THE LOCATIONS OF SECULAR RESONANCES AND THE EVOLUTION OF SMALL SOLAR SYSTEM BODIES

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Abstract. Generalizing the secular perturbation theory of Milani and Knežević (1990), we have determined in the $a - e - I$ proper elements space the locations of the secular resonances between the precession rates of the longitudes of perihelion and node of a small body and the corresponding eigenfrequencies of the secular perturbations of the four outer planets. We discuss some implications of the results for the dynamical evolution of small solar system bodies. In particular, our findings include: (i) the fact that the $g = g_6$ resonance in the inner asteroid belt lies closer than previously assumed to the Flora region, providing a plausible dynamical route to inject asteroid fragments into planet–crossing orbits; (ii) the possible presence of some low–inclination "stable islands" between the orbits of the outer planets; (iii) the fact that none of the secular resonances considered in this work exists for semimajor axes > 50 AU, so that these resonances do not provide a mechanism for transporting inwards possible Kuiper–belt comets.

1. Introduction

Resonance effects between the precession rates of perihelion and node of a minor body and the corresponding eigenfrequencies of the secular perturbations of the four outer planets are called *secular resonances*. The positions of these resonances in the whole semimajor axis range between 0.1 and 55 AU have first been determined by Heppenheimer (1979), who used a purely linear, first–order analytical theory. The resonance locations were found by comparison with values of the fundamental planetary frequencies computed by the secular perturbation theory of Brouwer and Van Woerkom (1950). This theory, based on low–degree truncations of the perturbative series (dropping most perturbative terms of order two and higher in the ratio between planetary and solar mass, and including only low–degree terms in the eccentricities and inclinations) provides only approximate values for the planetary frequencies. As a consequence of this, the results of Heppenheimer — as well as those of other investigators — on the locations of the secular resonances had a limited accuracy.

Several authors in the last 20 years have determined the positions of secular resonance surfaces, although only in the region of the main asteroid belt. Their

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secular perturbation theories were much more refined than the linear one, but still based on some truncations and approximations. In 1969 Williams developed a mixed analytical–numerical theory based on the Gauss averaging method, which can be applied to derive proper elements. By means of this theory, Williams and Faulkner (1981) derived detailed graphical maps of the three main secular resonances $g = g_5$, $g = g_6$ and $s = s_6$ in the asteroidal belt. However, this theory, of order one in the mass ratio, breaks down near the mean motion resonances and cannot describe the topology of secular resonance surfaces in these zones. More recently, Morbidelli and Henrard (1991) revived the use of semi–numerical averaging methods. They developed a theory which avoids any expansion of the main term of the Hamiltonian (the linear one in the masses) with respect to the eccentricity or inclination of the small body, in order to achieve results valid for any values of these elements. Introducing suitable action–angle variables, they took into account the dynamics related to the motion of the argument of perihelion of the small body, which is important at high inclinations. They also computed the correction due to the quadratic terms in the mass ratio, using a series expansion in eccentricity and inclination up to degree four. They then draw the surfaces of the main secular resonances $g = g_5$, $g = g_6$ and $s = s_6$ (and also some more complicated ones) in the semimajor axis interval $1.5 \leq a \leq 3.5$ AU.

In this paper we are going to discuss the locations of secular resonances in the $a - e - I$ proper elements space both in the asteroid belt and in the outer solar system, by using a theory based on updated planetary frequencies, and yielding accurate results up to moderate values of the eccentricities and inclinations. In Sec. 2 we shall recall briefly the main features of this theory (for a more detailed description, see Milani and Knežević, 1990, Knežević et al., 1991, and the papers referenced therein). Then (Sec. 3) we shall discuss some implications of the results for the dynamical evolution of different types of small solar system bodies.

2. Secular resonance locations.

The determination of the positions of secular resonances consists in the computation of the frequencies g and s of the perihelion and node of a minor body as functions of the proper orbital elements, and then in a search for values that satisfy a resonance condition ($g = g_j$, $s = s_j$; and also more complicate combinations). In this paper we discuss only the "linear" secular resonances, i.e. those involving just one secular frequency of the perturbed body (g,s) and one planetary frequency (g_j, s_j), which appear already in the linear first–order secular perturbation theory; for some recent results on other types of secular resonances, see Milani and Knežević (1990, 1991). To compute the secular frequencies we have used the following equations, generalized by Knežević et al. (1991) in such a way to accomodate any number of relevant perturbing planets:

$$g = g_0 + \frac{2}{L^*}\frac{\partial F_1^{**}}{\partial \nu^2}; \qquad s = s_0 + \frac{2}{L^*}\frac{\partial F_1^{**}}{\partial \mu^2} \tag{1}$$

