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A new class of nondiffracting pulses based on focusing leaky waves

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Abstract

In this work, we propose an azimuthally-invariant periodic leaky-wave (LW) radiator for the generation of Bessel beams and X-waves by means of backward cylindrical LWs at millimeter wavelengths. A rigorous framework is first outlined to understand the theoretical constraints of such a novel design. A specific attention is devoted to the impact of the attenuation constant on the focusing properties of the generated Bessel beams. On this basis, a practical design is then proposed to meet the previous requirements. Numerical results for different frequency spectra confirm the interesting capabilities of the considered structure, paving the way for the first generation of nondiffracting pulses produced by focusing LWs.

Introduction

During the last decade, we have witnessed the increasing development of millimeter-wave systems for focusing electromagnetic energy in the near field [1,2]. Indeed, diffraction and dispersion are well-recognized physical phenomena that limit the performance of millimeter-wave radiators. However, modern microwave and millimeter-wave applications, such as wireless power transfer, radiometry, thermal ablation, and so on, have pushed researchers to find solutions that exhibit either limited-diffractive or limited-dispersive features. Well-known examples of such solutions are Bessel beams and, more recently, X-waves (XWs) [3,4].

As is known [3], both solutions, viz., Bessel beams and XWs, can be interpreted as the monochromatic (*beams*) and the polychromatic (*pulses*) versions, respectively, of the more general class of nondiffracting waves. Despite the interest in these solutions dates back to the end of the 80s and the beginning of the 90s, when the first experimental realizations appeared in optics [5] and acoustics [6], there are still only a few implementations in the microwave regime (see, e.g., [7] and Refs. therein). Just very recently, theoretical and numerical results have been reported in [7–9], whereas experimental results just appeared in [10].

Indeed, most of the microwave Bessel-beam launchers were initially based on a scalar design approach [11], as is typical in optics where the paraxial approximation holds. As a consequence, the beams generated by such devices are characterized by relatively large spot-sizes at the expense of a rather poor resolution. In this connection, it has been shown that leaky waves (LWs) can profitably be used for generating Bessel beams with considerably narrower spot-sizes in both microwave [12,13] and millimeter-wave ranges [1]. However, both these leaky-wave antennas (LWAs) exhibit a narrow operating fractional bandwidth, and thus they are not suitable for the generation of polychromatic nondiffracting waves, such as XWs [7]. This narrow-band feature is due to the resonant character of the radiation mechanism, which is typically based on the superposition of an outward and an inward cylindrical wave [14].

Nevertheless, it has recently been shown that Bessel beams can conveniently be generated using inward cylindrical waves only [15–17], thus leading to the design of wideband radiators. (Note that in [15] analytical results are obtained for zeroth-order Bessel beams, whereas in [17] numerical results are extended to higher orders.) Even more interestingly, in [18] this principle has been successfully used to design a sinusoidally-modulated LWA based on the excitation of a backward leaky wave. However, in that work, neither the impact of the attenuation constant, nor a rigorous dispersion analysis has been provided. Furthermore, the wideband potential of using backward LWs has not yet been discussed in connection with the possibility of generating XWs.

In this work, we propose an azimuthally-invariant periodic LW radiator, namely a parallel-plate radial waveguide with annular slots (see Fig. 1) [19,20], suitably designed for the generation of Bessel beams and XWs at millimeter waves (around 60 GHz). A rigorous dispersion analysis is carried out in order to ensure limited-dispersion properties over a considerable fractional bandwidth (20%). The impact of the attenuation constant on the focusing capabilities of

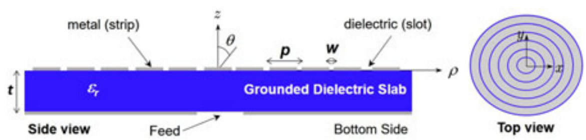

Fig. 1. Side and top views of the proposed azimuthally-invariant periodic leaky-wave antenna (LWA) for near-field focusing. Parameters in the text.

backward LWs is first evaluated on a theoretical basis. On this ground, the geometrical parameters of the structure are selected in order to meet the design requirements. Finally, numerical results confirm the promising capabilities of the proposed device for the generation of limited-diffractive leaky waves. Specifically, focusing pulses are generated by spectral superposition of Bessel beams over the entire theoretical bandwidth. A comparison is then performed between pulses generated with either a uniform frequency spectrum or a Gaussian one.

