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CONTINUOUS AND DISCRETE FLOWS ON OPERATOR ALGEBRAS

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Abstract

Let $(N, \mathbb{R}, \theta)$ be a centrally ergodic W* dynamical system. When N is not a factor, we show that for each nonzero real number t, the crossed product induced by the time t automorphism θ_t is not a factor if and only if there exist a rational number r and an eigenvalue s of the restriction of θ to the center of N, such that $rst = 2\pi$. In the C* setting, minimality seems to be the notion corresponding to central ergodicity. We show that if $(A, \mathbb{R}, \alpha)$ is a minimal unital C* dynamical system and A is not simple, then, for each nonzero real number t, the crossed product induced by the time t automorphism α_t is not simple if there exist a rational number r and an eigenvalue s of the restriction of α to the center of A, such that $rst = 2\pi$. The converse is true if, in addition, A is commutative or prime.

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1. Introduction

We say that a flow (Y, T) is a pair consisting of a compact metric space Y and an action $T : Y \times \mathbb{R} \to Y$. The time t map of the flow (Y, T) is the automorphism $T^t : Y \to Y$. We say that a flow (Y, T) is minimal if there are no nontrivial closed invariant subspaces of Y. If (Y, T) is a minimal flow then, for $t \neq 0$, [3, Proposition 1.5] shows that the time t map T^t is not minimal if and only if there exists a rational number r and an eigenvalue s of T such that $rst = 2\pi$. (The 2π term appears in this equality when we remove it from the definition of eigenvalue in [3, Definition 1.1].) In the noncommutative setting, a flow is a C* dynamical system $(A, \mathbb{R}, \alpha)$ consisting of a (unital) C*-algebra A and a one-parameter group of automorphisms $\alpha : \mathbb{R} \to \mathrm{Aut}(A)$. The action α is said to be *minimal* (or, equivalently, we say that A is α-simple) if A has no nontrivial invariant ideals. When $A = C(Y)$ is a commutative (unital) C*-algebra, a flow α on A is induced by a flow T on Y, that is, $\alpha_t(f) = f \circ T^t$ for all f in A and for all t in $\mathbb{R}$. Then α is minimal if and only if T is minimal in the classical sense.

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The aim in this paper is to extend some of the results in [3] to the noncommutative case, in other words, given a minimal C* dynamical system $(A, \mathbb{R}, \alpha)$, we try to relate the values of t for which the crossed product induced by the time t automorphism α_t is not simple, with the eigenvalues of (a restriction of) α. We are unable to answer this question in general. Our partial results are contained in Section 3. It is natural to ask what is the corresponding problem for von Neumann algebras. Just as the notion of a simple C*-algebra corresponds to the notion of factor in the W*-algebra setting, it turns out that minimality in the C*-algebra setting corresponds to central ergodicity in the W*-algebra setting (recall that a W* dynamical system $(N, \mathbb{R}, \theta)$ is said to be *centrally ergodic* if the restriction of θ to the center of N is ergodic). In fact, if $(A, \mathbb{R}, \alpha)$ is a minimal C* dynamical system then the crossed product $A \rtimes_\alpha \mathbb{R}$ is simple if and only if the strong Connes spectrum of α is $\mathbb{R}$ (see [4]); whilst if $(N, \mathbb{R}, \theta)$ is a centrally ergodic W* dynamical system then the crossed product $N \rtimes_\theta \mathbb{R}$ is a factor if and only if the Connes spectrum of θ is $\mathbb{R}$ (see [13, Corollary XI.2.8, p. 336]). In Section 2 we discuss this W* version of the problem, which we are able to solve satisfactorily. We conclude each of the sections with some open problems.

2. W* dynamical systems

Let $(N, \mathbb{R}, \theta)$ be a W* dynamical system. We say that a real number s is an eigenvalue for θ if there exists a nonzero a in N such that, for all t in $\mathbb{R}$, we have $\theta_t(a) = e^{ist}a$. When θ is ergodic (that is, the set of fixed points of θ is $\mathbb{C}1$), the set of eigenvalues, which we denote by $\Lambda(\theta)$, is a subgroup of $\mathbb{R}$, see [10, Lemma 2.1].

