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Secondary Bars and Triaxial Bulges within Primary Bars

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Abstract. Numerous galaxies clearly show two misaligned nested stellar bars or one bar plus one structure with twisted isophotes. Some galaxies seem even to have three distinct triaxial structures. Recent optical and near-infrared multi-band observations show that the angle between the primary and secondary bars does not take any peculiar value. The best way to reconcile such observations with theory certainly consists in postulating that the secondary bar rotates *faster* than the primary one. Numerical simulations have demonstrated the viability of such systems, and both *direct* and *counterrotating* models with bars within bars can be computed. Bar-within-bar systems could play a significant role in fueling AGN's and/or circumnuclear starbursts, since AGN's and/or blue or Hα nuclear rings are frequently hosted by such galaxies.

1. Introduction

Since the pioneering paper of de Vaucouleurs (1974) more than twenty years ago, more and more compelling evidence has been accumulated that some disk or lenticular galaxies host more than one triaxial structure. Table 1 presents a non-exhaustive list of observations devoted to this subject. In recent years, there is clearly a strong boost in the discovery of such galaxies due to the advent of optical CCD's and infrared arrays. Higher resolution data from, for example, HST could give us even more surprises in the coming years. These central isophotal deformations have thus been known for a long time and appear to be a widespread feature of disk galaxies although no reliable percentage can yet be given. Since any triaxiality should be efficient to bring material in the central region (Kormendy 1982; Norman & Silk 1983), one expects a link between central activity (starburst, AGN) and central mass deformation. In particular, systems with bars within bars have been advocated as a possible mechanism to fuel AGN's (Shlosman et al. 1989). However, the link between bars and central activity has been much debated (see e.g. Ho, these proceedings), and no definite answer has been given so far. Nevertheless, much light could certainly be shed on this problem by including the scale of the deformation in the scenario: large-scale primary bars should be related to the formation of secondary bars and/or ring-like nuclear starbursts whereas small-scale secondary (or nuclear) bars themselves could be more intimately related to AGN's.

Below, observational evidence of misaligned nested stellar bars and multiple triaxial structures in disk galaxies is first reviewed. Various theories and numerical simulations able to likely explain this phenomenon are then presented.

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Table 1. Observations of galaxies with multiple triaxial structures.

Author(s)	Year(s)	Band(s)	Object(s)
de Vaucouleurs [a]	1974	photographic plates	NGC 1291; 1326
Sandage & Brucato [a]	1979	photographic plates	NGC 1543
Kormendy [b]	1979, 1982	photographic plates	NGC 3945; +5
Schweizer [a]	1980	photographic plates	NGC 1317
Baumgart & Peterson [a,b]	1986	photographic plates	NGC 1433; +7
Jarvis et al. [a]	1988	*gr* + kinematics	NGC 1291; 1543
Bertola et al. [b]	1991	*ri*	>20
Buta & Crocker [a]	1993	*BVI*	NGC 6782; +12
Shaw et al. [a,c]	1993	*K*	NGC 1097; 4736; 5728
Wozniak et al. [a,b,c]	1995	*BVRI*, Hα	NGC 5850; >20
Quillen et al. [a]	1995	*BVRJHK*	NGC 1097
Shaw et al. [a,c]	1995	*JHK*	>20
Friedli et al. [a,b,c]	1995	*JHK*	>10
Márquez et al. [a]	1995	*BVR* + kinematics	NGC 6701

[a] Secondary bars / [b] Triaxial bulges / [c] Twisted isophotes

2. Observational Evidences

2.1. Terminology and Definitions

Let us start by defining a few terms and quantities (see also Wozniak et al. 1995). I suggest to use the term *secondary bar* (subscript s) when a distinct *primary bar* (subscript p) is present. The term *nuclear bar* should then be reserved for small-scale bars in absence of large-scale ones. Since a *thick secondary bar* is indistinguishable from a *triaxial bulge*, both terminologies will equally be used. The various structural parameters (surface brightness profiles μ, ellipticities $e = 1 - b/a$, position-angles PA's) are best determined by ellipse fitting in order to get accurate and observer-independent quantities. The length ratio between the two bars is $\beta = l_p/l_s$ whereas the luminosity ratio is $\gamma = L_p/L_s$. The respective maximum ellipticities are defined as $e_s^{\max}$ and $e_p^{\max}$. The projected angle between the two bars θ ($0° \leq |\theta| \leq 90°$) is positive if the secondary bar *leads* the primary bar and negative if the secondary bar *trails* the primary bar (Buta & Crocker 1993). Finally, the pattern speed ratio is $\alpha = \Omega_s/\Omega_p$.