Here g_0 and s_0 are respectively the free oscillation frequencies of the longitude of perihelion and of ascending node, in the assumption that the planets were moving along coplanar and circular orbits. Their values are derived from a second-order linear theory, namely a theory which takes into account in the disturbing function linear and quadratic terms with respect to the perturbing mass, and terms of second degree in eccentricity and inclination. While in the Lagrangian linear theory the free frequencies have the same values with just opposite signs (namely $s_0 = -g_0$), the consideration of quadratic terms in the masses makes $s_0 \neq -g_0$, even in the limit $e \rightarrow 0$, $i \rightarrow 0$.

The second terms of the right-hand sides of Eqs. (1) are the corrections applied in order to obtain the final values g and s. The quantity $F_1{}^{**}$ is the first-order fourth-degree secular part of the Hamiltonian. In the equations giving the derivatives of $F_1{}^{**}$ with respect to the proper amplitudes of the Poincaré variables ν and μ ($\frac{\partial F_1^{**}}{\partial \nu^2}$; $\frac{\partial F_1^{**}}{\partial \mu^2}$), singularities can occur (see Knežević et al., 1991, Eqs. (3) and (4)) when g_0, s_0 are equal to one of the planetary frequencies g_j, s_j. Singularities also occur for mean motion resonances of the form $(p+1):p$ and $(p+2):p$; due to the truncation of the theory in the second-order to degree two, only first and second order mean motion resonances are accounted for (see Knežević, 1989). While the latter singularities cannot be removed, the former are artifacts, resulting from the ordering of the different terms taken into account by the theory. Nevertheless, the formulae given above can be used to estimate the location of secular resonances, because these singularities in general occur at different positions in the phase space; of course, when a singularity is close to a resonance, the accuracy is somewhat degraded. To determine the secular resonance surfaces, we used in our computations the planetary frequencies from the *LONGSTOP 1B* dynamical model (Nobili et al., 1989), derived from the analysis of long-term numerical integrations of the orbits of the outer planets.

We then divided the solar system into five semimajor axis zones spanning the range from 2 to 50 AU, namely: the asteroid zone 1 ($2.0 \leq a \leq 4.0$ AU); three zones between the outer planets: zone 2 ($6.2 \leq a \leq 8.0$ AU), zone 3 ($11.3 \leq a \leq 16.2$ AU), zone 4 ($22.8 \leq a \leq 25.3$ AU); and the transneptunian zone 5 ($25.8 \leq a \leq 50$ AU). For each zone the secular resonances have been mapped in a proper inclination versus proper semimajor axis plane for a fixed proper eccenticity value of 0.1 (Knežević et al., 1991). Each resonance has been represented by the three contour lines corresponding to values of the associated divisor $g - g_j$ of $(-g_j/30, 0, g_j/30)$ $arcsec/yr$ for the perihelion resonances and of $(s_j/30, 0, -s_j/30)$ $arcsec/yr$ for the divisor $s - s_j$ of the node resonance; for the strong resonances $g = g_6$ and $s = s_6$, this corresponds approximatively to a width of 1 $arcsec/yr$. Of course, these contour lines are not the true borders of the "resonant strips", but are meant to just give an idea of the relative importance of the resonances (i.e., wider is the strip, more effective is probably the resonances. Plots of g_0 and $-s_0$ versus semimajor axis were also derived for each zone; on them, the positions of mean motion resonances are apparent.

We have tested the sensitivity of our results to changes in the proper eccentricity. A comparison between Figs. 1a and 1b show the relatively strong dependence of the location of the node resonance upon the proper eccentricity. The shift is about 3° passing from $e = 0$ to $e = 0.1$. On the other hand, the corresponding shift in the location of the perihelion resonances is very small. In both cases, anyway, no

