WHITE DWARF EVOLUTION IN REAL TIME: WHAT PULSATING WHITE DWARFS TEACH US ABOUT STELLAR EVOLUTION

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The variable white dwarfs repeatedly force theory to conform to their observed properties so that further progress can be made in understanding the structure and evolution of all white dwarfs. We use the term "understanding" in a loose sense here because, as we will show, both observational constraints and interpretation of the observations *vis-à-vis* theory contribute to uncertainties in our understanding at this time. In any case, recent progress in this field (sometimes called white dwarf seismology) has provided some fascinating insights into the evolutionary and structural properties of white dwarfs and their progenitors. This short review is our attempt to describe recent progress made in the interaction of theory with observations.

Because of space limitations, we do not include as many references as we would like. Instead, we offer the following list of texts and reviews that we have found useful: Cox (1980), Unno *et al.* (1979), Winget and Fontaine (1982), Van Horn (1984), Kawaler (1987), Winget (1988) and, finally, the forthcoming extensive review by Van Horn *et al.* (1989). The first two references are texts that contain most the material necessary for understanding the theory of nonradial pulsations as applied to stars in general, and in particular, the variable white dwarfs

1. The Observations and Some Matters of Theory

There are white dwarfs aplenty in this part of the Galaxy — some 25% of all stars near the Sun consist of these fascinating beasts. Among that plenitude there is a small subset which are obviously variable. Although they are usually dim and rather shy, they probably constitute the most populous class of variable star in the Universe. So much for those big, fat, yellow and red variables you hear so much about! Thus far, thirty-one of these white dwarf variables ("WDVs") have been discovered. We include in this total those variables that are well on their way to becoming white dwarfs. There are three major subclasses categorized primarily by effective temperature and spectral features. These are: the ZZ Ceti variables, or DAVs, ($T_{\rm eff} \approx 12000$K, DA hydrogen atmosphere), the DBVs ($T_{\rm eff} \approx 27000$K, DB helium atmosphere), and the DOV composite class consisting of very hot DO white dwarfs (the "PG 1159" stars) and the nuclei of planetary nebulae ($T_{eff} \approx 10^5$K with considerable uncertainty, HeII, CIV and, perhaps OVI spectral features — but no Balmer lines).

All these variables are multiperiodic, with periods ranging between $100s \leq \Pi \leq 1000s$. Periods less than about 100*s* are not found in any of these stars; it is possible that periods longer than 1000*s*, currently found in the pulsating planetary nebula nuclei ("PNNs"), could exist in the cooler variables. The problem of detecting long periods using Earth-based observations stems from the possible confusion of variations in sky transparency with those of the star, and the requirement of adequate phase coverage and cycle counts to unambiguously determine the periods present. Reliable decoding of the pulsation spectra of for these "long period" objects requires multi-site observing techniques such as those to be described by Ed Nather at this meeting.

The following two subsections will briefly review the photometric properties of the variable white dwarfs along with some pertinent theory. We shall try to emphasize those properties which must be attacked and explained by theory before we all can agree that we understand what is really going on inside these stars.

(a) "Simple" Variables

The threshold for detectability of a statistically significant amplitude in a power spectrum, that is, a pulsation period, for WDVs is now about one millimagnitude in visible light — and this requires long, clean observing run(s). The maximum amplitude observed in any WDV is close to 500 mmag. Among the ZZ Ceti variables, there appears to be a loose correlation between average amplitude, complexity of light curve, and effective temperature. The trend is that cooler ZZ Ceti stars near the red edge of the observed instability strip tend to have numerous high power peaks in their Fourier power spectra whereas variables nearer the blue edge are characterized by spectra showing only a few modest signals (Winget and Fontaine 1982). For the hotter variables the situation is unclear and we shall return to those after discussing the "simple" DAV variables.