2.2. Galaxies with Two Triaxial Structures

Much evidence for the existence of galaxies with a stellar bar within another stellar bar or galaxies with both a primary bar and a triaxial bulge can be found in the papers listed in Table 1. The evidence is mainly photometric since the kinematics of these galaxies is unfortunately still very poorly known. Nuclear molecular bars are reviewed by Turner (these proceedings). I also here restrict myself to discuss the most extensive study done so far, i.e. the one by Wozniak et al. (1995) for *BVRI*, Hα and Friedli et al. (1995) for *JHK*. The sample has been selected on the basis of well-known candidates from the literature as well as from a list of moderately inclined galaxies with nuclear rings (Buta 1984).

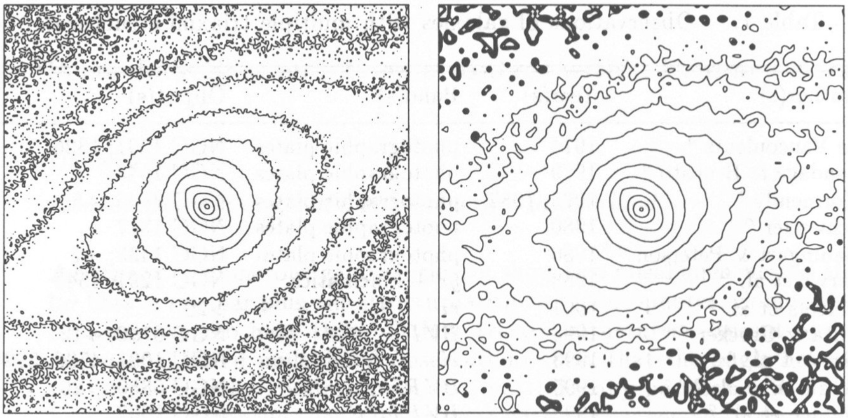

Figure 1. Isophote contours of the bar-within-bar prototype NGC 5850 for the I-band (left; $0.38''\text{pixel}^{-1}$) and K-band (right; $0.9''/\text{pixel}$). The frames are $80''$ wide. The contour scale is logarithmic with a spacing of 0.6 mag. The angle between the two bars is $\theta \approx -66°$ and the secondary bar length $l_s \approx 9.2''$.

