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TWO-DIMENSIONAL SPREADING AND THICKENING OF AUFEIS

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ABSTRACT. The growth of two-dimensional, or laterally confined (flume), *aufeis* is shown from laboratory data to depend primarily on seven, independent, dimensionless parameters. During the early, two-dimensional, phase of its growth, *aufeis* consists of a mixture of ice and water, or ice–water slush, forming on a frigid base. Its early growth depends on four parameters: those expressing position along *aufeis*, period of spreading, slope of frigid base over which *aufeis* forms, and magnitude of heat flux to air from the surface of *aufeis* relative to latent heat release during freezing. The influences of two of the three remaining parameters, those expressing magnitude of heat flux to air relative to heat flux to frigid base and confined width of *aufeis* growth, are not felt until after a transition time has passed. The transition time apparently coincides with the beginning of the processes by which the ice–water slush on the surface of *aufeis* freezes solid. After a slush layer on *aufeis* begins to freeze solid, a new slush layer forms over its frozen surface. The continuing, cyclic process by which slush layers form and eventually freeze results in the ice laminations that are a feature of *aufeis*. The influence of the seventh governing parameter, a Reynolds number, cannot be discerned in the laboratory data.

NOMENCLATURE

d	Water or slush depth on the surface of *aufeis*
d_{co}	Critical depth in wide channel
f	Designator for "function of"
g	Acceleration of gravity
ℓ	Streamwise spread length of *aufeis*
ℓ_e	Equilibrium length of two-dimensional *aufeis*
ℓ_s	Equilibrium length if ϕ_i is neglected in expression for ℓ_e
L	Latent heat of water fusion
M	Total mass per unit width of *aufeis*
q	Water discharge per unit width
Q	Total water discharge
Re	Reynolds number
s	Total thickness of *aufeis*
S_0	Longitudinal slope of refrigerated flume
t	Time
t_e	Time-scale coresponding to length scale ℓ_e
t_ℓ	Time at which *aufeis* length reaches ℓ
t_s	Time-scale corresponding to length scale ℓ_s
T	Temperature
w	Width of refrigerated flume
x	Longitudinal coordinate
y	Vertical coordinate
α	Thermal diffusivity
η	Thickness of *aufeis* underlying surface layer of slush
ρ	Density
ν	Kinematic viscosity
ϕ_i	Heat flux from *aufeis* to base
ϕ_{wa}	Heat flux from *aufeis* to air
Φ_r	Ratio of ϕ_{wa} to $(\phi_{wa} + \phi_i)$
ψ	General dependent variable

Subscripts and superscripts

a	Air
b	Frigid base
f	Freezing
i	Ice
0	Value of variable at $x = 0$ or $t = 0$
s	Water surface
w	Water
*	Normalized quantity

1. INTRODUCTION

Small-scale *aufeis* formations were observed and monitored as they grew under conditions of steady discharge and heat-flux rates in a recirculating flume located in a refrigerated laboratory at the Iowa Institute of Hydraulic Research (IIHR). In a previous paper, Schohl and Ettema (1986b) presented basic theoretical concepts, including appropriate length and time scales, and a detailed, composite description of the processes associated with *aufeis* growth. The present paper, a continuation of that paper, presents laboratory data on the spreading and thickening of *aufeis* in terms of seven significant independent, dimensionless parameters that influence *aufeis* growth. The forms of the key parameters are determined from theory and dimensional analysis.

Aufeis formations (also referred to as naleds or icings) are spreading and thickening ice accretions that grow in cold winter air when a shallow flow of water streams over a river ice cover, or over frozen ground, and freezes progressively to it. They can form initially on any frigid surface, but their subsequent growth always preceeds as a progressive accretion of ice. *Aufeis* begins forming when an insulated flow path under an ice cover or under ground is interrupted, causing water to emerge at the surface and flow in frigid air. In cross-section, *aufeis* is typically laminated, as evident in Figure 1.

Aufeis formations have long been of interest and concern to scientists and engineers because they cause a variety of engineering problems. For example, they may block drainage facilities, causing subsequent spring flooding and wash-outs of embankments; they may inundate roads, railroads, and airfields causing them to become unusable;

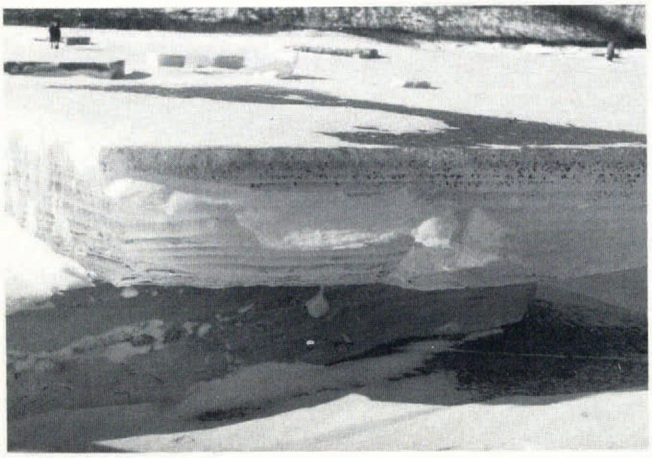

Fig. 1. A block of aufeis *formed on a shallow river. Note the ice laminations.*

they may engulf bridges spanning streams; they may threaten flood-plain communities and individual homes; and they may disrupt operation of tunnels and mines. Further information about *aufeis* formations occurring in Nature and the problems they cause has been provided by Schohl and Ettema (1986a, b), Ashton (1986), Kane (1981), Carey (1973), and Alekseyev and others (1973).

2. EXPERIMENTS

Aufeis was grown in a 0.60 m wide, 0.30 m deep, and 12 m long refrigerated, tilting flume. As *aufeis* spread and thickened down-stream along the flume, the profiles of the water and ice associated with it were recorded. Figure 2 illustrates schematically a simplified, or idealized, *aufeis* formation in the refrigerated flume. Because its longitudinal cross-section is invariant across the width of the flume, and its shape is defined by a typical longitudinal, two-dimensional cross-section, the *aufeis* formation in Figure 2 may be considered two-dimensional. In the experiments,