Possibly the best-studied DAVs are ZZ Ceti (a.k.a. R548), G117-B15A, G226-29, and GD385. All of these variables pulsate at relatively low amplitude. The power spectrum of ZZ Ceti contains two pairs (doublets) of peaks of average amplitude 5 mmag with periods of 213.132605, 212.768427, and 274.250814, 274.774562*s*, with uncertainties of around $4\mu s$ (Stover *et al.* 1980). The price paid for this remarkable precision was 58 observing runs spaced over nine years, and an exacting analysis of the data. Another result of that analysis is an upper limit on the secular rate of change of period of one signal of $\dot{\Pi} \leq 2 \times 10^{-13} ss^{-1}$. The significance of such measurements will be discussed shortly.

Another simple pulsator is GD385 (Kepler 1984). It has only three signals. Two of them make up a doublet with periods of 256.127*s* and 256.332*s*, while the third (a singlet) is at a frequency twice that of the average of the frequencies in the doublet. GD385 is similar in one respect to ZZ Ceti , but why isn't the singlet a doublet and what causes the near doubling of frequency? A similar pattern is seen in G226-29; Kepler *et al.* (1983) show that only three signals are present in this star. The triplet has a mean frequency (period) of 9.15 mHz (109.3*s*) and the three signals are spaced evenly in frequency by

Δf=1.614 $\times 10^{-2}$mHz within observational error. In addition, the amplitudes are symmetric about the center signal making GD385 perhaps the "neatest" of the well-studied low amplitude WDVs.

G117-B15A is an example of organized confusion. Kepler *et al.* (1982) have reported six well-resolved signals whose frequencies are related to each other by various sums and differences and mysterious factors of 3/4 (see their Table 4) — a theoreticians delight and nightmare. With all this going on, the star is remarkably steady in another respect; the strongest signal (at 215.2*s* period and 44 mmag amplitude) is found to be constant, with an upper bound of $9.9\times10^{-15} ss^{-1}$ on $\dot{\Pi}$ (Kepler *et al.* 1988).

None of the hot pulsators match the simplicity of the above stars except perhaps the DOV PG 1707+427, discovered by Bond *et al.* (1984, see also Grauer *et al.*, these proceedings). A first inspection of the power spectrum of PG 1707 reveals only two peaks — one at 447.9*s* and a much smaller one at 334.6*s*. Grauer *et al.*, however, find that the stronger peak varies in amplitude by as much as a factor of three between runs. Their analysis indicates that this peak is possibly composed of two (or more?) signals spaced very closely in frequency and these beat against each other to produce variations in the amplitude of the light curve. The individual signals are not resolvable in a single run. PG 1707 appears to be unique among the hot pulsators — the rest of these variables all show much more complex behavior (as do many DAVs).

(b) "Complex" Variables

There is a prejudice (probably well-founded) among theoreticians in the WDV community that associates low amplitude behavior with simplicity and sinusoidality of signal. This prejudice may be reinforced by the fact that we theorists can only handle such "linear" signals in our calculations; nonlinear calculations of nonradial pulsations of white dwarfs are intractable at present (whereas they have been part of the arsenal of tools used for many years in the study of purely radially pulsating Cepheids). But not all WDVs are linear pulsators and, in fact, most are probably not. Before discussing those observations which clearly indicate nonlinear behavior in WDVs, it is useful to review some results from linear theory that may carry over to the nonlinear regime.

If the dynamic behavior of a WDV is primarily governed by the mechanical response of the star to small (linear) perturbations of pressure, density, etc., (however produced) then the system acts like a coherent collection of springs or pendulums. And, as in a mechanical system, the star responds by pulsating in a ensemble of normal modes, each with a well defined frequency. For a WDV these modes are most certainly (we think!) gravity modes (*g*-modes), where the restoring force for displaced fluid is buoyancy. Also, as in a vibrating mechanical system (such as ocean waves), the modes differ in wavelength and in the geometry of crests, troughs, and nodal points (in three dimensions). It is a characteristic of *g*-modes that the lower frequency modes are the more complex in structure. In the simplest case, a particular mode is conveniently labeled by three indices that describe the nodal structure: in the radial direction the index is k, and in the two angular directions (θ and φ in spherical coordinates) these are l and m (from spherical harmonics... see, not one equation!).