The paper is organized as follows. In the section "Theoretical analysis", we envisage the theoretical aspects related to the generation of nondiffracting waves through both inward cylindrical waves (section "Generation of nondiffracting waves through inward cylindrical waves") and backward LWs (section "Generation of nondiffracting waves through backward leaky waves"); a preliminary design of a periodic LWA is proposed in the section "Design of a periodic LWA". In the section "Generation of limited-diffractive beams and limited-dispersive XWs through an LWA", the performance of the LWA design outlined in the section "Design of a periodic LWA" is extensively discussed in connection with the generation of nondiffracting beams (section "Generation of Bessel beams through an LWA") and nondiffracting pulses (section "Generation of XWs through an LWA"). Finally, conclusions are drawn in the section "Conclusion".

Theoretical analysis

Generation of nondiffracting waves through inward cylindrical waves

As is known [3,7], an ideal XW can be generated by taking the inverse Fourier transform of monochromatic zeroth-order Bessel beams

$$\chi(\rho, z; t) = \int_{-\infty}^{+\infty} W(f) J_0(k_\rho \rho) e^{-jk_z z} e^{j2\pi f t} df, \tag{1}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, t is the time, f the frequency, and $W(f)$ is a spectral weighting function; ρ and z are the radial and longitudinal coordinates of a cylindrical reference frame (see Fig. 1), and k_ρ and k_z the radial and longitudinal wavenumbers, respectively, related through the separation relation $k_0^2 = k_\rho^2 + k_z^2$, k_0 being the vacuum wavenumber. According to the definitions provided in [3,4], *ordinary* XWs are weighted with an exponentially-decaying frequency spectrum [3]. In this work, we will refer to two different kinds of spectral weights: *uniform* (UXWs) [7] and *Gaussian* (GXWs) [9].

From equation (1) it is clearly seen that XWs are nothing more than a weighted spectral superposition of Bessel beams. As a consequence, any radiating device which is able to efficiently generate Bessel beams over a certain frequency bandwidth is a potential candidate for generating XWs (some restrictions will be discussed in the section "Generation of limited-diffractive beams and limited-dispersive XWs through an LWA").

More generally, the vectorial formulation of Maxwell's equations in a cylindrical reference frame shows that a cylindrical aperture of infinite extent supports the generation of Bessel beams in the near-field region. Indeed, if we suppose to excite the structure with an azimuthally-symmetric source (e.g., a vertical coaxial feed), the electromagnetic field can completely be described by considering a transverse magnetic (with respect to the vertical z-axis) TM^z vector potential $A_z = H_0^{(2)}(k_\rho \rho) e^{-jk_z z}$ [21], which gives rise to the following electric field components [22]

$$E_z(\rho, z) \propto H_0^{(2)}(k_\rho \rho) e^{-jk_z z}, \tag{2}$$

$$E_\rho(\rho, z) \propto H_1^{(2)}(k_\rho \rho) e^{-jk_z z}, \tag{3}$$

$$E_\phi(\rho, z) = 0, \tag{4}$$

where $H_0^{(2)}(\cdot)$ and $H_1^{(2)}(\cdot)$ are the outward Hankel functions of order 0 and 1, respectively. We note that the assumption of infinite extent (which also holds for electrically large apertures provided that the field is sufficiently attenuated at the edge truncation, as customarily happens for LWAs [23,24]) allows for retaining only the outward components of a cylindrical wave (generally constituted by a superposition of outward and inward Hankel waves).

Since an inward cylindrical-wave aperture distribution is needed to focus a Bessel beam [15], and inward and outward Hankel functions are related through [25]

$$H_n^{(1)}(k_\rho \rho) = -H_n^{(2)}(-k_\rho \rho), \tag{5}$$

for $n \in \mathbb{Z}$, then equations (2) and (3) evaluated at z=0 reveal that k_ρ must be negative to recover the inward character of the aperture distribution, and in turn be able to generate Bessel beams. This principle has been profitably applied in [18] to backward LWs, for which $k_\rho = \beta_\rho - j\alpha_\rho$ with phase constant $-k_0 \leq \beta_\rho \leq 0$ and attenuation constant (or *leakage* rate) $\alpha_\rho > 0$.

Generation of nondiffracting waves through backward LWs

The analysis reported in [15] was derived under the assumption that $k_\rho \in \mathbb{R}$. In fact, for $\alpha_\rho = 0$, geometrical optics [15,26] predicts a Bessel beam in a diamond-shaped region delimited by the shadow boundaries (see white dashed lines in Figs 2(a)–2(d)) at an angle θ_0 called *axicon angle* (measured from the vertical z-axis) related to the wavenumbers through [26]

$$\tan\theta_0 = k_\rho / k_z. \tag{6}$$

This simple geometrical interpretation also allows for calculating the so-called *nondiffracting range*, i.e., the distance $z = z_{ndr}$ from the aperture plane z=0 at which the shadow boundaries intersect each others, and thus the beam intensity abruptly decays (the beam enters the shadow region), through the relation

$$z_{ndr} = \rho_{ap} / \tan\theta_0, \tag{7}$$

where ρ_{ap} is the aperture radius.