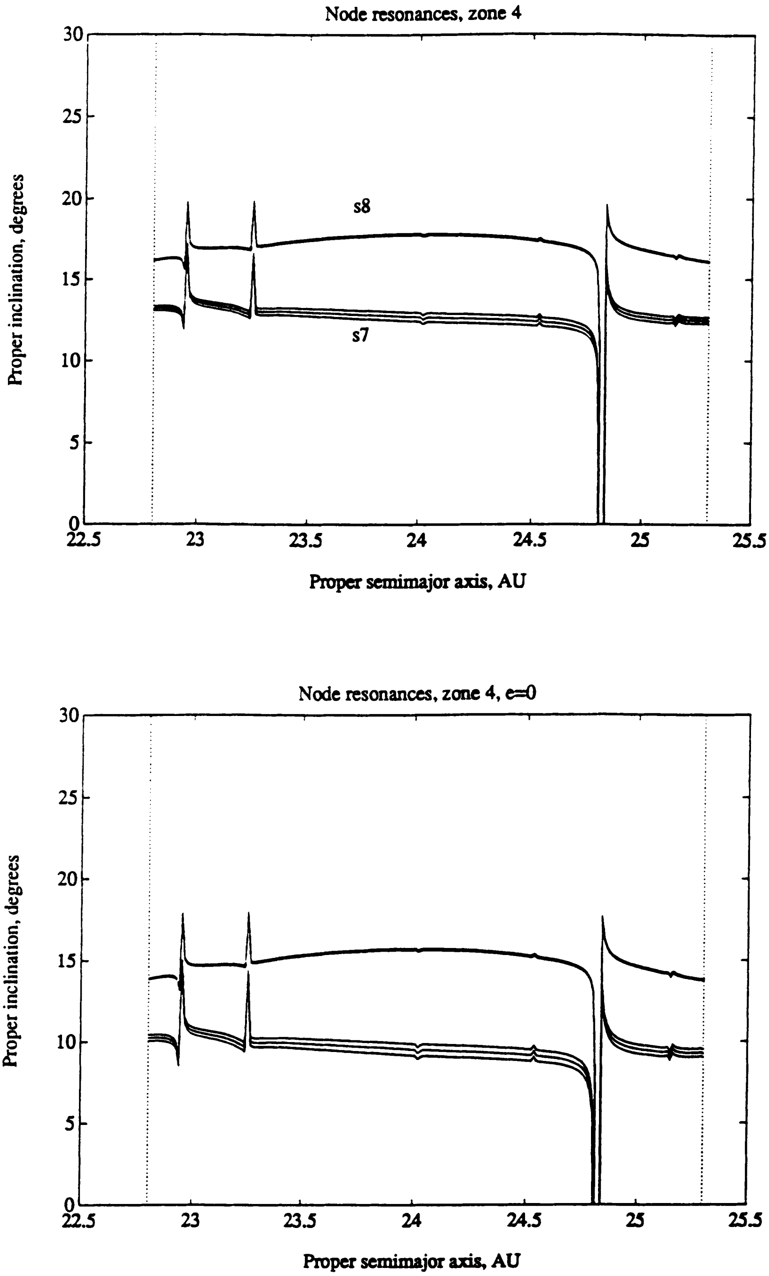

Fig. 1. Node secular resonances for a constant value of the proper eccentricity, in the proper inclination vs. proper semimajor axis plane. Resonances are labelled by the corresponding planetary eigenfrequency involved (i.e. s_7 stands for $s = s_7$). Singularities appear whenever a secular resonance crosses a mean motion resonance. 1a.proper eccentricity e=0.1, 1b.proper eccentricity e=0.0.