Our first lemma is known for the case where the underlying measurable space of the center of the von Neumann algebra in question is a probability space; see [1, Lemma 12.1.1, p. 326]. The proof there, however, cannot be adapted to a more general situation. Our proof here is essentially contained in the proof of [12, Theorem 10.6]. We also remark that this lemma is known when the action on the underlying space of the center of the von Neumann algebra in question is uniquely ergodic [2, Exercise 4.24.2].

LEMMA 2.1. *Let $(N, \mathbb{R}, \theta)$ be a centrally ergodic* W* *dynamical system and let t be a strictly positive real number. If θ_t is not centrally ergodic then there exists a nonzero eigenvalue s of the restriction of θ to the center $Z(N)$ of N, such that $e^{ist} = 1$.*

PROOF. Suppose that θ_t is not centrally ergodic. Then $\mathcal{A} = Z(N)^{\theta_t}$ is a commutative von Neumann algebra which is not reduced to the scalars. Furthermore, $\mathcal{A}$ is θ-invariant and the action of θ on $\mathcal{A}$ is ergodic and periodic. Hence the action of θ on $\mathcal{A}$ is transitive. Thus there exists $t_0 > 0$ such that the action of θ on $\mathcal{A}$ is isomorphic to the canonical action of $\mathbb{R}$ on $L^\infty(\mathbb{R}/t_0\mathbb{Z})$. Since $s = 2\pi/t_0$ is an eigenvalue for this action (with eigenfunction defined by $f(t + t_0\mathbb{Z}) = e^{ist}$, for all t in $\mathbb{R}$), then we have that s is an eigenvalue of the action of θ on $\mathcal{A}$, and so s is an eigenvalue of the action of θ on $Z(N)$. Since the action of θ on $\mathcal{A}$ is periodic with period t_0, there exists k in $\mathbb{Z}$ such that $t = kt_0$. Hence $s = 2\pi/t_0 = 2\pi k/t$ is a nonzero eigenvalue for the restriction of θ to $Z(N)$ and $e^{ist} = e^{i2\pi k} = 1$, as desired. □

The next result can be regarded as the W* version of [3, Proposition 1.5].

PROPOSITION 2.2. *Let $(N, \mathbb{R}, \theta)$ be a centrally ergodic* W* *dynamical system and let $\tilde{\theta}$ denote the restriction of θ to the center $Z(N)$ of N. Consider the map with domain $(1/2\pi)\mathbb{Q} \otimes \Lambda(\tilde{\theta})$ and codomain $\mathbb{R}$ defined by $(r/2\pi) \otimes s \mapsto (rs/2\pi)$. This map is a $\mathbb{Q}$-linear monomorphism with range equal to*

$$\{0\} \cup \{t \in \mathbb{R} \setminus \{0\} \mid \theta_{1/t} \text{ is not centrally ergodic}\}.$$

Hence the set above is a $\mathbb{Q}$-linear subspace of $\mathbb{R}$ isomorphic to $(1/2\pi)\mathbb{Q} \otimes \Lambda(\tilde{\theta})$.

PROOF. It is clear that the map $(r/2\pi) \otimes s \mapsto (rs/2\pi)$ is a $\mathbb{Q}$-linear monomorphism. We only need to show that the range is as proposed. Let $r = p/q$ be a nonzero rational number and let s be a nonzero eigenvalue for $\tilde{\theta}$. Put $t = rs/2\pi = ps/2\pi q$. We show that $(N, \mathbb{Z}, \theta_{1/t})$ is not centrally ergodic. Since $\Lambda(\tilde{\theta})$ is a group, we get that ps is also an eigenvalue. Therefore, there exists $a \in Z(N)$, $a \neq 0$, such that $\theta_t(a) = e^{ipst}a$, for all $t \in \mathbb{R}$. Notice that a is not a scalar since $a \neq 0$ and $ps \neq 0$. Hence

$$\theta_{1/t}(a) = \theta_{2\pi q/ps}(a) = e^{i2\pi q}a = a.$$

Thus $(N, \mathbb{Z}, \theta_{1/t})$ is not centrally ergodic. Conversely, assume that $(N, \mathbb{Z}, \theta_{1/t})$ is not centrally ergodic for some $t \neq 0$. By Lemma 2.1, there exists $0 \neq s \in \Lambda(\tilde{\theta})$ such that $e^{is(1/t)} = 1$. Therefore $s(1/t) = 2\pi k$ for some $k \in \mathbb{Z}$ and so $t = (1/2\pi k)s$, as desired. □