To extend such analysis to LW radiators we need to evaluate the impact of the attenuation constant on the Bessel-beam generation, a leaky mode being characterized by a radial complex

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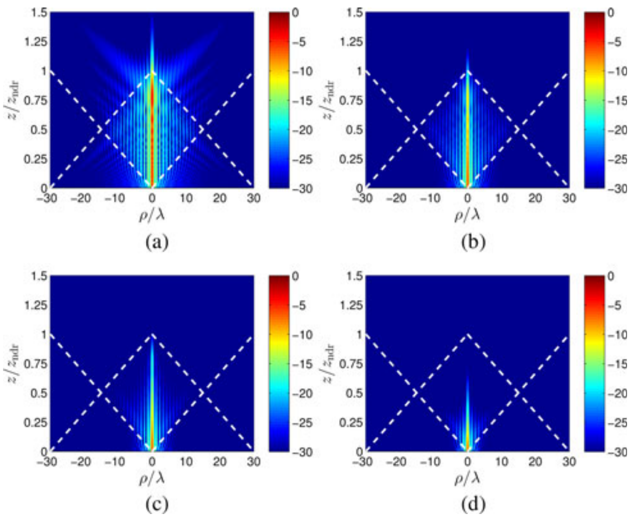

Fig. 2. Color maps of the normalized absolute value (in dB) of the E_z component of the electric field at 60 GHz along an arbitrary ρz plane limited by $|\rho| < 30\lambda$ and $0 < z < 1.5z_{ndr}$, for $\beta_\rho = -0.5k_0$ and (a) $\alpha_\rho = 0$, (b) $\alpha_\rho = 0.005k_0$, (c) $\alpha_\rho = 0.01k_0$, and (d) $\alpha_\rho = 0.02k_0$. In all cases, a zeroth-order Bessel-like beam is clearly distinguishable within the diamond-shaped region defined by the shadow boundaries (white dashed lines).

wavenumber $k_\rho = \beta_\rho - j\alpha_\rho$. As stated in [18], it is expected that the exponential decay of LWs (described by their attenuation constant) would produce Bessel beams over a smaller spatial region.

To give a proof of concept, in Figs 2(a)–2(d) the absolute value of the E_z component is reported in dB for a Bessel beam generated by a finite aperture with radius $\rho_{\rm ap} = 30\lambda$ at 60 GHz (being $\lambda = 0.5$ cm the free-space wavelength), assuming a *backward* leaky wave characterized by a normalized phase constant $\hat{\beta}_\rho = \beta_\rho/k_0 = -0.5$ (corresponding to an axicon angle $\theta_0 = 30^\circ$ through equation (6)) and considering four different values of the normalized attenuation constant $\hat{\alpha}_\rho = \alpha_\rho/k_0$, ranging from 0 to 0.02. Results have been obtained by numerically evaluating the radiation integral [7] using the aperture field distribution given by the non-zero tangential components of the electric field (viz., E_ρ in equation (3)) at the aperture plane z=0. Since $-1 < \hat{\beta}_\rho < 0$ due to the backward nature of the leaky mode, E_ρ is equivalently described by an inward Hankel distribution $H_1^{(1)}(\cdot)$ with a positive argument (see equation (5) for n=0), as required to focus a Bessel-like beam within the nondiffractive range [15].

As is seen, when $\hat{\alpha}_\rho = 0$ (see Fig. 2(a)), a zeroth-order Bessel beam is efficiently generated in a diamond-shaped region, which is determined by the corresponding shadow boundaries (white dashed lines) [15,16]. However, when $\hat{\alpha}_\rho$ increases [see Figs 2(b)–2(d)], the electric field exhibits an exponential decay, which limits the intensity of the field more and more as long as the leakage rate $\hat{\alpha}_\rho$ increases. This is in agreement with the experimental results recently reported in [18]. On one hand, it is seen that for small leakage rates, i.e., $\hat{\alpha}_\rho \leq 0.01$ (see Figs 2(b) and 2(c)), the impact of $\hat{\alpha}_\rho$ is even beneficial to 'smooth' the beam from the diffractive behavior of the field outside the diamond-shaped region (see Fig. 2(a)). On the other hand, for higher leakage rates, i.e., $\hat{\alpha}_\rho \geq 0.02$ (see Fig. 2(d)), the impact of $\hat{\alpha}_\rho$ has a detrimental effect, leading to a vanishing Bessel beam. As a consequence, in order to design an LWA for efficiently generating Bessel beams (and in turn XWs), the $\hat{\alpha}_\rho$ should preferably never be higher than 0.02.