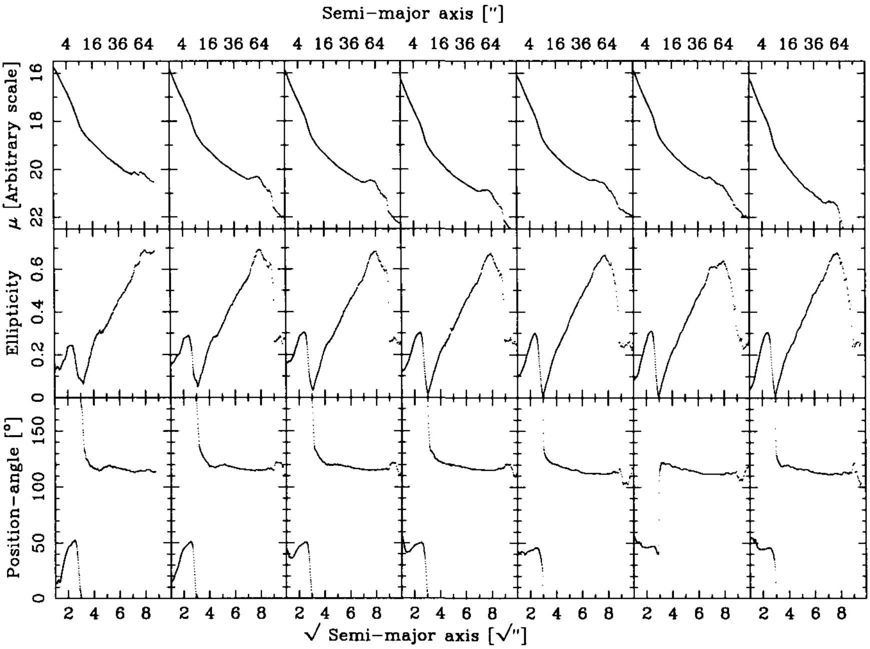

Figure 2. Ellipse fitting for $BVRIJHK$-bands of NGC 5850 (from left to right). The signatures of the two bars (two distinct PA's, maximum ellipticities, and surface brightness slopes) are clearly observed in all bands although the optical bands are affected by small amounts of dust. Note that the scale goes as the square root of the radius.

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The Prototype NGC 5850. Together with NGC 1291 (SBa) and NGC 1543 (SB0), NGC 5850 (SBb) can be viewed as one of the best examples of a bar-within-bar galaxy (see Figure 1). The signatures of the two bars are two distinct i) PA's, ii) maximum ellipticities, and iii) surface brightness slopes. These are clearly seen in all observed bands (see Figure 2). In the K-band, $\theta \approx -66°$, $\beta \approx 9.1$, $\gamma \approx 3.5$, $e_s^{\max} \approx 0.31$, and $e_p^{\max} \approx 0.68$.

Global Properties. Keeping in mind that the sample of double-barred galaxies is small (thirteen galaxies in the I-band), one has the following mean structural parameters: $\beta \approx 7.2$ with extreme cases of 3.7 and 18.0, $\gamma \approx 4.0$ with extreme cases of 2.0 and 7.5, $e_s^{\max} \approx 0.34$, and $e_p^{\max} \approx 0.55$. There may be an observational bias toward the lower values of β and γ which are the easiest to detect. The most striking features are that *no preferred angle θ has been found* as well as the presence of six leading and seven trailing secondary bars. This is a very important clue to understanding these objects (see Section 3). All of them have peculiar circumnuclear morphology like blue or red $B - I$ ring, or Hα hotspots, but this might be a selection effect. The high number of Seyfert nuclei amongst these galaxies (6 out of 13) is certainly more significant. In the $J - K$ color map, some secondary bars are clearly redder than the primary bar. Double-barred galaxies are preferentially early-type galaxies (Hubble type $T \leq 3$).

Projection Effects. One can ask if projection effects could seriously alter the above results: 1) Could the secondary structure simply be a round center seen in projection? No. This is clearly ambiguous when $PA_{\mathrm{disk}} \approx PA_{\mathrm{s}}$, but otherwise the secondary misaligned structure does exist and will be robust to deprojection. Another indication of central triaxiality comes from significant isophote twists in many cases. 2) Are all secondary bars perpendicular to primary bars, i.e. the deprojected angle $\theta_d = \pm 90°$ although θ could appear widely different? No. If this has to be considered for some galaxies (e.g. NGC 1317), this is impossible in cases with moderate inclination i and θ not close to 90° (e.g. NGC 1291). For the most part of bar-within-bar galaxies, $\theta_d \neq \pm 90°$. 3) Do all secondary bars lead primary bars after deprojection? No. There are still roughly half leading and half trailing secondary bars after deprojection. So, misaligned secondary bars are generally not generated by projection effects.

Dust and Star Formation Effects. The presence of high dust absorption and/or intense sites of star formation can mislead the observer by strongly highlighting dynamically insignificant or unphysical components. For instance along a nuclear ring, two opposite hotspots of star formation could look like a secondary bar in the ellipse fitting. Dust lanes and spiral arms strongly twist the isophotes and mimic the signature of triaxiality. Although these problems are already much less severe in the near-infrared, they can be best alleviated by using multi-band observations and color maps on which dust and star formation clearly emerge. In the sample of Wozniak et al. (1995), the overwhelming part of secondary bars were proved to be real structures according to the JHK study by Friedli et al. (1995) and not dust or star formation artifacts.