Let θ be a flow on a von Neumann algebra N. If N is a factor, then θ and θ_t are automatically centrally ergodic for every real number t. To determine whether the crossed product induced by the time t automorphism is a factor, one must examine the Connes spectrum of θ_t. If N is not a factor and θ is centrally ergodic, we will show that the crossed product by the time t automorphism is a factor if and only if θ_t is centrally ergodic. We first give the following easy lemma.

LEMMA 2.3. *An inner automorphism on a von Neumann algebra N is centrally ergodic if and only if N is a factor.*

PROOF. Straightforward. □

We are ready to prove the main result of this section.

THEOREM 2.4. *Let $(N, \mathbb{R}, \theta)$ be a centrally ergodic* W* *dynamical system where N is not a factor. Let t be a nonzero real number. Denote by $\tilde{\theta}$ the restriction of θ to the center of N. The following statements are equivalent.*

(1) *The* W* *dynamical system $(N, \mathbb{Z}, \theta_t)$ is not centrally ergodic.*
(2) *The von Neumann algebra $N \rtimes_{\theta_t} \mathbb{Z}$ is not a factor.*
(3) *There exists (r, s) in $\mathbb{Q} \times \Lambda(\tilde{\theta})$ such that $rst = 2\pi$.*

PROOF. (1) implies (2). This is well known; see [13, Corollary XI.2.8, p. 336] or [9, Theorem 8.11.15, p. 362].

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(2) implies (3). If $N \rtimes_{\theta_t} \mathbb{Z}$ is not a factor then either $(N, \mathbb{Z}, \theta_t)$ is not centrally ergodic or $\Gamma(\theta_t) \neq \mathbb{T}$; see [9, Theorem 8.11.15, p. 362]. If $(N, \mathbb{Z}, \theta_t)$ is not centrally ergodic, we use Proposition 2.2 to conclude that $(1/t) = (r/2\pi)s$ for some (r, s) in $\mathbb{Q} \times \Lambda(\tilde{\theta})$. Hence $rst = 2\pi$, and we are done. If not, assume that $(N, \mathbb{Z}, \theta_t)$ is centrally ergodic and $\Gamma(\theta_t) \neq \mathbb{T}$. Then $\Gamma(\theta_t)^{\perp} \neq \{0\}$. As $\mathbb{T}$ is compact, we may use [13, Theorem XI.2.9(ii), p. 336] to conclude that $\theta_t^n = \theta_{nt}$ is inner for some nonzero integer n in $\Gamma(\theta_t)^{\perp} \subset \mathbb{Z}$. Since N is not a factor, θ_{nt} is not centrally ergodic (see Lemma 2.3). Another application of Proposition 2.2 completes the proof.

(3) implies (1). Suppose that there exists (r, s) in $\mathbb{Q} \times \Lambda(\tilde{\theta})$ such that $rst = 2\pi$. Then $(1/t) = (r/2\pi)s$ and so, by Proposition 2.2, the W* dynamical system $(N, \mathbb{Z}, \theta_t)$ is not centrally ergodic. □

Recall that a von Neumann algebra M of type III induces a (unique, up to conjugacy) flow on a von Neumann algebra of type II_∞, which is called the *noncommutative flow of weights of M* or the *associated covariant system of M*; see [13, Definition XII.1.3, p. 368]. As an example, we specialize Theorem 2.4 to the type III_λ case, $0 < \lambda < 1$, to obtain the following result.