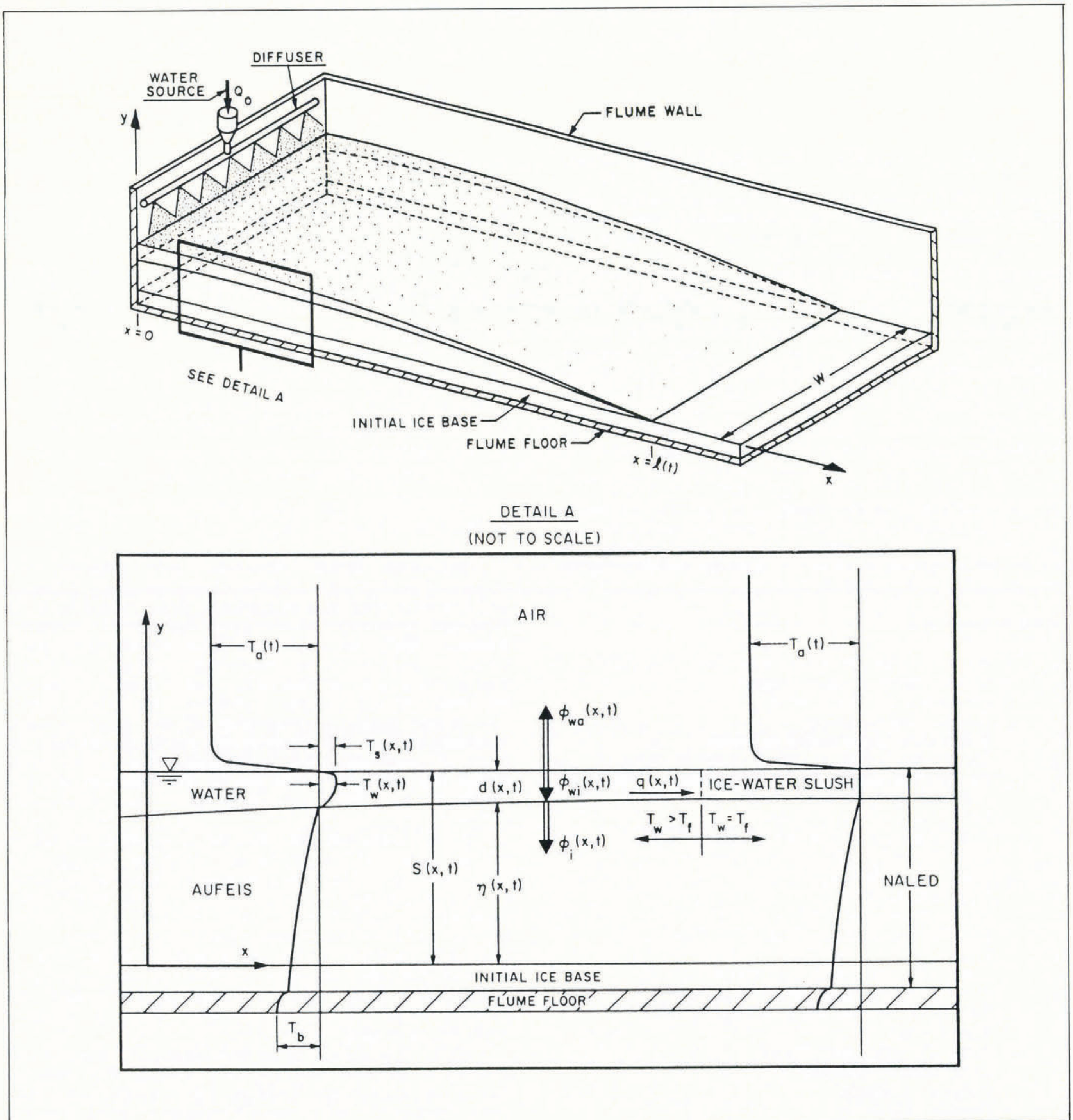

Fig. 2. Schematic of two-dimensional aufeis *formation in the refrigerated flume. Detail A shows the principal variables associated with* aufeis *formation.*

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aufeis spread and thickened in a complex, layer-by-layer manner. The early phase of its growth was two-dimensional, but its later growth was influenced by three-dimensional, or boundary, effects.

Figure 2 also illustrates many of the dependent and independent variables associated with *aufeis* growth. The variables controlled during the experiments include the source-water discharge, Q_0; the source-water temperature, T_{w0}; the air temperature, T_a (or heat flux, ϕ_{wa}); the flume-floor (circulating coolant) temperature, T_b (or heat flux, $\phi_i(x,0)$); and the flume slope, S_0. The dependent variables measured during the experiments include the spread length, $\ell(t)$, the thickness of the surface layer of water or slush, $d(x,t)$, and the thickness of *aufeis* underlying the surface layer, $\eta(x,t)$.

For each experiment conducted (36 total), Table I lists the values maintained for the controlled variables, the corresponding values of the key heat-flux components, and the values of four normalization scales (defined in subsequent sections). The heat flux ϕ_{i0} refers to the initial value of the heat flux, ϕ_i, from the water–ice interface into the underlying *aufeis*. This heat flux varied with both distance, x, and time, t, during the experiments, because in addition to depending on the flume floor temperature, T_b, it depends on the thickness of the initial ice base and on the thickness of *aufeis*, $\eta(x,t)$. One controlled variable not included in Table I is the source-water temperature. For all of the experiments, the temperature of the water supplied to the diffuser was maintained between about 1° and 3°C. Because the water cooled as it flowed through the diffuser, the temperature, T_{w0}, of the water in the pool under the diffuser was maintained between 0° and 0.5°C. All data obtained from the experiments have been documented by Schohl and Ettema (1986a).

Aufeis growth was initiated by introducing source water to the diffuser. An experiment was terminated either when the *aufeis* reached the down-stream (free overfall) end of the flume, or when the up-stream depth of *aufeis* approached the depth of the flume, whichever condition occurred first. As indicated in Table I, the time required for one experiment, from the first release of source water to the end of a test, varied from about 3 to 72 h. At the end of each test, the *aufeis* was cut to expose and measure the layers of solid and slush ice that characterize cross-sections of *aufeis*. The experimental apparatus and procedures have been described more fully by Schohl and Ettema (1986a, b).

3. DESCRIPTION OF *AUFEIS* GROWTH

The growth of a two-dimensional *aufeis* formation is portrayed sequentially in Figure 3, in which the vertical scale is distorted by approximately one order of magnitude. In Figure 3, T_f refers to the freezing temperature of water. The normalization scales t_s and ℓ_s, for which values are listed in Table I, are defined subsequently in section 4.

TABLE I. LIST OF EXPERIMENTS

Test No.	*Controlled variables*				*Independent variables*			*Normalization scales*				*Test duration*
	Q_0	T_a	T_b	S_0	q_0	ϕ_{wa}	ϕ_{i0}	ℓ_{e0}	t_{e0}	ℓ_s	t_s	
	l s^{-1}	°C	°C		l s^{-1} m^{-1}	W m^{-2}	W m^{-2}	m	h	m	h	h
1	0.00197	−4.4	−1.5	0	0.00392	34	89	10.7	80.7	38.7	1062	35.7
2	0.00195	−5.0	−2.7	0	0.00388	40	146	7.0	34.8	32.7	767	35.0
3	0.00200	−4.6	−0.3	0	0.00398	36	17	25.5	455	37.4	975	21.3
4	0.00203	−9.5	−1.3	0	0.00404	89	72	8.4	48.5	15.2	159	45.2
4b	0.00200	−9.6	−1.3	0	0.00398	90	70	8.3	48.4	14.8	153	32.7
5	0.00196	−9.0	−2.5	0	0.00390	83	128	6.2	27.2	15.7	176	44.7
6	0.00199	−9.4	−0.2	0	0.00396	87	11	13.5	127	15.2	162	43.5
7	0.00196	−12.7	−1.4	0	0.00390	132	72	6.4	28.9	9.9	69.5	52.7
8	0.00200	−12.1	−2.7	0	0.00398	123	149	4.9	16.6	10.7	80.6	72.3
9	0.00203	−13.6	−0.3	0	0.00404	144	17	8.4	48.1	9.3	60.1	25.2
10	0.00409	−5.1	−1.4	0	0.00813	40	76	23.4	187	68.3	1590	9.9
11	0.00399	−4.9	−2.3	0	0.00793	38	117	17.1	102	69.6	1700	13.4
12	0.00392	−5.0	−0.2	0	0.00779	39	11	51.3	937	65.9	1550	4.8
13	0.00395	−9.7	−1.5	0	0.00785	91	76	15.8	87.7	28.9	296	20.2
14	0.00397	−9.6	−2.3	0	0.00789	90	125	12.3	52.9	29.3	303	21.6
15	0.00403	−10.5	−0.3	0	0.00801	101	16	22.9	181	26.6	244	13.5
16	0.00399	−12.5	−1.5	0	0.00793	128	78	12.9	57.9	20.6	149	18.2
17	0.00396	−14.7	−2.8	0	0.00787	162	143	8.6	26.1	16.2	92.1	24.1
18	0.00395	−14.6	−0.3	0	0.00785	160	14	15.1	80.1	16.3	94.3	17.7
19	0.00301	−4.3	−0.6	0	0.00598	33	35	29.4	401	61.4	1750	10.6
20	0.00297	−5.1	−2.2	0	0.00590	40	122	12.2	69.4	48.8	1120	15.6
21	0.00300	−3.9	0.0	0	0.00596	29	0	66.2	2040	66.2	2040	7.7
22	0.00298	−10.5	−1.1	0	0.00592	102	58	12.4	72.3	19.4	177	14.0
22a	0.00295	−10.2	−0.9	0	0.00586	97	45	13.8	89.9	20.3	194	19.2
23	0.00294	−9.8	−2.8	0	0.00584	92	152	8.0	30.3	21.2	213	28.9
24	0.00300	−9.9	−0.1	0	0.00596	93	6	20.1	188	21.4	214	10.4
24a	0.00306	−10.5	−0.6	0	0.00608	101	35	14.9	102	20.1	185	17.7
25	0.00302	−13.0	−1.0	0	0.00600	135	53	10.7	52.7	14.8	102	23.5
26	0.00295	−13.2	−2.5	0	0.00586	139	135	7.1	24.0	14.0	93.2	37.0
27	0.00299	−12.6	−0.2	0	0.00594	130	9	14.3	95.6	15.2	109	20.3
28	0.00204	−5.0	−1.5	0.01	0.00406	39	86	10.9	80.7	34.6	822	16.7
29	0.00203	−13.6	−0.8	0.01	0.00404	145	43	7.2	35.2	9.3	59.1	43.7
30	0.00301	−3.9	−1.1	0.01	0.00598	29	59	22.6	237	68.1	2150	2.7
31	0.00299	−11.4	−1.4	0.01	0.00594	113	73	10.7	53.3	17.6	145	13.9
32	0.00196	−13.2	−5.2	0	0.00390	139	163	4.3	13.1	9.3	62.1	70.4
33	0.00201	−5.1	−4.7	0	0.00400	40	148	7.1	34.9	33.6	783	48.1

Note: the thickness of the initial ice base over which the *aufeis* grew was ≈0.03 m for tests 1–31 and 0.06 m for tests 32–33.