It is worth mentioning that the value of $\hat{\alpha}_\rho$ has in practice also a lower bound. Indeed, in order to let an LWA radiate the 90% of its power (this also guarantees that the numerical results would not differ much from the theoretical formulation underlying equations (2)-(3), which is based on the infinite-aperture assumption), the aperture radius is determined by the following design rule [23]:

$$\rho_{ap} \simeq 0.18\lambda/\hat{\alpha}_\rho. \tag{8}$$

A very small value of $\hat{\alpha}_\rho$ would produce impractically large apertures in terms of wavelengths, especially at microwaves. (Note that, in the example of Fig. 2, $\rho_{\rm ap}$ has been fixed to 30λ to approximately fulfill the aforementioned design rule for the smallest considered leakage rate, viz., $\hat{\alpha}_\rho = 0.005$.)

Design of a periodic LWA

These considerations have been used to design an annular strip-grating 'bull-eye' LWA, i.e., a parallel-plate radial waveguide (PPW) with annular slots (see Fig. 1) which can support a backward cylindrical LW [23]. This structure can easily be fed by a printed surface-wave launcher by coupling a coplanar waveguide feedline to a slot etched in the ground plane [19], or by a coaxial feed [27], if azimuthally symmetric fields are required.

When the size of the annular slots w is small with respect to the period p (see Fig. 1), the n=0 Floquet harmonic can be seen as a perturbation of the transmission electron microscopy (TEM) mode of the unperturbed PPW. As a consequence, the wavenumber of the radiating n=−1 Floquet harmonic is approximately given by $k_{\rho,-1}(f) = \beta_\rho^{TEM} + \Delta\beta_\rho - j\alpha_\rho - 2\pi/p$, where $\Delta\beta_\rho$ is the perturbation of the phase constant, α_ρ is the attenuation constant due to the presence of the slots, and $\beta_\rho^{TEM} = k_0\sqrt{\varepsilon_{\rm r}}$ is the wavenumber of the TEM mode supported by the unperturbed PPW filled by a dielectric with permittivity ε_r.

Once the frequency is fixed, β_ρ^{TEM} depends only on ε_r. Assuming small perturbations, even $\beta_{\rho,-1}$ depends only on ε_r. As a consequence, once the minimum and maximum values of $\beta_{\rho,-1} = \Re\{k_{\rho,-1}\}$ at the edges of the bandwidth [i.e., $\beta_{\rho,-1}(f_{min})$ and $\beta_{\rho,-1}(f_{max})$] are fixed, the fractional bandwidth (defined as $\Delta f = (f_{max} - f_{min})/f_{op}$) depends only on the slope of $\beta_{\rho,-1}$, which is solely determined by ε_r. In particular, here we fixed $\beta_{\rho,-1}$ to take values within the range $-0.4k_0 \leq \beta_{\rho,-1} \leq -0.1k_0$ over a fractional bandwidth of 20% centered around $f_{op} = 60$ GHz (viz., a 54–66 GHz bandwidth). Such constraints give us $\varepsilon_r = 1.2$ and p=3.716 mm, whereas the width of the slots has been preliminarily set to w=0.416 mm in order to get a 'moderate' leakage rate. The thickness of the substrate has been set to t=0.83 mm ($t \simeq \lambda/6$) to prevent the propagation of higher-order modes. With these values at hand, the dispersion curve of this structure has been obtained by means of a method-of-moments (MoM) in-house code (see, e.g., [28]).

As is shown in Figs 3(a) and 3(b), the phase constant is limited in the range $-0.47k_0 < \beta_{\rho,-1} < -0.18k_0$ over the entire bandwidth, whereas the attenuation constant is remarkably regular around the value of $\alpha_\rho \simeq 0.005k_0$; such a value fixes the aperture size to $\rho_{\rm ap} = 40\lambda = 20$ cm through equation (8).

In the following section "Generation of limited-diffractive beams and limited-dispersive XWs through an LWA", we will use the dispersion values given by the MoM code to assess the generation of limited-diffractive Bessel beams and limited-dispersive pulses through the proposed annular strip-grating LWA.