COROLLARY 2.5. *Let M be a factor of type III_λ, $0 < \lambda < 1$, with associated covariant system $(N, \mathbb{R}, \theta)$. Let t be a nonzero real number. The following statements are equivalent.*

(1) *The W* dynamical system $(N, \mathbb{Z}, \theta_t)$ is not centrally ergodic.*
(2) *The von Neumann algebra $N \rtimes_{\theta_t} \mathbb{Z}$ is not a factor.*
(3) *There exists a rational number r such that $t = r \log \lambda$.*

PROOF. Let $\tilde{\theta}$ denote the restriction of θ to the center of N. In this case $(2\pi/\log \lambda)\mathbb{Z} = T(M) = \Lambda(\tilde{\theta})$; see [13, Theorem XII.1.6, p. 369] or [11, 28.11, p. 425]. Therefore, using Proposition 2.2, if t is a nonzero real number then $(N, \mathbb{Z}, \theta_t)$ is not centrally ergodic if and only if there exist r' in $\mathbb{Q}$ and n in $\mathbb{Z}$ such that $1/t = (r'/2\pi)(2\pi n/\log \lambda)$ if and only if $t = r \log \lambda$, where $r = 1/r'n$ is rational. □

Let $(N, \mathbb{R}, \theta)$ be a centrally ergodic W* dynamical system. It could be the case that, for all nonzero t, the crossed product induced by the time t map θ_t is a factor: for example, if $(N, \mathbb{R}, \theta)$ is the associated covariant system of a factor of type III_0 (because $\Lambda(\tilde{\theta}) = \{0\}$; see [13, Theorem XII.1.6, p. 369] or [11, 28.11, p. 425]). On the other hand, it could be the case that, for every real number t, the crossed product induced by the time t map θ_t is not a factor: for example, if $N \rtimes_\theta \mathbb{R}$ is semifinite, where N is a properly infinite semifinite von Neumann algebra which admits a faithful semifinite normal trace τ such that $\tau \circ \theta_t = e^{-t}\tau$, for all t in $\mathbb{R}$ (because $\Lambda(\tilde{\theta}) = \mathbb{R}$; see [12, Theorem 8.6]).

Theorem 2.4 is no longer valid when N is a factor. The author is grateful to Professor A. Kishimoto for communicating to him the following example (see [5, before Corollary 1.3]). We adapt it here to the W* setting. Let N be the von Neumann algebra generated by two unitary operators u and v satisfying $uv = e^{2\pi i s}vu$, where s is an irrational number. It follows that N is a factor. Let θ be the flow

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on N defined by $\theta_t(u) = e^{2\pi i t}u$ and $\theta_t(v) = e^{2\pi i s t}v$, for all t in $\mathbb{R}$. Then θ is centrally ergodic and $\Lambda(\tilde{\theta}) = \{0\}$. However, the crossed product $N \rtimes_{\theta_1} \mathbb{Z}$ induced by the time 1 automorphism is not a factor (because θ_1 is inner). It is worth mentioning that, in fact, the von Neumann algebra $N \rtimes_\theta \mathbb{R}$ is a factor. Therefore, this example satisfies a stronger condition than that required in Theorem 2.4.

One may ask whether a result similar to Proposition 2.2 is true if we substitute the centrally ergodic condition by ergodicity, that is, suppose that θ is an ergodic flow on a von Neumann algebra. It is true that the map $(r/2\pi) \otimes s \mapsto (rs/2\pi)$ is a $\mathbb{Q}$-linear monomorphism from $(1/2\pi)\mathbb{Q} \otimes \Lambda(\theta)$ into the subset of $\mathbb{R}$ consisting of $\{0\}$ together with the set of all nonzero real numbers t such that $\theta_{1/t}$ is not ergodic. By results of Størmer [10], if the kernel of θ is different from $\{0\}$ (or, equivalently, $\operatorname{Sp}\theta \neq \mathbb{R}$; see [10, Theorem 3.2]), then N must be abelian [10, Theorem 3.5]. Therefore, if $\operatorname{Sp}\theta$ is not $\mathbb{R}$, this map is also onto, by Proposition 2.2. We do not know if this map is onto when $\operatorname{Sp}\theta = \mathbb{R}$. We conclude this section with some open questions.

PROBLEM 1. Suppose that $(N, \mathbb{R}, \theta)$ is a centrally ergodic flow. If N is a factor, characterize the values of t for which the crossed product associated to the time t automorphism θ_t is a factor.

PROBLEM 2. If $(N, \mathbb{R}, \theta)$ is an ergodic flow, characterize the values of t for which the time t automorphism θ_t is ergodic.