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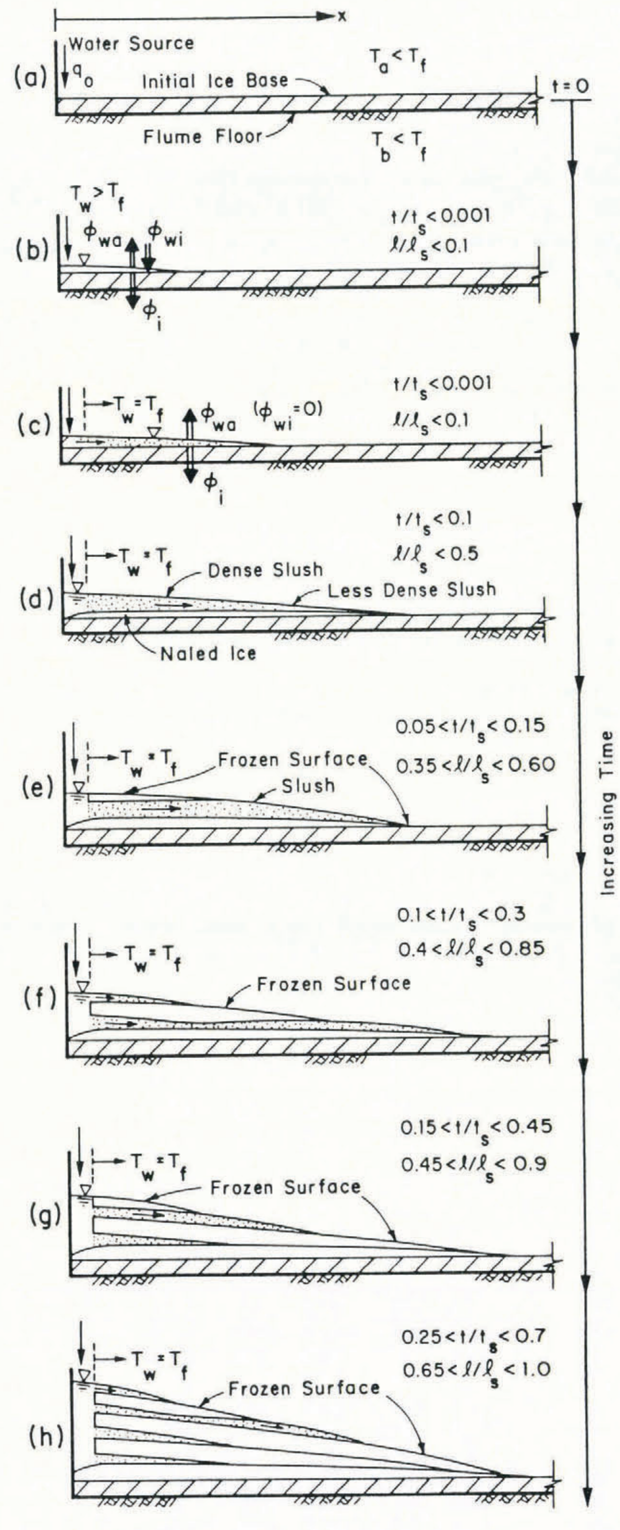

Fig. 3. Sketches illustrating two-dimensional aufeis *formation.*

Water cooled rapidly to its freezing temperature as it flowed over, and fed, an *aufeis* formation. Ice crystals, in the form of platelets anchored to the base ice surface, grew into the flow. The accumulation and growth of the ice platelets transformed the free-surface laminar flow of water into a flow through a porous medium composed of ice. Herein, this mixture of ice and flowing water is referred to as *aufeis* slush. During the experiments, *aufeis* slush eventually froze at its surface to form a solid crust of ice. Water continued to percolate through the layer of permeable slush, between the thickening ice crust over the slush and the underlying ice surface, until the permeability of the layer was sufficiently reduced to force water to flow over the frozen crust. Another slush layer then began to form. A second crust of ice would eventually form on the new surface flow. The continuing, cyclic process by which ice layers formed on the surface of slush layers resulted in the ice laminations that are a feature of *aufeis* cross-sections. Depending on the amount of time that the surface layers of ice had for thickening before they were covered by a fresh layer of slush, the laminations sometimes consisted entirely of solid layers of ice and sometimes consisted of alternating solid layers and layers of slush ice.

The flume constrained *aufeis* so that its initial growth was two-dimensional. However, the surface of the slush always began freezing and solidifying in the central third of its width rather than uniformly across the flume. As the central part hardened and thickened, constricting the flow through the underlying slush, the water was diverted to the sides of the flume so that the slush near the side walls continued to develop. The subsequent *aufeis* surface was uneven, with a rough, dry center and wet, slushy sides. Strictly speaking, the *aufeis* was no longer two-dimensional.

4. NORMALIZED PARAMETERS INFLUENCING *AUFEIS* GROWTH

The dependent variables that describe the growth of laterally confined *aufeis*, for example, the streamwise spread length (ℓ), the depth of ice–water slush on the surface (d), or the slope of the *aufeis* surface (S), depend on at least 14 independent variables. If a general dependent variable is designated as ψ, the following functional relationship can be written:

$$\psi = f_1(x, t, q_0, \phi_{wa}, \phi_i, S_0, w, \rho_i, \rho_w, \nu, g, L, \alpha_i, \alpha_w) \qquad (1)$$

in which (see Fig. 2) x is streamwise distance, t is time, q_0 is unit discharge of water, ϕ_{wa} and ϕ_i are heat fluxes from *aufeis* surface to air and from surface layer to underlying *aufeis*, respectively, S_0 is slope of base on which *aufeis* forms, w is lateral width of confined *aufeis* formation, ρ_i and ρ_w are densities of ice and water, respectively, ν is kinematic viscosity of water, g is gravitational acceleration, L is latent heat of water fusion, and α_i and α_w are thermal diffusivities of ice and water, respectively. Because temperature effects are incorporated into the heat-flux components, only three dimensions (length, mass, and time) are contained in the variables in Equation (1). Therefore, the 14 independent variables combine into 11 dimensionless groups, or normalized parameters.

Schohl and Ettema (1986a) derived, from conservation principles, depth-integrated equations for flow through *aufeis* slush. Rewritten in appropriate non-dimensional form, these equations indicate the correct form and importance of six of the 11 normalized parameters. The forms of the remaining five parameters are determined from dimensional analysis. In non-dimensional form, Equation (1) becomes

$$\psi^* = f_2\left(\frac{x}{\ell_e}, \frac{t}{t_e}, S_0, \phi_{wa}^*, \Phi_r, \frac{w}{\ell_e}, \mathrm{Re}, \frac{d_{co}}{\ell_e}, \frac{\nu}{\alpha_w}, \frac{\rho_i}{\rho_w}, \frac{\alpha_i}{\alpha_w}\right) \qquad (2)$$

in which ψ^* is a normalized dependent variable, $\Phi_r = \phi_{wa}/(\phi_{wa} + \phi_i)$, $\phi_{wa}^* = \phi_{wa}(10^{10})/\rho_w L^{3/2}$, where the constant 10^{10} is included merely for convenience, $d_{co} = (q_0^2/g)^{1/3}$, the critical depth in a wide rectangular channel conveying a unit discharge q_0, and $\mathrm{Re} = q_0/\nu$, a Reynolds number.