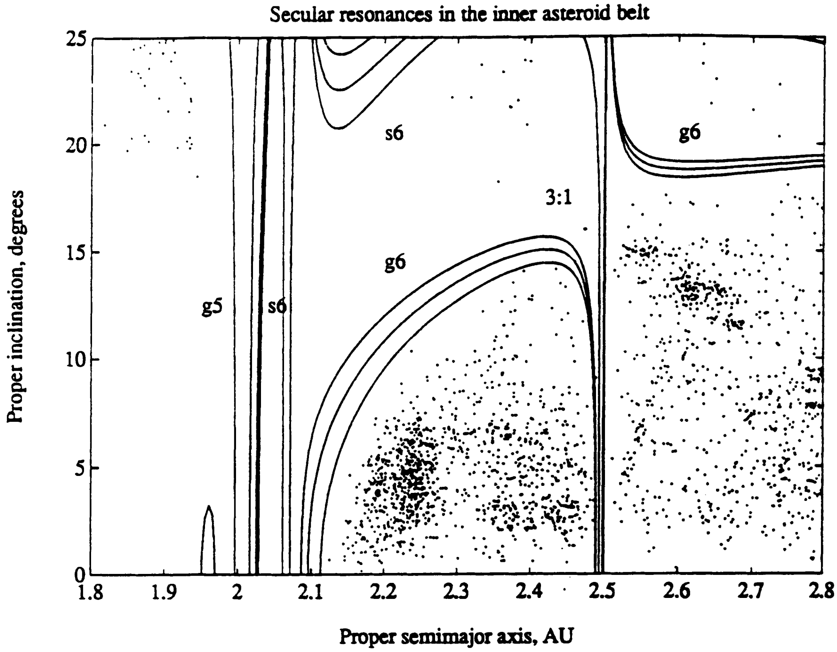

Fig. 2. An enlargement of the inner asteroid belt. The dots represent the position in proper elements space of all asteroids with proper eccentricity $e < 0.2$. The resonance lines are drawn for a constant proper eccentricity $e = 0.1$.

qualitative change in the shape of the resonance curves appears for low to moderate values of the proper eccentricity.

3. The potential importance of secular resonances for the dynamical evolution of small solar system bodies

Let us now discuss some implications of the positions of main secular resonances for the dynamical evolution of the small bodies orbiting the Sun. Most of these issues have to be considered only at a qualitative level, since the theory described above did not allow us to derive the real "strengths" and "widths" of the resonances, i.e. their capability to induce strong variations in eccentricity and inclination at various distances from the exact resonance surfaces. On the other hand, in order to obtain quantitative results, extensive numerical experiments should be carried out for each zone, and this has not yet been done.

(a) An important application concerns the delivery of meteorites and Earth–approaching small asteroids from the inner edge of the asteroid belt. We have found (see Fig. 2) that the $g = g_6$ secular resonance at low inclinations occurs at a semi–major axis of about 2.10 AU, rather than at 2.05 AU as previously assumed (Williams and Faulkner, 1981; Wetherill, 1988). This difference is due in part to the inclusion of higher–order and/or –degree terms in the theory, and in part to the improved planetary frequency g_6 adopted in our dynamical model (we took $g_6 = 28.25$ *arcsec/yr* instead of 27.77 *arcsec/yr*, the value of Brouwer and Van Woerkom adopted by Williams and Faulkner). Although the exact location of the resonance is also affected by the inner planets (which are not taken into account by our theory), and therefore at the level of a few hundredths of *AU* must be considered model–dependent, there is no doubt that $g = g_6$ is closer than previously assumed to the Flora region of the inner asteroid belt, where a large number of small asteroids are crowded. We conjecture that when some of these asteroids are broken up by a collision, a significant fraction of the resulting fragments may — either directly or after close approaches with Mars — enter the secular resonance, undergo large increases in eccentricity and become Earth–crossers within a time span of the order of 1 *Myr* (Scholl and Froeschlé, 1991).

The mean motion resonances of order higher than two are not accounted for by our theory, so the shape of the curves representing $g = g_6$ and $s = s_6$ between 2.0 and 2.1 *AU* are probably not very meaningful, since at 2.065 *AU* we find the strong 4 : 1 mean motion resonance with Jupiter. Numerical experiments (Scholl and Froeschlé, 1991) have actually shown that bodies injected in the region from 2.0 to 2.13 *AU* may have a complex dynamical behavior, jumping from the secular resonance $g = g_6$ to the mean motion resonance 4 : 1. According to these calculations, the 4 : 1 resonance broadens significantly the Earth–crossing region and decreases the transport time scales.

(b) As shown by Fig. 2, several asteroids lie close (or possibly inside) the $g = g_6$ resonance between 2.3 and 2.5 *AU*. Of course, the presence of the 3 : 1 resonance may limit the accuracy of the proper elements in the vicinity of 2.5 *AU*. However, recent numerical integrations (Froeschlé and Scholl, 1991) of fictitious bodies with $a = 2.3$ and 2.4 *AU*, starting eccentricity 0.14 and inclination $I = 14°$ and $16°$ show that during the whole integration time span of 2.7 *Myr* they were always or temporarily located in the secular resonance, suffering large variations in eccentricity (up to ~ 0.85) and becoming planet–crossers in a time scale of $\sim 10^6 yr$. Taking into account the results mentioned previously, Froeschlé and Scholl have conjectured that the secular resonance $g = g_6$ is a good candidate for producing planet–crossing objects at least up to a semi–major axis of ≤ 2.4 *AU*.