3. C* dynamical systems

Let $(A, \mathbb{R}, \alpha)$ be a unital C* dynamical system. We say that a real number s is an eigenvalue for α if there exists a nonzero a in A such that, for all t in $\mathbb{R}$, it follows that $\alpha_t(a) = e^{ist}a$. We denote by $\Lambda(\alpha)$ the set of eigenvalues of α. If $A = C(Y)$ is commutative and α is induced by a flow T on Y, one may check that the eigenvalues for α are the same as the eigenvalues of T in the classical sense. (We remark that, in [3, Definition 1.1] and elsewhere in the literature, a 2π term appears in the classical definition of eigenvalue. We compensate for such a constant in the results below.)

Using results by Olesen and Pedersen, we obtain the following proposition.

PROPOSITION 3.1. *Let $(A, \mathbb{R}, \alpha)$ be a C* dynamical system. Assume that for each s in $\mathbb{R}$ and for each nonzero ideal I of A we have $I \cap \alpha_s(I) \neq \{0\}$. If α is minimal then, for all nonzero real numbers t, the automorphism α_t is minimal.*

PROOF. Assume that α is minimal and let t be a nonzero real number. If α_t is not minimal then there exists a nontrivial α_t-invariant ideal J of A. Hence J is invariant under $G_0 = t\mathbb{Z}$. Since $\mathbb{R}/G_0$ is compact, we may use [7, Proposition 2.2] to conclude that J contains a nonzero α-invariant ideal I, and so the ideal I is nontrivial because it is contained in the nontrivial ideal J. This contradicts the minimality of α and completes the proof. □

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Observe that if A is prime then the hypothesis of the proposition is satisfied. Furthermore, this hypothesis is equivalent to saying that the dual action $\widehat{\alpha}$ has full Connes spectrum; see [6, Lemma 3.2].

The following lemma is well known. We include a proof for completeness. For the case where A is commutative, this is [14, Theorem 5.3].

LEMMA 3.2. *Suppose that* $\alpha\colon A \to A$ *is a minimal automorphism of a unital* C*-*algebra* A. *Then* α *is centrally ergodic.*

PROOF. If α is not centrally ergodic, there is an element a in the center of A which is not a multiple of the identity, such that $\alpha(a) = a$. Let λ be an element in Sp (a) and let I be the ideal in A generated by $a - \lambda$. We have that I is nonzero, as a is not a scalar. Since $a - \lambda$ is a noninvertible element in $Z(A)$, then no element of the form $(a - \lambda)b$, with $b \in A$, is invertible. Thus $\|(a - \lambda)b - 1\| \geq 1$. Hence I is a proper ideal. Furthermore, I is α-invariant. We conclude that α is not minimal, as desired. □

We can now state the following result similar to Proposition 2.2.

PROPOSITION 3.3. *Let* $(A, \mathbb{R}, \alpha)$ *be a unital and minimal* C* *dynamical system and let* $\widetilde{\alpha}$ *denote the restriction of* α *to the center of* A. *Consider the map with domain* $(1/2\pi)\mathbb{Q} \otimes \Lambda\,(\widetilde{\alpha})$ *and codomain* $\mathbb{R}$ *given by* $(r/2\pi) \otimes s \mapsto (rs/2\pi)$. *This map is a* $\mathbb{Q}$-*linear monomorphism into*

$$\{0\} \cup \{t \in \mathbb{R} \setminus \{0\} \mid \alpha_{1/t} \textit{ is not minimal}\}.$$

Furthermore, the map is onto if A *is either commutative or prime. Hence, in this case, the above set is a* $\mathbb{Q}$-*linear subspace of* $\mathbb{R}$ *isomorphic to* $(1/2\pi)\mathbb{Q} \otimes \Lambda(\widetilde{\alpha})$.

PROOF. Let s be a nonzero eigenvalue of $\widetilde{\alpha}$ and let r be a nonzero rational number. We show that $\alpha_{2\pi/rs}$ is not minimal.

Suppose that $r = p/q$. Since ps is also an eigenvalue of $\widetilde{\alpha}$, there exists a nonzero a in $Z(A)$ such that $\alpha_t(a) = e^{ipst}a$ for all t in $\mathbb{R}$. Therefore, a is a fixed point for the time $2\pi q/ps$ automorphism. Since both a and ps are not zero, we obtain that $\alpha_{2\pi q/ps} = \alpha_{2\pi/rs}$ is not centrally ergodic and hence it is not minimal, by Lemma 3.2.