One result from theory is that for nonrotating, nonmagnetic (theorist-style) stars, modes with sufficiently large k , but fixed l and m , are spaced equally in period for successive integer values of k; i.e., $\Pi \propto k$. Is this kind of period spacing seen in real stars? And what if it is not? Well, first of all, the preceding discussion has to do with what modes are *possible* in a WDV; not all (or any) may be present. Some guidance comes from a "nonadiabatic" analysis of pulsations which looks into the linear response of a potential variable with respect to *thermal* effects; i.e., is the star "stable" or "unstable" in certain modes? It may be that all, none, or some of a set of modes in a sequence of equally spaced modes are actually present. A nonadiabatic analysis may indicate this but, to warn the reader, such analyses are difficult and full of pitfalls. We shall review some of the nonadiabatic successes and failures later on.

The prototype DOV, PG 1159-035 (a.k.a. GW Vir), discovered by McGraw *et al.* (1979), is a multiperiodic, large amplitude, pulsator as seen in the optical (Winget *et al.* 1985) and X-ray (Barstow *et al.* 1986). One of us (Kawaler 1988a) has examined the power spectra from extensive optical observations of this star and concludes that some eight signals spanning periods from 390*s* to 833*s* yield a statistically significant mean period spacing of either 21.0±0.3*s* or 8.8±0.1*s*. A comparison with evolutionary models then yields the happy result that the first (second) figure corresponds to a set of l=1 (l=3) modes in a $0.6M_0$ white dwarf — a mass that is nearly the average for all single white dwarfs (Kawaler 1986; and see Weidemann and Koester 1984, and Oke *et al.* 1984 for traditional determinations of WD masses). However, some modes in the succession of k are never seen (missing intermediate modes), and some modes are not always present in all runs — they come and go.

Linear theory, at its present stage of development, cannot always adequately explain why some modes in a sequence are present and some not. The coming and going of modes is even more sinister. Is the star really in the process of stopping in its tracks and then starting again, or, as is the case with some of the simple pulsators, do some of these modes consist of finely spaced multiplets that periodically beat down the signal? A case can be made for either of these possibilities (and more). Rotation or magnetic fields (see below) can split signals into multiplets, but testing this idea requires long observation to resolve the splitting. Intrinsic appearance and disappearance of normal modes is not out of the question; energy could be transferred in a nonlinear way between widely separated modes, thus extinguishing one and reinforcing another. Present-day theory, at the level needed, is of no help here.

Both rotation and magnetic fields destroy the spherical symmetry of a star and cause a single pulsation mode to split in frequency into a multiplet. Slow, uniform, rotation splits a mode evenly in frequency. The triplet observed in G226-29 (see above) probably falls into this category. So may the two doublets in ZZ Ceti — although Jones *et al.* (1989) make a strong argument that a weak magnetic field of around 10^5 G may be responsible. Direct evidence for splitting is virtually absent in almost all other WDVs. However, stars do rotate, and white dwarfs probably rotate slowly for the most part, and there is no theoretical reason why they should not have magnetic fields of some sort. Hence, *we expect fine splitting of signals to be present in most WDVs — the problem of detecting that splitting is one of resolution.*

The DBV class of variable white dwarfs was predicted and then discovered by Winget and his collaborators (Winget *et al.* 1982a; Winget *et al.* 1982b). The class prototype is GD358 whose power spectrum shows tightly clumped groups of signals in the period ranges 550–950*s*, 300–420*s*, and on to higher frequencies, with pulsation amplitudes of as high as 300 mmag. In the discovery runs, Winget *et al.* (1982a) found that the peaks in each clump were spaced evenly in frequency by $\Delta f=1.86\times10^{-4}s^{-1}$, leading them to suggest that GD358 was rotating with a period of around $(\Delta f)^{-1}\sim1.5$ hours. Subsequent observations by J.A. Hill (1986) make this interpretation doubtful. He finds that the spacing varies with run length (typically 2--6 hours) and that not all signals are reproducible. This sounds familiar; beating is probably taking place and run lengths are not long enough to resolve the details. H. Saio, of the University of Tokyo, has suggested that perhaps the signals from GD358 are evenly spaced in period rather than frequency. As has been discussed here, the two types of spacing arise from different underlying causes and tell us quite different things about the star. P.W. Jones and one of us (CJH) have examined data kindly provided by Hill and find (unfortunately and paradoxically enough) that equal spacing in period or frequency are statistically equally likely. Sigh!