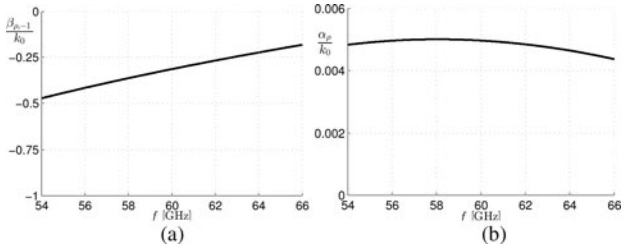

Fig. 3. (a) Normalized phase $\beta_{\rho,-1}/k_0$ and (b) attenuation α_ρ/k_0 constants versus f, for the proposed LWA. The dispersion curves have been obtained through a method-of-moments (MoM) in-house code [28].

Generation of limited-diffractive beams and limited-dispersive XWs through an LWA

Generation of Bessel beams through an LWA

To have an ideal XW, k_ρ and k_z appearing in equation (1) should change linearly with frequency, whereas in any microwave radiating device (as those considered here) they generally exhibit a nonlinear behavior, thus causing the well-known frequency dispersion of the wavenumber. Since the ratio between k_ρ and k_z defines the *axicon angle* θ_0 through equation (6), the frequency dispersion of the wavenumber is also referred to as *cone dispersion* [29] to emphasize the variation of θ_0 with the frequency. This aspect is particularly important for practical realization of XWs, as we will see in the next section "Generation of XWs through an LWA".

According to the dispersion curves of the proposed 'bull-eye' LWA (see Fig. 3(a)), the axicon angle is expected to change within the range $10^\circ < \theta_0 < 30^\circ$ (through equation (6)) over the considered bandwidth (i.e., for $f_{min} \leq f \leq f_{max}$ with $f_{min} = 54$ GHz and $f_{max} = 66$ GHz). As a consequence, the nondiffractive range $z_{ndr}(f)$ attains the following values: $z_{ndr}(f_{min}) \simeq 37$ cm, $z_{ndr}(f_{op}) = 60$ cm, and $z_{ndr}(f_{max}) = 108$ cm, at 54 GHz, 60 GHz, and 66 GHz, respectively. Numerical results for E_z (see Figs 4(a)–4(c)) corroborate the theoretical prediction.

As is clearly seen, the shadow boundaries (white dashed lines) stretch out as the frequency increases according to the dispersion relation of $\beta_{\rho,-1}$. It also manifests that the intensity distribution of E_z resembles that of a Bessel beam characterized by an attenuation constant of the order of $\alpha_\rho = 0.005k_0$, previously evaluated (compare Fig. 4(b) with Fig. 2(b)). Even more interestingly, the Bessel-beam profile is maintained over the whole frequency range of interest (results for intermediate frequencies are similar), thus opening to the possibility of generating polychromatic nondiffracting waves such as XWs, as continuous frequency superpositions of nondiffracting beams such as Bessel beams. This fundamental issue is addressed next.

Generation of XWs through an LWA

Since in the previous section "Generation of Bessel beams through an LWA" we have seen that the considered LWA allows us for generating Bessel-like beams for the longitudinal component of the electric field over the entire frequency range, it is expected that the same structure would also be able to generate a focusing propagating pulse closely similar to an XW.

In fact, it has recently been established [7] that the efficient generation of UXWs through finite apertures at microwaves is subjected to the fulfillment of specific criteria regarding the bandwidth capabilities, the aperture size, and the frequency dispersion of the device.

These considerations originally suggested the use of radial line slot array (RLSA) antennas for generating both UXWs and GXWs [7,9], but the discussion is still valid for any mm-wave radiator, provided that the mechanism of radiation is described by a single propagating mode characterized by a real wavenumber, i.e., $\alpha_\rho = 0$. However, we have extensively shown in the previous Subsections that the only effect of the imaginary part of the leaky wavenumber is to produce a decay of the beam intensity for $0 \leq z \leq z_{ndr}$. Since Bessel beams are the main constituents of XWs it is expected that an XW generated by means of backward LWs will exhibit a similar behavior along the propagating z-axis.

