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ADDENDUM TO

'COHOMOLOGY AND PROFINITE TOPOLOGIES FOR SOLVABLE GROUPS OF FINITE RANK'

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Abstract

We remedy an omission in the proof of Proposition 2.7 of the paper 'Cohomology and profinite topologies for solvable groups of finite rank', *Bull. Aust. Math. Soc.* **86** (2012), 254–265. This proposition states that a solvable group with finite abelian section rank has merely finitely many subgroups of any given index.

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In the paper [1], a *solvable FAR-group* is a solvable group with finite abelian section rank. Moreover, $\mathcal{FS}$ denotes the class of all groups G such that, for each natural number n, G has only finitely many subgroups of index n. Proposition 2.7 in the paper states that every solvable FAR-group is a member of the class $\mathcal{FS}$; however, the argument provided applies only when the group is abelian. The purpose of this brief note is to fill that gap.

PROPOSITION. *Every solvable FAR-group belongs to $\mathcal{FS}$.*

PROOF. As in the paper, we write $H \leq_f G$ whenever H is a subgroup of finite index in the group G.

The proposition is proved by induction on the length of the derived series of the group, the abelian case having been established in the paper. Let G be a solvable FAR-group whose derived series has length >1, and suppose that n is a natural number. Take A to be the last nontrivial term in the derived series of G, and write $\epsilon : G \to G/A$ for the quotient map. By the inductive hypothesis, A and G/A both contain only finitely many subgroups of index $\leq n$. Hence, it will follow that G has merely finitely many subgroups of index n if we can establish that, for any $B \leq_f A$ and $Q \leq_f G/A$, the number of subgroups $H \leq G$ such that $H \cap A = B$ and $\epsilon(H) = Q$ is finite. To show this, set $K = \epsilon^{-1}(Q)$ and observe that, if $H \leq G$ with $H \cap A = B$ and $\epsilon(H) = Q$, then $B \lhd K$ and H/B is a complement to A/B in K/B. But $H^1(Q, A/B)$ is finite by

Proposition 2.8 in the paper, implying that K/B contains only finitely many such complements. Therefore, the number of subgroups H of G such that $H \cap A = B$ and $\epsilon(H) = Q$ must be finite. □

Reference

[1] K. Lorensen, 'Cohomology and profinite topologies for solvable groups of finite rank', *Bull. Aust. Math. Soc.* **86** (2012), 254–265, doi:10.1017/S0004972711003340.

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