Other stars that have been extensively observed show problems similar to the above. PG 1159 has been discussed previously (and will be brought up again in a different context) and K1-16 (a PNN) is another example. The latter, discovered by Grauer and Bond (1984), is a very complex pulsator whose pulsation spectrum has not been completely deciphered. Grauer informs us that the best that can be said for it is that bands of power are definitely present. Since K1-16 may be the hottest and fastest evolving of the WDVs, it is essential that we find out what's going on. At the other end of the temperature and evolution scale, much the same might be said for HL Tau 76 — which is the first discovered WDV (Landolt 1968)!

Many of the WDVs show evidence for nonlinearities besides just high amplitudes and temporal changes in amplitude. A striking feature that is often present are signals whose frequencies are linear combinations of the frequencies other signals; i.e., if signals at f_1, f_2, and f_3 are seen, then, for example, f_3 might be equal to f_1+f_2 to the limits of resolution. At this level we might question what constitutes a normal mode for the star. GD358 may fall into this class. Jones and CJH, following a suggestion by Hill, have found that the second group of signals in the power spectrum from that star consist exclusively of linear combinations of signals in the first group. (This shows up clearly in long runs only.) Could it be that the star is excited to initially produce only the first group of modes but, through nonlinear coupling of undetermined origin, then goes on to produce all the rest of what is seen? If so, then simple linear seismological theory only tells part of the story.

Other evidence of a striking nature for nonlinearities comes from (at least) four other WDVs. The best documented is from GD154 (a DAV; Robinson *et al.* 1978) and PG 1351+489 (a DBV; Winget *et al.* 1987, Hill 1986). At first glance, the power spectra from most runs of these two stars look straightforward. There is one strong peak at low frequency — call it f_o — followed by a dribble of lower power peaks at higher frequencies. Closer examination, however, reveals the following: either the higher frequency peaks are exact harmonics of f_o ($2f_o$, $3f_o$, *etc.*), or they are very close to odd half-integer multiples of f_o ($\frac{3}{2}f_o$, $\frac{5}{2}f_o$, etc.). The harmonics are probably due to the periodic, but nonsinusoidal, shape of the light curve. The other signals are right out of a textbook on nonlinear dynamics. And, just so things don't look too simple, their exact frequencies are: $1.53f_0$, $2.53f_0$, etc., for GD154, and $1.47f_0$, $2.47f_0$, etc., for PG 1351. In addition, apparently just for spite, the $1.53f_0$ signal in GD154 occasionally is the strongest, and $1.47f_0$ sometimes disappears in PG 1351. What gives?

PG 1351 is perhaps the best studied of the DBVs — and, as was once thought, for good reason. Because of the stability of its main peak (at 489.5*s*), researchers at UT (Austin) and CU (Boulder) have attempted to detect a secular change in period of that peak. The data should be sufficient to have done this but the analysis has not yet yielded a consistent result — for unknown reasons — and we blame the star (or our imagination) . Perhaps the nonlinear behavior introduces effects that defeat the analysis. Goupil *et al.* (1988), for example, have suggested that PG 1351 is on its way to chaos. We have examined some of their observational arguments for this conclusion and find that, while tentative, the suggestion has merit and deserves further study.

