the estimates of both judge and advisor are independent and drawn from a normal distribution centered on the true value x_T with variances σ_J^2 and σ_A^2. Since x_J and x_A are drawn from independent distributions, the density function is given by

$$f_{JA}(\tilde{x}) = f_J(\tilde{x}) \cdot f_A(\tilde{x}) = \left(e^{-\frac{(x_J-\tilde{x})^2}{2\sigma_J^2}} \sqrt{\frac{1}{2\pi\sigma_J^2}}\right) \cdot \left(e^{-\frac{(x_A-\tilde{x})^2}{2\sigma_A^2}} \sqrt{\frac{1}{2\pi\sigma_A^2}}\right) \tag{20}$$

$$= e^{-\frac{1}{2}\left(\frac{(x_J-\tilde{x})^2}{\sigma_J^2}+\frac{(x_A-\tilde{x})^2}{\sigma_A^2}\right)} \cdot \frac{1}{2\pi\sigma_J\sigma_A} \tag{21}$$

Optimizing with respect to $\tilde{x}$ gives

$$\frac{d}{d\tilde{x}}\Big(\log(f_{JA}(\tilde{x}))\Big) = -\frac{1}{2}\frac{d}{d\tilde{x}}\left(\frac{(x_J-\tilde{x})^2}{\sigma_J^2}+\frac{(x_A-\tilde{x})^2}{\sigma_A^2}\right) \tag{22}$$

$$= \frac{1}{\sigma_J^2}(x_J-\tilde{x}) + \frac{1}{\sigma_A^2}(x_A-\tilde{x}) = 0 \tag{23}$$

Solving (23) for $\tilde{x}$ gives

$$\tilde{x} = \frac{x_J\sigma_A^2 + x_A\sigma_J^2}{\sigma_J^2+\sigma_A^2} \tag{24}$$

which is a weighted average of x_J and x_A.

$$\tilde{x} = \frac{x_J\sigma_A^2 + x_A\sigma_J^2}{\sigma_J^2+\sigma_A^2} \tag{25}$$

4.2 Weighting almost always outperforms averaging

We compare the weighted average (2) with the arithmetic (non-weighted) average $\bar{x}$.

$$\bar{x} = \frac{1}{2}(x_A + x_B) \tag{26}$$

First, let us recall that for any random variable X and a real number a holds

$$\mathrm{Var}(aX) = a^2\mathrm{Var}(X) \tag{27}$$

Further, if X and Y follow independent Gaussian distributions $(\mu_X, {\sigma_X}^2)$ and $(\mu_Y, {\sigma_Y}^2)$, respectively, then also $X+Y$ follows a Gaussian distribution with expected value $\mu_{X+Y} = \mu_Y + \mu_Y$ and variance ${\sigma_{X+Y}}^2 = {\sigma_X}^2 + {\sigma_Y}^2$.

Now we look at the distributions of $\tilde{x}$ and $\bar{x}$. Since they are both linear transformations of x_J and x_A we can directly apply the above two rules. Thus, $\tilde{x}$ and $\bar{x}$ follow a Gaussian distribution with expected value x_T and the respective variances

$$\sigma_w^2 = \frac{\sigma_J^2\sigma_A^2}{\sigma_J^2+\sigma_A^2} \tag{28}$$

$$\sigma_a^2 = \frac{1}{4}(\sigma_J^2+\sigma_A^2) \tag{29}$$

where σ_w^2 is the variance of the weighted mean and σ_a^2 is the variance of the arithmetic mean. Then $\sigma_w \le \sigma_a$ with equality only if $\sigma_A = \sigma_B$, because

$$\sigma_w^2 \leq \sigma_A^2 \tag{30}$$

$$\frac{\sigma_J^2 \sigma_A^2}{\sigma_J^2 + \sigma_A B^2} \leq \frac{1}{4}(\sigma_J^2 + \sigma_A^2) \tag{31}$$

$$4\sigma_J^2\sigma_A^2 \leq \left(\sigma_J^2 + \sigma_A^2\right)^2 \tag{32}$$

$$4\sigma_J^2\sigma_A^2 \leq \sigma_J^4 + 2\sigma_J^2\sigma_A^2 + \sigma_A^4 \tag{33}$$

$$0 \leq \sigma_J^4 - 2\sigma_J^2\sigma_A^2 + \sigma_A^4 \tag{34}$$

$$0 \leq \left(\sigma_J^2 - \sigma_A^2\right)^2 \tag{35}$$