In Equation (2), ℓ_e, termed the equilibrium length, is a length scale and t_e is a commensurate time-scale. The equilibrium length is a theoretical spread length for two-dimensional *aufeis*, derived by integrating the depth-integrated conservation of mass equation over the length of the *aufeis*:

$$\ell_e = \frac{q_0 \rho_w L}{\phi_{wa} + \phi_i}. \qquad (3)$$

Subject to simplifying assumptions (see Schohl and Ettema, 1986a, b), for this length of spread, the source-water discharge is equal to, or in equilibrium with, the rate at which water is freezing along the *aufeis*. The time-scale t_e

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is defined in terms of the equilibrium length and the source-water discharge, q_0, as follows:

$$t_e = \frac{\ell_e^{\,2}}{100 q_o} \, . \qquad (4)$$

With the value of 100 included, t_e represents the time required to cover a unit width of *aufeis* surface, ℓ_e in length, with water of average depth $\ell_e/100$. For all the experiments, average water depths were several orders of magnitude less than values of ℓ_e.

The length scale ℓ_s, referenced in Figure 3, is defined by Equation (3) with ϕ_i omitted. As *aufeis* accumulates layer by layer, the flow on its surface is often insulated from the heat flux ϕ_i. Consequently, the influence of ϕ_i, though initially significant, diminishes with *aufeis* thickening. The time-scale t_s is defined by Equation (4) with ℓ_s substituted for ℓ_e.

Four of the 11 normalized parameters in Equation (2) may be omitted. The three parameters ν/α_w, ρ_i/ρ_w, and α_i/α_w are omitted because they are essentially constant for ice and water at a temperature near the freezing temperature. The parameter d_{co}/ℓ_e is omitted because it is associated with the acceleration term in the normalized momentum equation, and this term is negligible for seeping flow through *aufeis* slush. The functional relationship containing the key variables influencing *aufeis* growth is

$$\psi^* = f_3\left(\frac{x}{\ell_e}, \frac{t}{t_e}, S_0, \phi_{wa}^*, \Phi_r, \frac{w}{\ell_e}, \mathrm{Re}\right). \qquad (5)$$

The laboratory data on the spreading and thickening of *aufeis* are presented below in terms of the parameters in Equation (5). Because, as mentioned above, ϕ_i varied with x and t, the initial values of ℓ_e and t_e, designated as ℓ_{e0} and t_{e0}, are used to normalize length and time variables.

5. *AUFEIS* SPREADING

In its initial phase of growth (Fig. 3b), *aufeis* spread relatively quickly. The growth and accumulation of ice platelets, which removed water mass and impeded the flow, gradually slowed *aufeis* spreading (Fig. 3c and d). As its surface froze into a layer of solid ice and a second layer of slush formed (Fig. 3e and f), *aufeis* spread intermittently, stopping for periods of time and then continuing. *Aufeis* sometimes stopped spreading for relatively long periods of time when either its length approached its time-varying equilibrium length, or as the slush froze solid near its down-stream end (Fig. 3g and h). Often, a fresh slush layer would later spread over and beyond the down-stream frozen surface, thereby continuing the *aufeis* expansion.

Aufeis spreading is described by Equation (5) with ψ^* replaced by normalized spread length, ℓ/ℓ_{e0}. Because it is not a function of x/ℓ_e, spread length is influenced by six of the seven independent parameters in Equation (5). In Figure 4a–c, the experimental data for *aufeis* spreading on initially horizontal slopes ($S_0 = 0$) are plotted against log of normalized time. In each plot, the values of base slope, S_0, and normalized heat flux, ϕ_{wa}^*, are the same for all tests represented. Therefore, in accordance with Equation (5), the data in each plot should separate consistently with variations in Φ_r, w/ℓ_{e0}, and Re for those cases in which these variables influence spreading. Curves representing the data plotted in Figure 4a–c are plotted together in Figure 5 in order to illustrate the influence on *aufeis* spreading of the normalized surface heat flux, ϕ_{wa}^*.

During the early phase of *aufeis* growth (Fig. 3b–d), before the surface of the slush layer freezes solid, the normalized rate at which *aufeis* spreads is influenced by variations in ϕ_{wa}^* but it is not affected by variations in Φ_r, w/ℓ_{e0}, or Re, at least not for the ranges over which these parameters were varied during the experiments. Figure 5 illustrates the influence of variations in ϕ_{wa}^* on the early phase of spreading. *Aufeis* growing under colder air (larger ϕ_{wa}^*) spreads slower because ice platelets accumulate faster. The insensitivity of the early spreading rate to variations in Φ_r, w/ℓ_{e0}, or Re is illustrated in Figure 4a, in which all the data collapse approximately to form a single curve, and

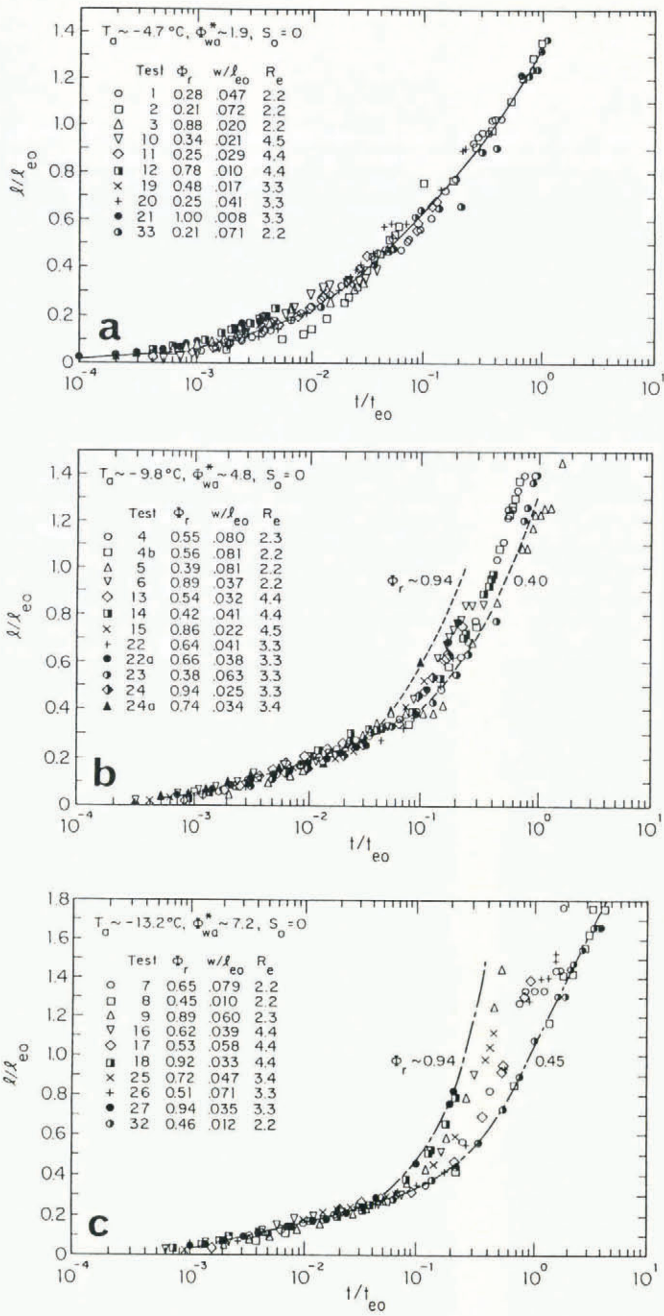

Fig. 4. Experimental data on aufeis *spreading over a horizontal slope: (a)* $T_a = -4.7\,°C$; *(b)* $T_a = -9.8\,°C$; *(c)* $T_a = -13.2\,°C$.

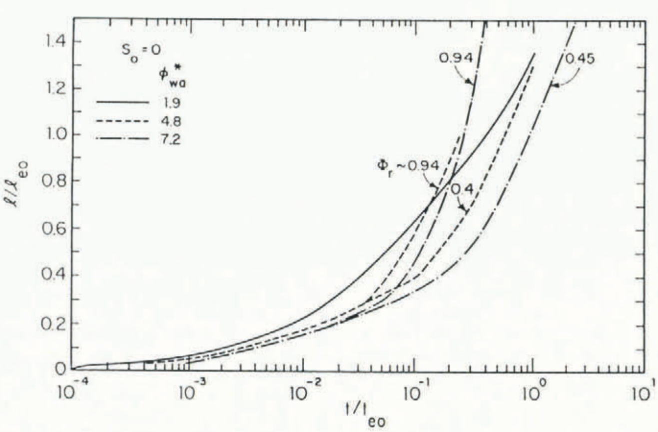

Fig. 5. Influences of ϕ_{wa}^* *and* Φ_r *on rate of* aufeis *spreading.*

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in Figure 4b and c, in which the data for t/t_{e0} less than about 0.05 form a single curve. (The slush on *aufeis* grown in air temperatures near −5 °C did not freeze solid during the experiments.)