If A is commutative, this is [3, Proposition 1.5]. For the case where A is prime, $Z(A) = \mathbb{C}^1$ and so $\Lambda(\widetilde{\alpha}) = \{0\}$. This, together with Proposition 3.1, complete the proof. □

We now prove a statement analogous to Lemma 2.3.

LEMMA 3.4. *An inner automorphism on a* C*-*algebra* A *is minimal if and only if* A *is simple.*

PROOF. Let α be an inner automorphism on a C*-algebra A. If A is simple then it is clear that α is minimal. To prove the converse, suppose that α is minimal. Let I be an ideal of A. Then I is α-invariant because α is inner. Thus I is trivial because α is minimal. Hence A is simple. □

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The following is the main result of this section.

THEOREM 3.5. *Let $(A, \mathbb{R}, \alpha)$ be a unital and minimal* C* *dynamical system where A is not simple. Assume, in addition, that A is either commutative or prime. Let t be a nonzero real number. Denote by $\widetilde{\alpha}$ the restriction of α to the center of A. The following statements are equivalent.*

(1) *The* C* *dynamical system $(A, \mathbb{Z}, \alpha_t)$ is not minimal.*
(2) *The* C**-algebra $A \rtimes_{\alpha_t} \mathbb{Z}$ is not simple.*
(3) *There exists (r, s) in $\mathbb{Q} \times \Lambda(\widetilde{\alpha})$ such that $rst = 2\pi$.*

PROOF. (1) implies (2). This is well known; see [6, Theorem 6.5] or [4, Theorem 3.5 and Proposition 3.8].

(2) implies (3). If $A \rtimes_{\alpha_t} \mathbb{Z}$ is not simple then either α_t is not minimal or $\Gamma(\alpha_t) \neq \mathbb{T}$; see [6, Theorem 6.5]. If α_t is not minimal then, using Proposition 3.3, we can find a rational number r and an eigenvalue s in $\Lambda(\widetilde{\alpha})$ such that $(1/t) = (r/2\pi)s$. Hence $rst = 2\pi$, as desired. Else, assume that α_t is minimal and $\Gamma(\alpha_t) \neq \mathbb{T}$. Then $\Gamma(\alpha_t)^{\perp} \neq \{0\}$. As $\mathbb{T}$ is compact, we may use [8, Theorem 4.5] to find a nonzero n in $\Gamma(\alpha_t)^{\perp} \subset \mathbb{Z}$ such that $\alpha_t^n = \alpha_{nt}$ is inner. Since A is not simple, α_{nt} is not minimal; see Lemma 3.4. Another application of Proposition 3.3 completes the proof.

(3) implies (1). Suppose that there exists (r, s) in $\mathbb{Q} \times \Lambda(\widetilde{\alpha})$ such that $rst = 2\pi$. Then $(1/t) = (r/2\pi)s$ and so, by Proposition 3.3, α_t is not minimal. □

As an example, we specialize to the case where A is prime but not simple to obtain the following corollary.

COROLLARY 3.6. *Let $(A, \mathbb{R}, \alpha)$ be a unital* C* *dynamical system. Assume that A is prime but not simple. If α is minimal then $A \rtimes_{\alpha_t} \mathbb{Z}$ is simple for all $0 \neq t \in \mathbb{R}$.*

PROOF. This follows from Theorem 3.5 since $Z(A) = \mathbb{C}1$ and so $\Lambda(\widetilde{\alpha}) = \{0\}$. □

Theorem 3.5 (and Corollary 3.6) fails for simple C*-algebras, as one can see from the (C*-version of) the example described after Corollary 2.5. We conclude this section with some open problems.

PROBLEM 3. Is the map of Proposition 3.3 onto if we remove the condition on A being either commutative or prime?

PROBLEM 4. Suppose that $(A, \mathbb{R}, \alpha)$ is a minimal C* dynamical system and assume that A is simple. Characterize the values of t for which the crossed product associated to the time t automorphism α_t is a simple C*-algebra.

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