Here, we aim at showing that the LWA design outlined in the section "Design of a periodic LWA" also fulfills the criteria established in [7] to efficiently generate UXWs. In fact, according to [7], the confinement ratios along the radial C_ρ and the longitudinal C_z axes (defined as the ratios between the -3 dB widths of the main spot along the respective ρ, z directions) and the maximum nondiffracting extensions (i.e., $2\rho_{ap}$ along the radial axis, and z_{ndr} along the longitudinal axis) are given by the following two exact analytic expressions:

$$C_\rho = \frac{j_{0,1}}{\pi \sin\theta_0 \rho_{ap}/\lambda}, \tag{9}$$

$$C_z = \frac{2\sin\theta_0}{\Delta f \cos^2\theta_0\, \rho_{ap}/\lambda}, \tag{10}$$

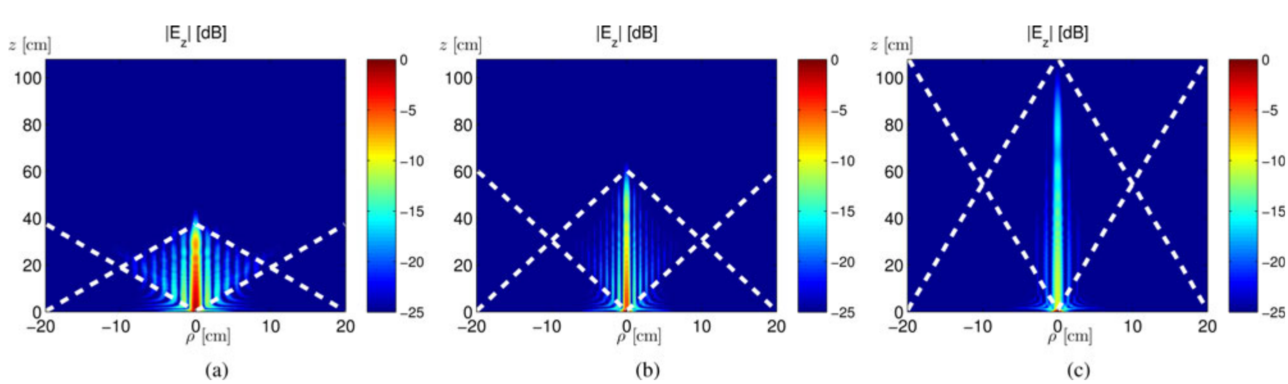

Fig. 4. Color maps of the normalized absolute value (in dB) of the E_z component of the electric field at (a) f=54 GHz, (b) f=60 GHz, and (c) f=66 GHz, along an arbitrary ρz plane limited by $|\rho| < \rho_{ap}$ and $0 < z < z_{ndr}(f_{max})$. Due to the unavoidably dispersive character of the backward leaky wave, the shadow boundaries (white dashed lines) change as f and hence $\beta_{\rho,-1}$ changes.

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where $j_{0,1} \simeq 2.405$ is the first zero of $J_0(\cdot)$, and Δf is the fractional bandwidth. Through equations (9) and (10), it is easily seen that, when $\Delta f = 20\%$ and $\rho_{ap} = 40\lambda = 20$ cm, a good confinement of the pulse along both the transverse and the longitudinal axes (i.e., both $C_z \ll 1$ and $C_\rho \ll 1$) is obtained for axicon angles within the range $5^\circ \leq \theta_0 \leq 30^\circ$. Such a cone dispersion is larger than that exhibited in Fig. 3(a) (viz., $10^\circ \leq \theta_0 \leq 30^\circ$), thus confirming the consistency of the proposed LWA design.

It should be stressed that equations (9) and (10) are exact only in the nondispersive case, i.e., $\theta_0(f) = \theta_0$, but they still work as lower-bounds for the dispersive case. In general, the minimization of the cone dispersion is highly desirable since it determines the frequency-dependent character of z_{ndr} [through equation (7)], which is responsible of the spatio-temporal spreading of XWs: the higher the dispersion, the faster the pulse will broaden as it propagates [7,8].

To further verify our design, numerical results have been obtained by replacing the ideal Bessel beams appearing in equation (1) with the longitudinal component of the electric field E_z effectively radiated by the LWA over the considered frequency range. This has led to the following expression:

$$\mathcal{E}_z(\rho, z; t) = \int_{-\infty}^{+\infty} W(f) E_z(f) e^{j2\pi f t} df. \tag{11}$$

Since we were interested in propagating the pulse over a distance $z = z_{ndr}(f_{max}) = 108$cm, and the longitudinal group velocity v_z of the pulse is approximately equal to $v_z = 0.696 c_0$ (c_0 being the speed of light in vacuum), the pulse has been observed over a period T=5 ns. To avoid fictitious replicas for $0 \leq t \leq T$ due to aliasing we have then numerically evaluated the integral of equation (10) using $N\lceil f_0 \Delta f T \rceil = 60]] >$ frequency samples (where $\lceil x \rceil$ returns the smallest integer $\geq x$), according to the Nyquist-Shannon theorem.