Figure 4b and c show that the data eventually diverge from the common curves corresponding to the early phase of *aufeis* growth. Figures 4 and 5 together indicate that the point of divergence is influenced primarily by the heat flux ϕ_{wa}; that is, *aufeis* freezes over (or reaches the phase shown in Figure 3e) more rapidly when formed under conditions of larger ϕ_{wa}. Figure 4b and c show that, although other influences may be present, the point of divergence for each curve varies with Φ_r: the larger the value of Φ_r, or the larger value of ϕ_{wa} relative to the total heat loss $\phi_{wa} + \phi_i$, the sooner, in terms of t/t_{e0}, the data diverge from the initially common curve. Figure 5 shows that, for equal Φ_r, the data diverge more rapidly with larger values of ϕ_{wa}*, or ϕ_{wa}. The strong dependence of the divergence point on ϕ_{wa} suggests examining the time of divergence in terms of time normalized by t_s (defined in section 4) rather than t_{e0}. The data suggest that the divergence point in each case corresponds to a unique, transition value of approximately 0.035 for t/t_s. This value is determined by examining the geometric aspect ratio (average overall thickness, q_0t/ℓ, divided by length of spreading, ℓ) of *aufeis* as a function of t/t_s, as shown in Figure 6. The transition time, the time at which the spread

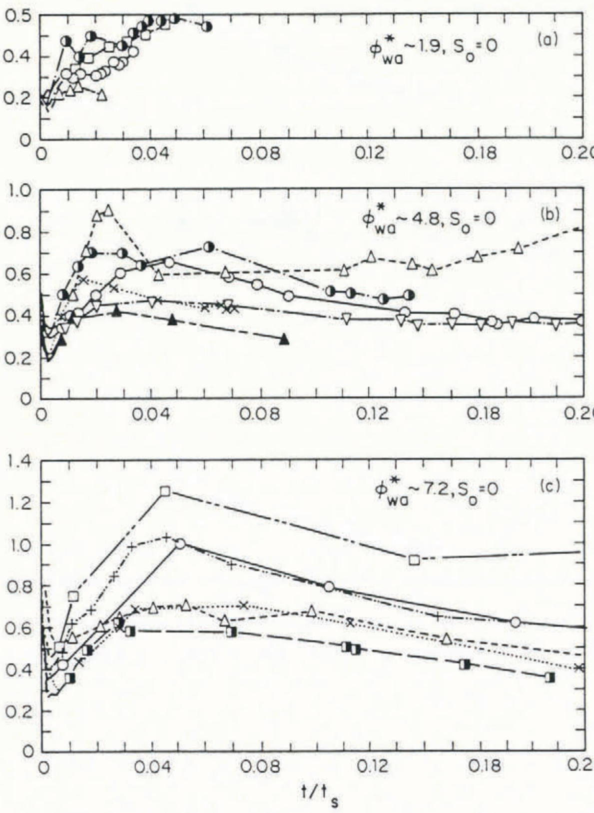

Fig. 6. Aspect ratio, defined as q_0t/ℓ^2, related to t/t_s (for (a), (b), and (c), symbols are defined in Figure 4a–c, respectively).

rate increases relative to the initial common rate, corresponds to the time at which the geometric aspect ratio reaches a maximum value. The value of t/t_s = 0.035 is the midpoint of the range 0.02–0.05 over which transition times estimated from the data are scattered.

Apparently, near the transition value of t/t_s, the magnitude of the heat flux ϕ_{wa} is effectively reduced over part of an *aufeis* formation's surface. Consequently, less of the water supplied to the *aufeis* freezes and the rate at which *aufeis* spreads is increased relative to the common curve representing the early phase of growth. The exact reason for the effective reduction in ϕ_{wa} when t/t_s passes 0.035 is not clear from the data. However, because the transition time is in terms of t_s, the reduction must be related to the increasing accumulation of ice platelets. It seems that, as time increases beyond the transition value of t/t_s, an increasingly significant part of the heat flux from the surface of *aufeis* acts to reduce the temperature within exisiting ice platelets rather than acting to cause more ice to grow within the slush. This effect leads to a decrease in the rate at which ice platelets grow. The transition value of t/t_s possibly represents the beginning of a continuous transition from the early, maximum rate of growth of ice platelets to the eventual slower rate of growth that occurs after the slush surface freezes over and partially insulates the underlying wet slush from the cold air above. If this is the case, the transition value of t/t_s may correspond to the start of the process by which the slush freezes solid.

At and beyond their divergence points, the curves in Figure 4b and c do not, in all cases, consistently separate in accordance with variations in Φ_r. This is attributed primarily to the influence on *aufeis* spreading of lateral boundaries (flume side walls) after the slush on the *aufeis* surface begins to freeze solid. As already mentioned, the *aufeis* slush usually froze first along the central one-third of the flume's width. When this occurred, some of the up-stream water flow spread over the frozen area, while the remainder was diverted toward the flume's side walls, away from the frozen area. The water in the slush near the flume's side walls could flow past the frozen slush in the center of the flume and contribute to the spreading of the *aufeis* down- stream. The parameter w/ℓ_{e0} accounts, at least partially, for the influence of lateral boundaries. Figure 4b and c reveal that, if two sets of data with nearly the same value of Φ_r are compared (such as tests 5 and 23 in Figure 4b or 17 and 26 in Figure 4c), the data associated with lesser values of w/ℓ_{e0} lie above the data for which w/ℓ_{e0} is larger. These results indicate that narrower *aufeis* formations spread faster, in normalized terms, than do wider *aufeis* formations.

For the relatively narrow range of Reynold's numbers, Re, attained during the experiments, no consistent effect due to variations in Re are discernible in the data. The effect of Re would cause variations in the resistance to flow through *aufeis* slush, which is most significant during the early phase of growth. However, the data indicate that the early phase of *aufeis* growth is not affected by variations in Re.

Data collected for two initial slopes of the laboratory flume are compared in Figure 7a and b. Figure 7a compares data from *aufeis* formed when S_0 = 0.01 and average ϕ_{wa}* = 1.75 with a curve representing data from *aufeis* formed when S_0 = 0 and average ϕ_{wa}* = 1.9. The data, which apply only to the early phase of *aufeis* growth, indicate that, all else equal, *aufeis* spreads faster on steeper slopes. Figure 7b compares data from *aufeis* formed when S = 0.01 and average ϕ_{wa}* = 6.7 with a curve representing data from *aufeis* formed when S_0 = 0 and average ϕ_{wa}* = 7.2. These data indicate that, in colder air, *aufeis* may not spread significantly faster on steeper slopes. Compared with *aufeis* grown on horizontal surfaces, the *aufeis* grown in test 31 spread significantly faster while the *aufeis* grown in test 29 spread only slightly faster during the early phase of *aufeis* growth. During the later, layer-by-layer, phase of growth, the data in Figure 7b suggest that *aufeis* formed on sloping surfaces may spread at about the same rates as *aufeis* formed, under similar conditions, on horizontal surfaces. However, the data suggest that the transition value of t/t_s may exceed 0.035 for *aufeis* grown on sloped surfaces, but further work is needed to verify this tentative conclusion.

6. *AUFEIS* PROFILE

Aufeis thickening is described by Equation (5) with ψ* replaced by normalized thickness, s/ℓ_{e0}. The longitudinal profile of a two-dimensional formation of *aufeis* is described in terms of overall thickness, s, as a function of streamwise distance, x (Fig. 2). Thickness s includes only the water and ice that accumulated during a test; it does not include the thickness of the underlying ice base preformed over the flume floor.