Well, there you have it. The range of behavior of the WDV pulsations is large indeed, ranging from simple "linear" pulsators with pulsation spectra that are well described by linear theory to stars which undergo complex light variations that border on non-periodic. In all of these pulsators, however, the observed pulsation characteristics do show, to some degree, underlying regularity that can be attributed to linear normal mode pulsation. In the remainder of this paper, we will describe how these regularities allow us to determine structural and evolutionary properties of white dwarf stars. Clearly, we only understand the tip of the iceberg; in working from the fundamental behavior alone we are ignoring the rich complexity that one day will provide even more detailed information about the workings of the white dwarf stars.

2. Physical Interpretation of White Dwarf Pulsation Spectra

As introduced in the previous section, the fundamental nature of the variable white dwarfs is that they are nonradial *g*-mode pulsators. The periods are much longer than the radial pulsation periods one expects from white dwarfs (i.e. 1-10 seconds) and they are multi-periodic. These facts, taken together, are the sole evidence for *g*-mode pulsation. Parameters relevant to *g*-mode pulsation reflect the physical conditions of the host star in an averaged sense; it is this line of thought that leads us to the exploitation of the pulsations in seismological inquiry. These "solid" seismological results are rooted in simple linear adiabatic pulsation theory; other more tentative conclusions can be drawn from the actual existence of observable pulsations. Since the pulsation modes are self-excited, there must be some interesting physics happening to drive the pulsations. Using techniques for examining nonadiabatic pulsations that have been developed over the past 60 years in the study of those yellow and red variables, we can make additional claims about the interior conditions that must exist in these stars.

(a) Mass and Composition from Period Spacings

Normal modes for *g*-mode pulsation are, in limiting cases, equally spaced in period for successive values of *k*. In practice, the mode spacing is not uniform for realistic models of stratified white dwarfs.

In this case, subsurface zones of rapidly changing composition such as the hydrogen-helium interface can "trap" certain modes in the outer layers, so that they are not global modes in a true sense. As shown in Kawaler (1987), this trapping effect can, in DOV models, cause the spacing to vary from mode to mode by up to 25% for low order modes, with the deviation decreasing as k increases. At the observed periods of around $500s$, the deviation is less than about 10%, or 2-$3s$, in the models. In the theorists dream case where each mode in a series were present, we could use the departures from uniform spacings to diagnose the compositional structure of the outer layers of WDVs. The mean spacing over several modes, however, is uniform even in these models. In real stars, where not all modes are present, the spacings look even more uniform than we have the right to expect based on the models!

The period spacing for g-modes in DOVs is very insensitive to the internal composition or exact luminosity. In essence, for standard DOV models, the spacing is most sensitive to total stellar mass for two orders of magnitude in luminosity surrounding the observed DOV stars. Thus, the identification of period spacings in PG 1159 and PG 0122 (Hill, Winget and Nather 1987) have allowed their masses to be measured. In the near future, the resolution of the pulsation spectra of additional DOV star promises to give us a statistically significant (in the astronomer's sense: 5 objects is a respectable sample in this game) sample of white dwarf masses at the hot end of the cooling sequence.

At the cooler temperatures relevant to the DAV and DBV stars, the period spacings are sensitive to the composition of the outer layers as well as to the mass and luminosity of the star. In the DOVs, the whole star contributes about equally to setting the pulsation periods; since white dwarfs differ chemically only in the outermost layers comprising less than 1% of the mass, the DOV period spacings are very insensitive to the precise composition of the outer layers. In cool white dwarfs, the mass motions for each normal mode are large only in the outer, non-degenerate regions. These additional dependencies could result in correspondingly large differences in the fundamental period spacings from star to star. However, the pulsation spectra of some DBVs and DAVs do show surprising systematics (G. Fontaine, private communication) that are currently under investigation.