Farinella et al. (1991) have recently modeled the ejection of fragments from cratering or break–up events undergone by real asteroids as a consequence of impacts. They computed the fraction of escaped fragments falling in the g_6 resonance. For some asteroids between $2.4 \leq a \leq 2.5$ *AU* (6 Hebe, $a = 2.42$ *AU*; 304 Olga, $a = 2.40$ *AU*; 623 Chimaera, $a = 2.46$ *AU*; 930 Westphalia, $a = 2.43$ *AU*) the percentage of ejected fragments inside the $g = g_6$ resonance is of the order of 50%. Moreover, in the region $2.55 \leq a \leq 2.8$ *AU* several other asteroids lie near or inside $g = g_6$ (for example, 475 Ocllo, $a = 2.59$ *AU*; 631 Philippina, $a = 2.79$ *AU*; 759 Vinifera, $a = 2.62$ *AU*; 907 Rhoda, $a = 2.80$ *AU*), and also for them the fraction of fragments ending up in the resonance is $\geq 50\%$. These preliminary results suggest that the $g = g_6$ resonance is a potential source of planet–crossing fragments also in the region beyond the 3 : 1 mean motion resonance. In order to obtain quantitative results on the efficiency of the $g = g_6$ resonance in producing planet–crossing objects and also on the transport time scales, we plan to study the orbital evolution of some fictitious fragments by purely numerical computations. This will also allow us to explore the effects of the overlapping between the $g = g_6$ and $3:1$ resonances, which probably triggers chaotic behavior and may enhance the production of planet–crossers, as already found for the interaction between $g = g_6$ and $4:1$.

(c) The inner part of the asteroid belt at inclinations $I \geq 10°$ and with $2.0\ AU \leq a \leq 2.25\ AU$, bordering from inside the Phocaea group, appears to be strongly depleted. As explained by Knežević et al. (1991), this depletion is probably due to the fact that this region is criss–crossed by several main secular resonances ($g = g_6$, $g = g_7$, $g = g_8$, $s = s_6$ and also by other resonances like $2g = 2s$ (see Morbidelli and Henrard, 1991, Fig.9). Due to resonance overlapping, asteroid orbits in this region suffer large perturbations in both eccentricity and inclination, possibly ending into planet–crossing orbits, at least in some cases,as confirmed by the numerical experiments by Knežević et al. (1991). A more extensive numerical exploration and/or a better theory (for high inclinations and eccentricities) would be needed to give a complete explanation of the observed depletion.

(d) We consider now the zones between the outer planets. Recent extensive numerical experiments (Franklin et al., 1989; Weibel et al., 1990; Gladman and Duncan, 1990) have shown that the majority of test particles orbiting between the giant planets are perturbed to a close approach on timescales of millions of years. Our results are consistent with these studies. The region between Jupiter and Saturn (Knežević et al. 1991,figs.2) is crowed with many low–order mean motion resonances which — by analogy with the Kirkwood gaps — are probably effective in causing chaotic behavior. A possible region of "relative stability" (i.e., where small bodies could escape short–term ejection), found also in the afore–mentioned numerical studies, lies between about 7 and 7.5 AU, for small inclinations; however, secular resonances are present at inclinations between 10° and 20°, and may affect the dynamical evolution of any small body injected there. A similar situation (Knežević et al., 1991, figs.3) is observed between Saturn and Uranus, with a possible "stable island" at semimajor axes near 14 AU, below $i \simeq 15°$ of inclination. A third such "island" is possibly located between Uranus and Neptune at $23.5 \leq a \leq 24.5 AU$, with a possible gap appearing at the overlapping order–two $7:5$ and $5:7$ mean motion resonances with Neptune and Uranus, respectively (Knežević et al. 1991, figs.4). However, all such "islands" are unlikely to contain any more surviving primordial bodies if the orbits of the giant planets underwent significant semimajor axis changes after their formation (e.g. due to ejection of cometary material into the Oort cloud), with corresponding shifts in the resonance positions.