As is the case for *aufeis* spreading, for times less than the transition value of t/t_s (about 0.035), *aufeis* is

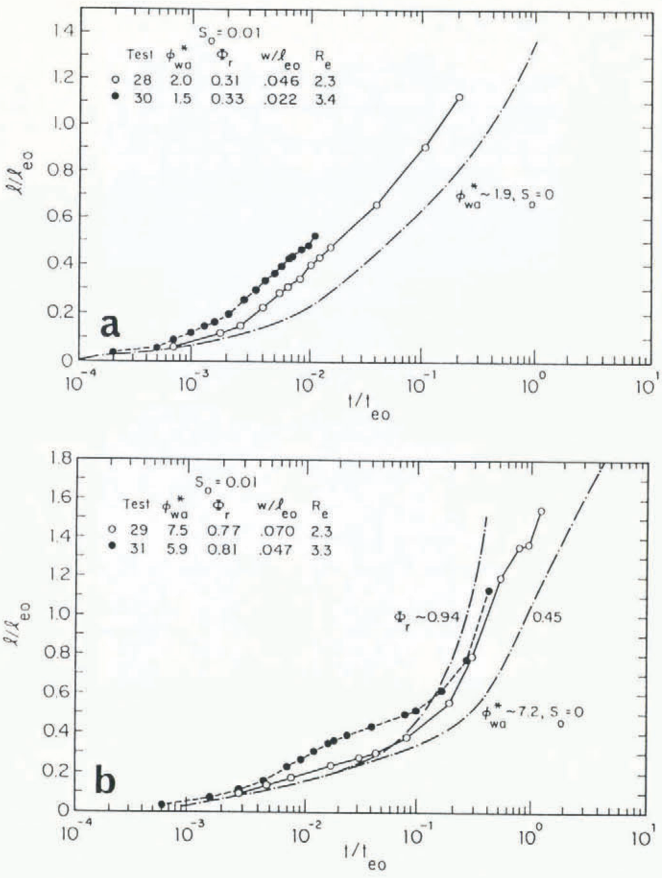

Fig. 7. Comparison of data on aufeis *spreading over two slopes,* $S_0 = 0$ *and* 0.01*; (a)* $\phi_{wa}^* = 1.5$–1.9*; (b)* $\phi_{wa}^* = 5.9$–7.5.

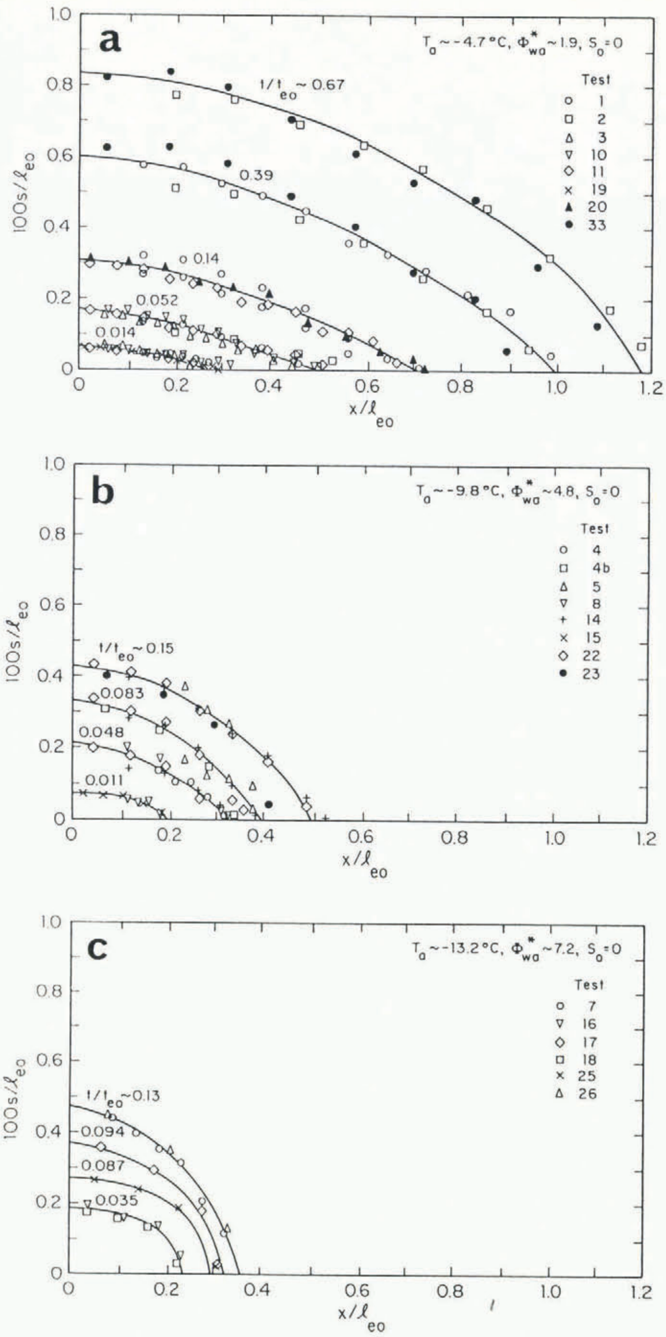

Fig. 8. Profiles of aufeis*, presented in terms of normalized parameters for times not exceeding transition time: (a)* $T_a = -4.7$°C*; (b)* $T_a = -9.8$°C*; (c)* $T_a = -13.2$°C.

influenced by ϕ_{wa}^* but is not affected by variations in Φ_r, w/ℓ_{e0}, or Re. This is evident in Figure 8a–c, which illustrate that the longitudinal shapes of *aufeis* formed in the flume for normalized times less than the transition time depend only on t/t_{e0} and ϕ_{wa}^*. Comparison of Figure 8a–c reveals that, for a given value of t/t_{e0}, *aufeis* thickness relative to *aufeis* length increases with increasing ϕ_{wa}^*. In each plot in Figure 8, the values of base slope, S_0, and normalized heat flux, ϕ_{wa}^*, are the same for all data included. Therefore, the data in each plot represent *aufeis* spreading at identical normalized rates (see Figs 4 and 5). The indicated values of t/t_{e0} are averages of the normalized times associated with the data on each curve. The point at which each curve intersects the abscissa corresponds to the appropriate spread length taken from Figure 4.

Figures 9a and b, and 10a and b illustrate that, for times larger than the time associated with the transition value of t/t_s, the shape of an *aufeis* formation is influenced by Φ_r in addition to t/t_{e0}, ϕ_{wa}^*, and S_0. For each plot in these figures the parameters S_0, ϕ_{wa}^*, and Φ_r are the same for all data included. Consequently, if the possible influence of w/ℓ_{e0} and Re are neglected, the data in each plot represent *aufeis* formations spreading at identical normalized rates. For average $\phi_{wa}^* = 4.8$ and a given value of t/t_{e0}, Figure 9a and b indicate that longer and thinner *aufeis* formations occur for larger values of Φ_r. The same result is shown in Figure 10a and b for average $\phi_{wa}^* = 7.2$

The possible influence on the data of the parameters w/ℓ_{e0} and Re is not evident in Figures 8–10. However, the parameter w/ℓ_{e0} must influence the longitudinal shape of *aufeis* because, as discussed in section 5, it influences the length of *aufeis* spreading.

As illustrated by the data associated with the largest values of t/t_{e0} in Figures 9 and 10, the surface of *aufeis* often became uneven after the slush on its surface began to freeze solid. A feature of *aufeis* formation that is not evident in the profiles presented in Figures 8–10 is ledging, or the stepped profile which characterizes the front of many *aufeis* formations. In the refrigerated flume, under conditions of constant air temperature and water flow, ledging was observed only at the down-stream front of spreading *aufeis*, as illustrated in Figure 11 (an *aufeis* formation formed under conditions similar to those in tests 8 and 32).

7. *AUFEIS* LAMINATION THICKNESS

As described in section 3, *aufeis* grows as successive layers of slush form and freeze solid into laminations of ice. Comparisons of measured values of lamination thickness with measured values of slush depth indicate that the thickness of an ice lamination is equal to the maximum depth attained by the slush from which the lamination developed. Therefore, data on slush depth indicate thicknesses of ice laminations in *aufeis*.

Besides being influenced by each of the key parameters given in Equation (5), the depth of a layer of *aufeis* slush may be affected by its position in the sequence of formation of slush layers. The data presented in this section were taken from the first layer of slush that developed after *aufeis* growth was initiated in the refrigerated flume.