2.3. Galaxies with Three Triaxial Structures

There are strong indications that some disk galaxies could have *three* different triaxial components (Wozniak et al. 1995).

The Prototype NGC 3945. The nearly face-on SB0 galaxy NGC 3945 is certainly one of the firmer candidates. The three triaxial structures are robust to deprojection. If the components are numbered from 1 (larger scale) to 3 (smaller scale), the structural parameters are in the I-band: $\theta_{12} \approx +85°$, $\theta_{13} \approx +67°$, $\beta_{12} \approx 2.1$, $\beta_{13} \approx 13.6$, $\gamma_{12} \approx 1.5$, $\gamma_{13} \approx 7.7$, $e_1^{\max} \approx 0.32$, $e_2^{\max} \approx 0.35$, and $e_3^{\max} \approx 0.15$. The semi-major axis of the smallest bar is only $3.1''$ long and its weakness could be explained by seeing and pixel size effects which tend to round the central region. This small structure has to be confirmed by higher resolution observations and would be an excellent target for HST.

Global Properties. Wozniak et al. (1995) have only found six galaxies likely to own three triaxial structures so that no serious statistics can be performed. All of them are however very early-type galaxies. By comparison with the double-barred case, it appears also that component 3 could be the counterpart of the secondary bar whereas component 1 is the primary bar. Component 2 is apparently an intermediate structure much larger than a typical secondary bar. It is not yet clear if the same mechanisms could be at play in these galaxies and in the double-barred ones, or if they could represent different stages of evolution, or if these are completely different phenomena.

3. Theory and Numerical Simulations

3.1. Secondary Gaseous Bars within Primary Stellar Bars

In order to explain the fueling of AGNs, Shlosman et al. (1989) have invoked the formation of a small-scale gaseous bar ($\lesssim$ 1 kpc) within a large-scale primary bar ($\lesssim$ 10 kpc). Simulations by Heller & Shlosman (1994) have shown that this process does exist but is transient since fragmentation and dynamical friction quickly dissolve the gaseous bar. So, if this mechanism could be very effective in powering AGNs, it is however not obvious to link it to the persistent misaligned secondary stellar bars discussed in Sect. 2.2.

3.2. Secondary Stellar Bars within Primary Stellar Bars

Let us now review the various possibilities able to explain the phenomenon of misaligned nested stellar bars. The cases of permanent (Case A) and time-dependent (Case B) misalignments are in turn examined:

Case A $\rightarrow \alpha = 1$, *i.e.* $\theta =$ constant.

- $\theta = 0°$. Galaxies with primary and secondary bars nearly aligned include NGC 4314 (Benedict et al. 1993) and NGC 4321 (Knapen et al. 1995). This can be interpreted with a single bar having two inner Lindblad resonances (ILR's) where the secondary bar is made of orbits trapped by the x_1 family inside the inner ILR (see e.g. Contopoulos & Grosbøl 1989 for a review of periodic orbit families). Nevertheless, in the case of two different