(e) Another interesting issue concerns the possible existence of an inner "Kuiper cometary belt", acting as an effective source of low–inclination short–period comets. To confirm this hypothesis, a viable dynamical mechanism to transport the comets into planet–crossing orbits needs to be identified. As shown in Fig. 3, beyond 50 AU the secular frequencies are always too small to allow for the existence of secular

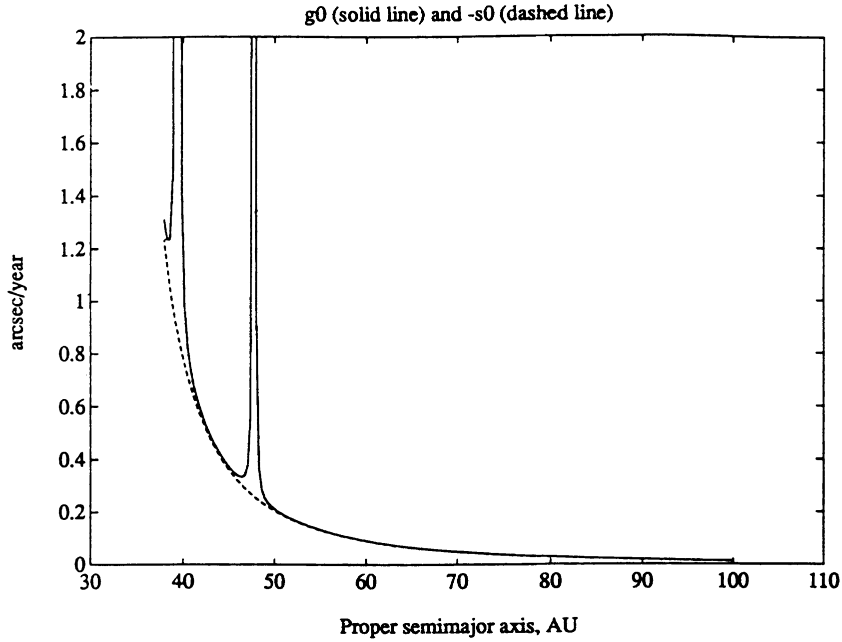

Fig. 3. Frequencies g_0 and $-s_0$ in the trans–Neptunian region up to $100AU$. Notice that for semimajor axis $a > 50AU$ both frequencies remain smaller than any planetary ones.

resonances, which therefore cannot provide the required mechanism. Does this rule out the Kuiper belt comet source? The problem is subtle, since it is necessary not only to show that objects formed in the Kuiper belt can become Neptune–crossers, but also to determine the time scale for this process, which must be on the order of the age of the solar system. Numerical experiments performed by Gladman and Duncan (1990) and Torbett and Smoluchovski (1990) are not at all conclusive, as they exhibit orbits becoming Neptune–crossers in less than 10^7 *yr*, but for semi-major axes less than 50 AU. Only Levison (1991), who has treated the dynamical evolution of small gravitationally non–interacting objects within the solar system as a Markov process, claims to have found evidence that small objects can diffuse and leave the region between 30 and 100 AU with a lifetime of $\approx 5 \times 10^9$ *yr*. Our results do not contradict such a low stochasticity, which would be unlikely in a region full of strong secular resonances. However, the problem remains on where does the stochaticity comes from. The diffusion within the Markov chain may simply arise as an artifact of the model, due for instance to very long oscillations of a quasi–periodic integrable motion. Such an artifact was harmless for the diffusion problem of the Jupiter family treated already as a Markov process by Rickman and Froeschlé (1979), since in this case the overall stochasticity comes from close approaches with Jupiter.

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Discussion

J. Wisdom – The experiments of Gladman and Duncan found a difference in the typical lifetimes of non-inclined and inclined orbits between Jupiter and Saturn. However, they only studied non-inclined orbits between the other outer planets. Do your calculation indicate any zone of stability or instability which they missed because they only studied non-inclined orbits? Do your calculations give any insight into the different lifetimes of slightly inclined and non-inclined orbits between Jupiter and Saturn?

Ch. Froeschlé – Qualitatively our results show that if such zones exist they are smaller for inclined orbits, which have not been studied by Gladman and Duncan.

However, since we didn't determine the strengths of the secular resonances, numerical experiments using your mapping would be very useful. The same kind of answer holds for the second question.

G.Tancredi – If the secular resonances are not an appropriate mechanism to bring comets from the Kuyper belt to planet-crossing orbits, do you hint any other long-term mechanism that could play this rôle?

Ch.Froeschlé – Unfortunately, this mechanism seems to be ruled out and, for the present time, I don't see any other mechanism, except, may be, the Arnold diffusion, since the KAM tori do not separate or confine the phase space for systems with many degrees of freedom.

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