(b) Rotation Rate and Magnetic Field Strength from Frequency Splittings

As mentioned above, another property of nonradial oscillations is that pulsation modes with the same l and k, but with different values of m all have the same frequency in the absence of effects which remove spherical symmetry. Some processes which can lift this degeneracy include rotation and global magnetic fields. Slow rotation and weak magnetic fields both result in uniform frequency spacing of modes with successive m. By "slow" we mean that the rotation period is much longer than the pulsation periods (i.e. the rotational kinetic energy is smaller than the kinetic energy associated with the pulsation), and by "weak" we mean that the field doesn't affect the equilibrium structure of the star. As mentioned above, it is often difficult to unambiguously determine frequency spacing; even when successful, it is unclear whether rotational effects or magnetic fields are responsible for the splitting. In any case, this facet of seismological study has begun to yield measurements of rotation periods (i.e. 17.2 hours for G226-29) and magnetic field strengths (Jones *et al.* 1989) for a variety of white dwarfs.

(c) Evolutionary Time Scale from Rates of Period Change

Since the normal modes of *g*-mode pulsation depend on the global properties of the star, as the star evolves the periods of the normal modes will change. As indicated in the first section, the pulsation periods of WDVs can be measured to extraordinary accuracy; thus we can hope to measure the change in these periods in a relatively short time. The time scale for white dwarf cooling increases quickly with decreasing temperature. The DOV stars cool on time scales of 10^6 years or less, while the DAVs cool a thousand times more slowly. Theoretical computations of the rate of period change for DOV models indicate expected period *increases* of about $10^{-11} s s^{-1}$ (Kawaler, Hansen, and Winget 1985) for modes with periods of about 500*s*. Winget, Hansen and Van Horn (1983) show that cooling tends to increase WDV periods while contraction leads to period decrease with time. The models show a period increase; since they have already reached the constant-radius white dwarf cooling curve, their evolution is dominated by cooling effects.

Using data spanning 5 years, Winget *et al.* (1985) measured a secular period change for the dominant mode of PG 1159 of the same magnitude as expected from theoretical models; however, they found the period to be *decreasing* with time. The negative $\dot{\Pi}$ indicates that contraction dominates cooling in the evolution of the pulsation properties of PG 1159. While it is possible that the luminosity of PG 1159 is much higher than we think, and it is still approaching the white dwarf cooling track, the models say that it would then be difficult to match the magnitude of $\dot{\Pi}$. Rotation may play a role here, though. If PG 1159 rotates (and why shouldn't it ... it has never been very obliging to us simple-minded theorists!) then conservation of angular momentum requires that it spin up as it contracts. Thus if the 516*s* mode has a non-zero value of *m*, then its period will change in response to the spin-up. Computations of this effect indicate that it can account for the observations if the rotation period is fairly, but not unreasonably, short... say a few hours.

While rotation can explain the observed sign discrepancy between the simple models and the observations, it is possible that other factors (i.e. nonadiabatic effects, magnetic fields) are also at work. In particular, modes that are trapped near the surface by composition transition regions will be more sensitive to envelope conditions than the more global untrapped modes. The contraction of a hot white dwarf is really a surface effect. Since the degeneracy of the core is much higher than the envelope, it is already about as small as it is going to get. Modes with significant amplitude in the degenerate core will therefore be affected by cooling only. Those modes with small core amplitudes but large envelope amplitudes will be much more sensitive to changes in the envelope; and it is there where most of the radius change occurs. Thus trapped modes will show period decreases at cooler effective temperatures than the untrapped modes. The fact that trapped modes are also more easily excited suggests that the 516s mode of PG 1159 could be one of these modes; the observed secular decrease of its period supports this idea. Despite these possible complications, since the effects of cooling, contraction, and spin-up all occur on comparable time scales, we can say that the current agreement in the size of $\dot{\Pi}$ between PG 1159 and the models confirms the evolutionary status and input physics of models of PG 1159 stars. This means that we are able to calibrate the neutrino emission rates relevant to hot white dwarfs to about a factor of two, since plasmon neutrinos play an important role in the cooling of white dwarf interiors.