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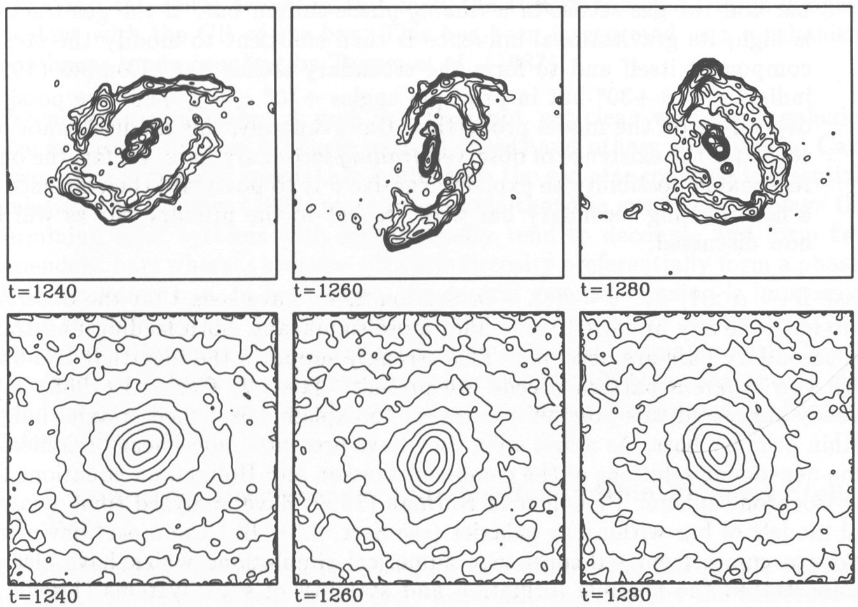

Figure 3. Time evolution of the face-on surface isodensity contours *of the gas (top) and the stars (bottom) of a bar-within-bar model. The* time is indicated below each frame (in Myr). The frames are 4 kpc wide. The contour scale is logarithmic with a spacing of 0.2 dex. The primary bar is always horizontal and the rotation is anti-clockwise. Note the CO-like bar and twin peaks.

bar pattern speeds (see Case B below), parallel bars within bars have also to be present at a given time and this might just be chance seeing the secondary bar aligned at the present time.

- $\theta = \pm 90°$. One of the few galaxies which could have the secondary bar perpendicular to the primary bar is NGC 1317 (Schweizer 1980; Wozniak et al. 1995). This configuration can be explained in the classical picture of one bar with an ILR, i.e. the secondary bar is supported by orbits trapped by the anti-bar x_2 family which exists and is stable inside the ILR. However, the mass trapped by this family cannot increase that much for fear of destroying the stability of the x_1 family and consequently to dissolve the primary bar. As for the above case, if two different bar pattern speeds exist, perpendicular bars within bars should also be observed.

- $\theta \neq 0°, \pm 90°$. The overwhelming fraction of observed double-barred structures falls in this category. Shaw et al. (1993) have presented models with gas and stars which produce two misaligned stellar bars rotating at the same pattern speed. First the star-gas disk is unstable and quickly forms a primary bar with two ILR's. Inside the outer ILR, dissipative collisions gradually shift gas orbits from parallel to perpendicular to the primary bar and the gas settles in a *leading* phase-shifted bar. If the gas fraction is high, its gravitational influence is then sufficient to modify the stellar component itself and to form the secondary stellar bar. Combes (1994) indicates $\theta \approx +30°$ but in principle angles $+10° \lesssim \theta \lesssim +80°$ are possible depending on the model properties. Unfortunately, this model cannot account for the existence of observed *trailing* secondary bars. So far, the only reasonable possibility to explain negative θ is to postulate the existence of a fast rotating secondary bar with respect to the primary bar as will be now discussed.

Case B $\rightarrow \alpha > 1$, *i.e.* $\theta = \theta(t)$. In Section 2, we have seen that the observed θ do not take any preferred value and, more significant, both leading and trailing secondary bars are observed. This strongly supports the idea that two bars with two *different* pattern speeds are present. Louis & Gerhard (1988) have already suggested this possibility in order to explain misaligned triaxial bulges within primary bars. As a first step, they have presented non-linear periodically time-dependent solutions of the combined Poisson and Boltzmann equations for the isochrone sphere. Pfenniger & Norman (1990) have analyzed some analytical models of bar-within-bar galaxies (see Sect. 3.5). But the most convincing evidence comes from self-consistent numerical simulations which have clearly demonstrated the possible formation and stability of such systems (Friedli & Martinet 1992, 1993; Combes 1994).