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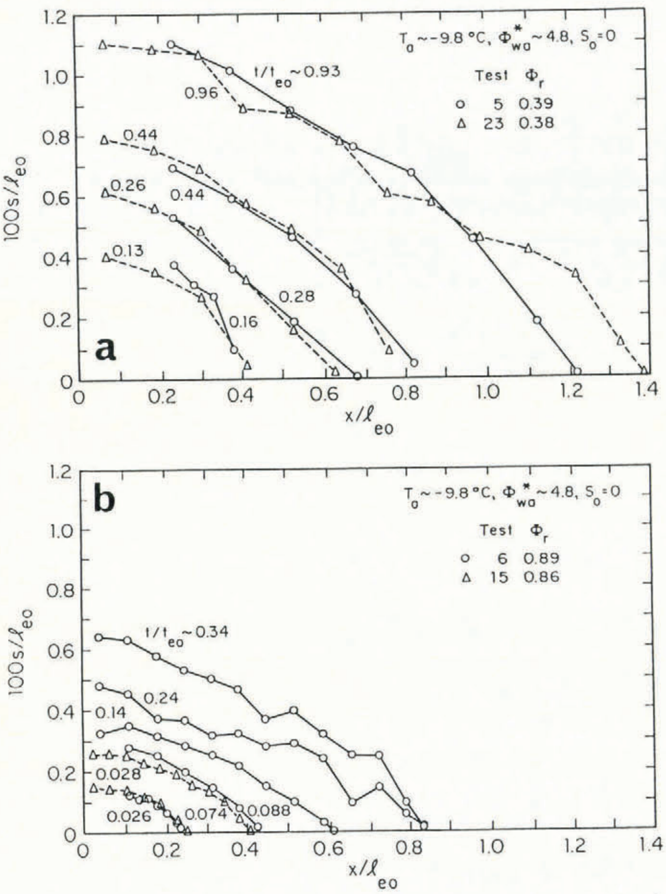

Fig. 9. Aufeis profiles, presented in terms of normalized parameters, for times exceeding transition time: $T_a = -9.8\,°C$; *(a)* $\Phi_r \simeq 0.38$; *(b)* $\Phi_r \simeq 0.87$.

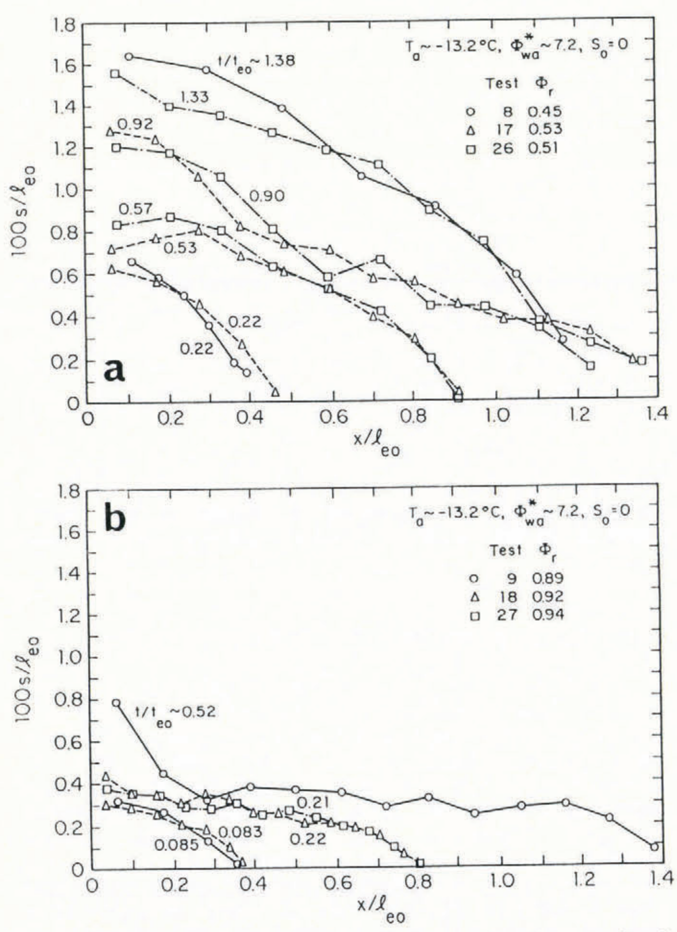

Fig. 10. Aufeis *profiles presented in terms of normalized parameters, for times exceeding transition time:* $T_a = -13.2\,°C$; *(a)* $\Phi_r = 0.45–0.53$; *(b)* $\Phi_r = 0.89–0.94$.

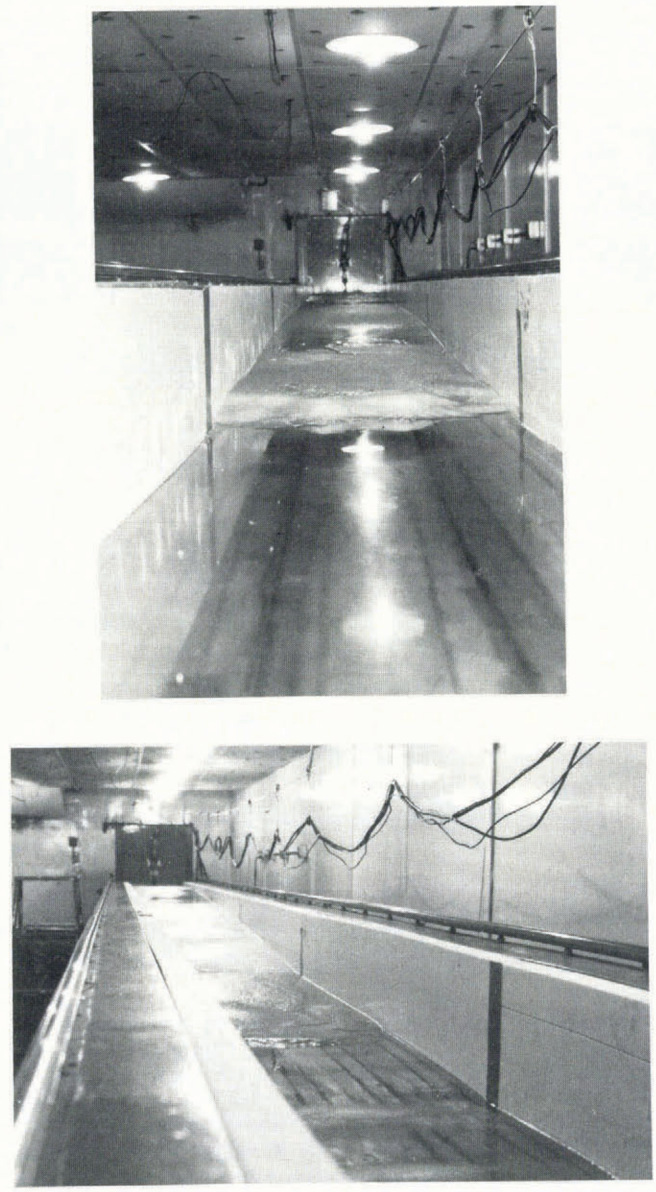

Fig. 11. Views of aufeis *formed in the flume. Mild ledging is evident at the front of the* aufeis.

The depth of the initial layer of slush, d, equals the thickness of the *aufeis*, s, minus the thickness of accumulated bottom ice, η (Fig. 2). The normalized depth of slush, d/ℓ_{e0}, varies with x/ℓ_{e0}, t/t_{e0}, and ϕ_{wa}^* in essentially the same manner s/ℓ_{e0} varies: slush depth was observed to decrease with increasing x and increase with increasing t and increasing ϕ_{wa}^*. Schohl and Ettema (1986a), though, show that d/ℓ_{e0} depends on Φ_r both before and after the transition value of t/t_s, unlike both ℓ/ℓ_{e0} and s/ℓ_{e0}.