Two cases of interests are then examined: *(i)* the generation of UXWs, i.e., XWs weighted with a uniform frequency spectrum:

$$W(f) = \begin{cases} 1 & f_{min} < f \leq f_{max}, \\ 0 & \text{elsewhere} \end{cases} \tag{12}$$

and *(ii)* generation of GXWs, i.e., XWs weighted with a Gaussian frequency spectrum:

$$W(f) = \sqrt{\frac{1}{\sigma_f \sqrt{\pi}}} e^{-((f-f_0)^2/2\sigma_f^2)}, \tag{13}$$

where σ_f can be related to the frequency bandwidth through the following relation

$$\sigma_f = \frac{f_{max} - f_{min}}{2\sqrt{\ln 1/s}}, \tag{14}$$

where s is an arbitrary threshold parameter which depends on the bandwidth definition (e.g., the standard choice of -10 dB bandwidth made in Fig. 5 is given by s=0.1). Note that the amplitude factor in front of the exponential function in equation (13) is needed to have a normalized power spectrum, i.e., $|W(f)|^2 = 1$.

We chose these two kinds of frequency spectra for a two-fold reason. On one hand, uniform frequency spectra, even if they cannot constitute a 'true' signal due to their finite spectral extent, are more amenable for validating results provided by a frequency-domain measurement campaign. As a matter of fact, any experimental validation based on measurements of the frequency components in the near field can be performed only over a band-limited frequency spectrum. On the other hand, Gaussian frequency spectrum (which of course can represent a true signal) is the most employed frequency spectrum for characterizing the reference pulses in time-domain measurements based on autocorrelation techniques and optical gating [30,31].

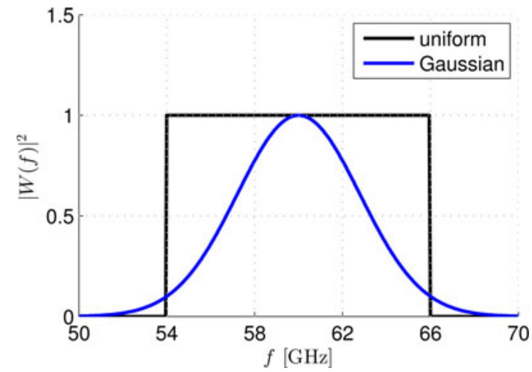

Fig. 5. Normalized frequency power spectra $|W(f)|^2$ for a uniform weight (black solid line) and a Gaussian weight (blue solid line) with s=0.1.

The resulting focusing LW pulses have been reported in Figs 6(a)–6(c) for UXWs and in Figs 6(d)–6(f) for GXWs. In both sets of figures the intensity distribution of $\mathcal{E}_z(\rho, z; t)$ has been captured at three different time instants to illustrate the movement of the pulse from $z < z_{ndr}(f_{min})$ to $z > z_{ndr}(f_{op})$ ($z_{ndr}(f_{max})$ corresponds to the upper-limit of the vertical axis). As is seen, the main spot exhibits the peculiar 'X-shape' that characterizes all types of XWs. Furthermore, as long as the pulse is in $z < z_{ndr}(f_{min})$ the intensity of the maximum as well as the spot size are not strongly perturbed. However, as the pulse travels beyond $z_{ndr}(f_{min})$, the shape of the main spot is progressively distorted and the intensity starts to fade out; beyond $z_{ndr}(f_{op})$ the intensity of the pulse abruptly decays and the main spot is barely appreciable.

Differences between UXWs and GXWs are hardly noticeable because the main features of the pulse come from the spectral content close to the operating frequency [7] where the Gaussian power spectrum is maximum (see Fig. 5). However, a closer look reveals that the GXW has a slightly larger -3 dB spot-width along the longitudinal axis and exhibits a slightly lower dispersive behavior. These two facts are both in agreement with equation (10) since a Gaussian weight would reduce the 'effective' fractional bandwidth of the pulse, thus increasing C_z and in turn worsening the confinement along the z-axis. Moreover, the dispersion effects are less important over a reduced fractional bandwidth, thus determining the weaker dispersion of GXWs with respect to UXWs.