The expected rate of period change for DBV stars is about $10^{-13} ss^{-1}$ (Kawaler *et al.* 1986a). Since DBV stars are much cooler than DOV stars, contraction is unimportant throughout the star. Thus the periods of DBV stars must increase with time if evolutionary effects are the only factor. Though smaller than the $\dot{\Pi}$ for DOVs, the quality of its measurement increases, as often sung by Ed Nather, "as time *squared* goes by." However, as detailed in the first section, the best candidate for this measurement, PG 1351, has thwarted attempts to obtain consistent results. The DAV white dwarfs cool on a time scale of a few billion years; Winget (1981) computes a rate of period change for these stars as about $2\times10^{-15} ss^{-1}$, or only a factor of 5 smaller than the upper limit determined by Kepler *et al.* (1988) for G117-B15A. As Winget (1988) points out, this upper limit already has told us that the core composition of G117-B15A cannot be heavier than oxygen; otherwise the models show that it would have a cooling rate, and thus a rate of period change, higher than the observed limit. In effect, the two star photometer has been used as a mass spectrometer in constraining the interior composition of G117-B15A.

(d) Thermal Structure and Composition from Nonadiabatic Considerations

The culprit responsible for driving the pulsations we see in DBV and DAV stars is the partial ionization zone in their outer layers. The cyclical ionization of the dominant species, when it occurs at the proper depth, can cause pulsational instability in modes with the observed periods. We will not go into further detail with respect to the driving in these cool white dwarfs here; we refer the reader to the reviews by Winget and Fontaine (1982), Winget (1988), and Starrfield (1987, and these proceedings) for further discussions. The important facet of all this is that resonances between the thickness of the surface layers of different composition and the nodal structure of the pulsation modes leads to an effective mode selection mechanism. That is, trapped modes are preferentially excited in the DAV and DBV stars.

The success of the partial ionization mechanism in explaining the excitation of DAV and DBV pulsations led Starrfield, Cox, and their collaborators to suggest that the DOV stars are driven by the same process operating in carbon and/or oxygen (Starrfield *et al.* 1983, 1984, 1985). They find, using static C/O envelopes appropriate to PG 1159 and K1-16, that excitation of modes with the correct periods can occur. However, further work by Starrfield (1987) shows that the effect disappears when the helium content of the surface layers increases beyond only 20% or so. This helium "poisoning" suggest to them that not only are the DOV stars hydrogen deficient, but that they are helium deficient as well!

Unfortunately, avoiding this helium poisoning problem removes one of the desirable features of the driving mechanism. Without composition gradients, there is no obvious way for the star to select only a few modes in which to pulsate. Indeed Starrfield *et al.* find that while modes with the correct period are unstable, many more modes are unstable in the models than are seen in the DOV stars. The possibility remains that nonlinear interactions, or the interaction between pulsation and convection, could select only a few modes... but who knows?

Another possible mechanism for driving the DOV pulsations is associated with nuclear burning. Currently, all standard evolutionary models of PNNs and hot white dwarfs have active nuclear burning shells. Sienkiewicz (1980), and Kawaler *et al.* (1986b) show that nuclear burning drives low-*k* *g*-mode pulsation in models appropriate to planetary nebula nuclei. In models with helium-rich surfaces, the unstable periods are a factor of 2-5 shorter than the periods observed in the pulsating PG 1159 stars. It is possible that nuclear burning is responsible for the observed pulsations, but only if some additional physics left out of Kawaler *et al.*'s models is responsible for lengthening the observed periods. For example, the inclusion of mass loss in a self-consistent way into the structure of the the models and also into the stability analysis could provide a stabilizing influence. On the other hand, it could also lengthen the unstable periods to a degree where the pulsations of the DOV stars could be ascribed to modes driven by nuclear burning.