Table II indicates the maximum depths attained by slush layers near the up-stream end of the flume. These depths should equal the thickness of subsequent ice laminations. The time at which the slush up-stream reached a maximum depth ranged from $t/t_s = 0.06$ to 0.10. At some time within this range, the surface of the slush up-stream

TABLE II. MAXIMUM DEPTH OF SLUSH UP-STREAM ($x/\ell_{e0} < 0.1$)

ϕ_{wa}^*	*Range of* $100d/\ell_s$
1.9	$0.10 < 100d/\ell_s < 0.24$
4.8	$0.20 < 100d/\ell_s < 0.24$
7.2	$0.26 < 100d/\ell_s < 0.30$

froze sufficiently to stop the thickening of the initial layer of slush. The maximum depth of slush decreased with increasing distance over *aufeis* but enough data to describe this variation are available only for $\phi_{wa}^* = 7.2$, for which the following measurements were made: at x/ℓ_{e0} near 0.5, the maximum d/ℓ_s ranged from 0.23 to 0.29 and at x/ℓ_{e0} near 1.0, the maximum d/ℓ_s ranged from 0.14 to 0.23.

Maximum slush depth, d, and corresponding time, t, are normalized by ℓ_s and t_s rather than ℓ_{e0} and t_{e0} in order to minimize the influence of Φ_r. In fact, the quantity of data is not sufficient to identify any consistent variations with Φ_r in either the maximum value of d/ℓ_s or in the corresponding value of t/t_s.

The slush layers developing after the initial layer were nearly as deep as the initial layer. This observation suggests that little water flowed through slush layers under the surface of an *aufeis* formation. The succeeding layers formed from nearly as much water flow as did the initial layer.

8. SCALES

In section 4, ℓ_{e0} and t_{e0}, the initial values of ℓ_e and t_e, are introduced as length and time scales for normalizing and, therefore, describing *aufeis* formation. Only the initial values of ℓ_e and t_e are convenient to use because the heat flux to the frigid base, ϕ_i, diminishes over time, gradually increasing the values of ℓ_e and t_e until, in the limit, these scales are equivalent to ℓ_s and t_s, which are length and time scales derived by neglecting ϕ_i. The usefulness of ℓ_{e0} and t_{e0} in describing *aufeis* geometry, particularly during the early phase of *aufeis* growth, is verified in the figures presenting the laboratory data on spreading and thickening of two-dimensional *aufeis* (e.g. Fig. 4). Use of ℓ_s and t_s, as is done in Figure 3, may be appropriate for describing the layer-by-layer phase of *aufeis* growth, after the transition time has passed, or, more generally, when ϕ_{wa} is much greater than ϕ_i. By themselves, however, neither ℓ_{e0} and t_{e0} nor ℓ_s and t_s are sufficient to describe *aufeis* formation.

The scales ℓ_{e0} and t_{e0} are derived from integration of the conservation-of-mass equation for flow over the surface of *aufeis*. Consequently, these scales normalize differences in total mass between *aufeis* formations, where normalized total mass per unit width of *aufeis*, M^*, is defined as follows:

$$M^*(t) = \frac{\rho_w q_0 t}{\rho_w \ell_{e0}^2} = \frac{t}{100 t_{e0}} . \qquad (6)$$

As is evident from Equation (6), at any given value of normalized time, t/t_{e0}, all two-dimensional *aufeis* formations consist of the same quantity of normalized mass. Furthermore, for all *aufeis* formations spreading at the same normalized rate (ℓ/ℓ_{e0} as a function of t/t_{e0}), the division of the total normalized mass into ice mass, M_i^*, and water mass, M_w^*, is the same. This is evident from the following equation for normalized ice mass, M_i^*, derived by Schohl and Ettema (1986a) for constant total heat flux, $\phi_{wa} + \phi_i$:

$$M_i^*(t) = \frac{1}{100} \int_0^{\ell/\ell_e} \left[\frac{t}{t_{e0}} - \frac{t_\ell(x)}{t_{e0}} \right] d \left[\frac{x}{\ell_{e0}} \right] \qquad (7)$$

in which M_i is ice mass per unit width and $t_\ell(x)$ is time at which the spreading, down-stream edge of *aufeis* reaches streamwise position x. Because the mass of unfrozen water is the difference between the total mass and the total ice mass, *aufeis* formations consisting of the same M^* and M_i^* also contain the same M_w^*.

While the total quantity of ice is accounted for using the scales ℓ_{e0} and t_{e0}, they do not take into account the division of ice into ice platelets, which grow to balance the heat flux ϕ_{wa}, and bottom ice, which grows to balance the heat flux ϕ_i. Effects that depend on one or the other of these heat fluxes, rather than the total heat flux, vary with Φ_r and ϕ_{wa}^*. For example, Figures 4 and 5 show that *aufeis* formations spread at different rates for different values of Φ_r and ϕ_{wa}^*. The effects on *aufeis* formation of both Φ_r and ϕ_{wa}^* are attributable to two influences of ϕ_{wa} that are separate from the influences of the total heat flux, $\phi_{wa} + \phi_i$. First, the ice platelets that grow to balance ϕ_{wa}, or ϕ_{wa}^*, increase resistance to water flow and, thereby, impede *aufeis* spreading. Secondly, of the total heat flux, the time at which the slush on *aufeis* begins to freeze solid is influenced primarily by ϕ_{wa}, an influence appropriately represented by the ratio of ϕ_{wa} to the total heat flux, or Φ_r.

Figure 8 shows that *aufeis* formations spreading at the same normalized rate have the same longitudinal shape at any given time, t/t_{e0}, before the transition time has passed. That is, not only do they have the same normalized mass, as mentioned above, but the distribution of the mass over x/ℓ_{e0} is also the same. However, Figure 8 also shows that, before the transition time has passed, *aufeis* thickness relative to *aufeis* length increases with increasing ϕ_{wa}^*. Explained within the context of the current discussion, different *aufeis* formations have the same total mass at a common value of t/t_{e0}, but their normalized spread lengths, ℓ/ℓ_{e0}, are different, depending on ϕ_{wa}^*. For two-dimensional formations, equivalence in total mass corresponds to equivalence in the cross-sectional areas under the curves (Fig. 8) delineating the shapes of different *aufeis* formations. Consequently, as would be expected, at the same value of t/t_{e0}, a shorter *aufeis* formation is thicker than a longer one.

The foregoing discussion deals with definition of appropriate scales for describing two-dimensional formations of *aufeis*. For description of three-dimensional formations, such as may form at the exit of a culvert or over a broad, shallow river, different length and time scales would apply. Appropriate scales would entail use of a theoretical spread area from which would be determined length as well as time-scales.

9. CONCLUSIONS

The many factors influencing *aufeis* growth reduce to a set of seven key independent parameters as expressed in Equation (5).

A transition time is identified which apparently coincides with the beginning of the processes by which the initial layer of ice–water slush on *aufeis* freezes solid. In terms of the time-scale t_s, this transition time is approximately $t/t_s = 0.035$. However, the transition time may vary slightly with ϕ_{wa}^* and Φ_r, and significantly with S_0.

The early phase of *aufeis* formation, before the transition time has passed, is influenced by only the first four key parameters listed in Equation (5). For the same values of S_0 and ϕ_{wa}^*, *aufeis* formations in their early phase spread at the same normalized rate (Fig. 4) and have the same normalized shape (Fig. 8). *Aufeis* formed under large values of ϕ_{wa}^* spreads more slowly and, consequently, is shorter and thicker than *aufeis* formed under smaller values of ϕ_{wa}^* (Figs 6 and 8). The small amount of data collected from *aufeis* formed on a sloped surface suggests that, during the early phase of formation, *aufeis* spreads faster on steeper surfaces (Fig. 7).

After the transition time has passed, Φ_r and w/ℓ_e of the remaining three parameters in Equation (5) come into play. The normalized rate of *aufeis* spreading increases with increasing Φ_r and with decreasing w/ℓ_e (Fig. 4). At a given value of t/t_e, *aufeis* forming under large Φ_r is both longer and thinner than *aufeis* formed under smaller Φ_r (Figs 4, 9, and 10). The data from *aufeis* formed on a sloped surface suggest that, after the transition time has passed, the rate at which *aufeis* spreads may no longer depend on slope, S_0 (Fig. 7).

Although it is possible that variations in Reynolds number, Re, also influence *aufeis* formation, for the relatively narrow range of Reynolds numbers attained during the experiments, no consistent effect is discerned.

The thickness of *aufeis* laminations corresponds to the thickness attained by slush layers before they freeze solid.

The present study is limited to two-dimensional *aufeis* formations such as those, for example, that may develop in culverts. Considerable work remains before relationships are available for use in the design of culverts and other watercourses for the possibility of *aufeis* formation. Further work is also needed on *aufeis* formations which spread and thicken three-dimensionally.

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