As a final comment, we should stress that our results are very similar to those experimentally obtained at microwaves with a broadband Bessel beam radiator [10]. Differently from [10], the XWs reported here exhibit a gradual broadening and decay of the energy as long as the pulse overcomes the theoretically predicted nondiffractive range. Furthermore, we should remark that XWs reported here are of *subluminal* type, as opposed to the *superluminal* ones reported in [10]. Such different behaviors are due to the wavenumber dispersion, almost absent in [10], but no longer negligible in waveguiding structures such as our

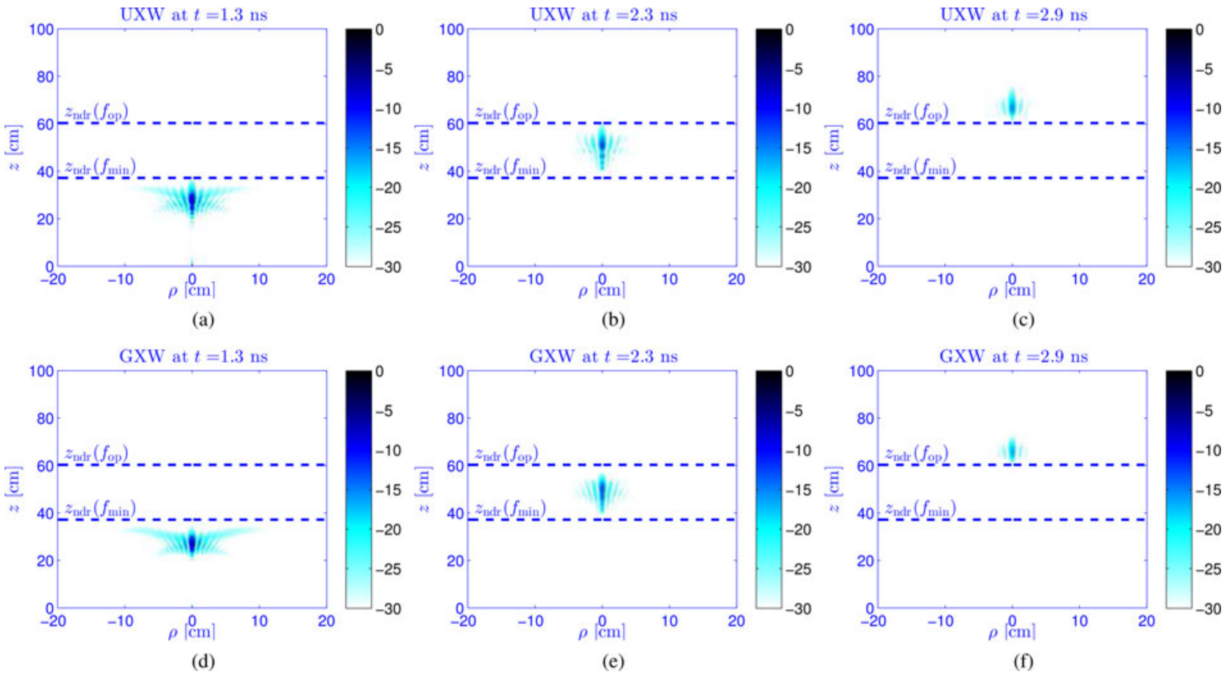

Fig. 6. Time evolution of (a)-(c) UXWs and (d)-(f) GXWS generated by means of focusing leaky waves. The intensity of $\mathcal{E}_z(\rho,\ z; t)$ has been captured at (a) and (d) t=1.3 ns, (b) and (e) t=2.3 ns, (c) and (f) t=2.9 ns, to show the distortion and the decay of the main spot of the pulse before and beyond the minimum and the operating nondiffractive ranges.

periodic LWA and RLSA antennas, as extensively commented in [7] and [9]. However, the pulse evolution within the nondiffractive range agrees well with the theoretical 1-D envelopes along the radial and the longitudinal axes, originally obtained in [7].

Conclusion

In this work, the role of the leakage rate in the generation of limited-diffractive beams (i.e., Bessel beams) and limited-dispersive pulses (i.e., XWs) has been carefully addressed for the first time. A preliminary theoretical analysis is proposed to derive simple criteria for designing an azimuthally-invariant periodic LW radiator, namely a PPW with annular slots. According to these criteria, a simple layout is proposed at 60 GHz and the dispersion analysis of the structure is obtained over a frequency range of 20%. These results are then used for evaluating the performance of the considered device in relation to the generation of Bessel beams and XWs. The effects due to an ideal uniform frequency spectrum and a real Gaussian frequency spectrum have been checked. Numerical results confirm the feasibility of the proposed approach, opening extremely interesting possibilities for the future generation of focusing LWs.

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