Recently, one of us (SDK) has investigated the pulsation properties of hydrogen-burning PNN models, finding that they too are unstable to short period *g*-mode pulsation. In particular, if the standard PNN models are correct, the hottest PNN should be pulsating with periods of about 50 to 200*s* (Kawaler 1988b). Extensive ground-based optical surveys of the best PNN candidates have been carried out by Hine (1988), and Hine and Nather (1987). Despite detection limits of order 0.5 mmag, as compared with the observed amplitudes of several mmag seen in the DOV stars, Hine and Nather have found no such pulsators. It is possible that the observed stars are, in fact, pulsating, but the amplitudes in the optical are too small to have been detected. If this is the case, then time series X-ray photometry will provide a much more stringent test of the pulsational stability of these objects.

A likely explanation for the important null result of Hine and Nather (1987) is that the standard PNN models used in the vibrational stability analysis are somehow in error. The precise structure of PNNs and PWDs depend sensitively on their prior evolutionary history. Models of PNNs based on this picture suggest that most PNNs have hydrogen-burning shells; this conclusion is based on the fact that models which burn hydrogen show the correct time scales of evolution across the H-R diagram (Schönberner 1987, and references therein). Similar models which burn helium evolve too slowly across through the region of the H-R diagram populated by planetary nebulae.

The null result of Hine and Nather (1987) indicates that current models which contain nuclear-burning shells are inadequate for detailed study of the pulsation and evolution of PNN and hot white dwarfs. For example, if nuclear burning were extinguished in a prior evolutionary stage, then the PNN would remain vibrationally stable. Models which do not contain burning shells but which evolve through the H-R diagram with acceptable time scales can, in principle, be constructed (D'Antona *et al.* 1987). Models of PNN evolution that satisfy the observational constraint that they are pulsationally stable result in white dwarfs with thin surface hydrogen layers. Such hot white dwarfs would be formed with total hydrogen masses less than $10^{-5}M_{0}$. However, because the tail of the hydrogen distribution reaches deep into the interior, a small fraction of hydrogen remains even if a large amount of mass is lost from the surface. This scenario for PNN evolution satisfies the pulsation constraint, and results in very few hot hydrogen rich white dwarfs; the surface hydrogen mass fraction falls below 0.1 when mass loss has reduced the total amount of hydrogen remaining to below $10^{-6}M_{0}$. The appearance of a hydrogen-rich spectrum in hot white dwarfs would occur only after gravitational settling of heavy elements has floated sufficient hydrogen to the surface, by which time the white dwarf would have cooled to a lower temperature. It is perhaps significant that the luminosity function of hot hydrogen-rich white dwarfs shows a sharp drop above 70,000K (Fleming, Liebert, and Green 1987). This temperature may provide a clue to total amount of hydrogen that remains at the surface of a cooling PNN. Clearly, the chemical evolution of white dwarf surfaces depends strongly on the initial thickness of the surface hydrogen layer (Fontaine and Wesemael 1987); the lack of observed pulsations in hot PNNs is an important clue to how much hydrogen hot white dwarfs have when they are born.

3. Conclusions

The frontier of stellar evolution represented by the white dwarfs is being pushed back thanks to observational mapping of the pulsation properties of the WDVs over the past 20 years; it is worthwhile to note that the most important tools in all this work have been modest 1-meter class telescopes. Using the observed pulsation spectra of the WDVs, pulsation theory has been able to determine some important quantities describing white dwarfs, such as rotation velocities and total masses. The evolutionary time scale for hot white dwarfs has been measured; it is just a matter of time before the more gradual changes in cooler white dwarfs are also detected. With a little additional work on both observational and theoretical sides, we may soon measure moderate magnetic fields in WDVs. We will also soon be able to quantitatively determine the compositional structure of their outer layers as modelling progresses. Even the lack of observed pulsations in the hottest white dwarfs is leading to re-evaluation of models of their structure. All of this success is rooted in the analysis of the basic pulsation spectra; the rich observed variations on this "linear" theme promise to provide additional detailed information on white dwarf structure and evolution. The WDVs are garden variety white dwarfs that have simply been caught in the act of pulsation; they are currently in one of the relatively brief phases of their evolution in which they are vibrationally unstable. By studying the properties of the pulsators, then, one is sampling the properties of all white dwarfs.

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