The formation of the secondary bar follows a significant primary bar-driven gas fueling of the center. Both the resulting central mass concentration which increases the rotation speed and the ILR_p favor the dynamical decoupling of the central part of the galaxy. The latter then evolves almost independently from the rest of the galaxy. The secondary bar is born. The eventual embarrassing chaos produced by the main resonances in the case of strong bars is minimized by the fact that the corotation of the secondary bar (CR_s) and the ILR_p are located at a similar radius. A striking gas feature of these galaxies is the frequent formation of a nuclear ring close to this radius, i.e. between the two bars (see Figure 3). The shape of this ring evolves on short time-scales in particular because gas self-gravitating effects are significant. The gas density along the ring is generally not constant but higher at both ends of the secondary bar major axis. This is very similar to the observed CO twin peaks (or $\mathrm{H}\alpha$ hotspots) and in some cases this phenomenon might be related to a double-barred structure rather than to a single bar with an ILR as usually assumed. Along the secondary stellar bar, there is also a distinct but shorter gaseous bar.

These types of systems can survive for many rotations of the secondary bar. However, a highly viscous gas can accumulate so much to the center that the secondary bar (or even both bars) could be dissolved. Using the ellipse fitting technique on the numerical simulations shown by Friedli & Martinet (1993) typically gives $\beta \approx 5.4$, $\gamma \approx 2.9$, $e_s^{\max} \approx 0.39$ and $e_p^{\max} \approx 0.27$. One also has $\alpha \approx 79.0/25.5[\mathrm{km\,s^{-1}\,kpc^{-1}}] \approx 3.1$. This is comparable to the observations excepted for the primary bar ellipticity which is too low. But a new set of simulations by Friedli et al. (1995) indicates that it is possible to produce much stronger primary bars as well.

Multiple pattern speeds have already been observed in numerical simulations by Sellwood & Sparke (1988) who have found different rotation figures for the bar and the spiral arms. In this case, the ILR of the spiral arms is again coincident with the CR of the bar. This has been interpreted as a mechanism of non-linear mode coupling by Tagger et al. (1987).

Differences between Cases A and B. It is not yet clear why some galaxies evolve according to Case A (single pattern speed) and others according to Case B (two different pattern speeds). In both cases, the gas-star coupling is essential. Simulations by Combes (1994) seem to indicate that the gas viscosity plays the determining role: systems with high viscosity tend to decouple and form two independent bars whereas systems with low viscosity preferentially form a phase-shifted secondary bar. In both cases, the central gas mass fueling is impressive and can reach many $M_\odot$ per year, high enough to power low or moderate AGN's or starbursts. However, the central gas fueling seems to be smaller with Case B than with Case A (decrease of gravity torques; Combes 1994) but this should also depend in an intricate way on the secondary bar axis ratio, dynamical friction, star formation, etc.

3.3. Counterrotating Secondary Stellar Bars within Primary Stellar Bars

After the prograde bar-within-bar case, let us examine the retrograde one. At first sight, this could appear as a purely academic exercise but the existence of disk galaxies with a large fraction of counterrotating stars (see e.g. Kuijken 1993) or gas (Ciri et al. 1995) raised the interesting question of "retrograde dynamics". Another point is that many galaxies with bars within bars are early-type galaxies although the processes described in Sect. 3.2. require large amounts of gas. If a secondary bar can easily rotate in the opposite sense of a primary bar in a purely collisionless system this mechanism could then be relevant for gas-poor galaxies. For instance, accretion of retrograde satellite(s) by a (barred) disk galaxy could lead to produce a central counterrotating secondary bar. Note however that a large fraction of stars on retrograde orbits would seem to strongly inhibit the formation of the primary bar (Zang & Hohl 1978). Another major drawback is the resulting strong disk heating which could be able to dissolve both bars. Simple models first can be computed and below two different cases are in turn examined:

Case A $\rightarrow \alpha = -1$. In a general study concerning bending mode instabilities of flat, coplanar, equal-mass, counterrotating disks, Sellwood & Merritt (1994) have reported the case of two bars with the same extent which were rotating in the opposite sense, i.e. $\alpha \approx -1$ and $\beta \approx 1$. This type of system has also been computed by Friedli (1995). The bars are generally weak, slow and can rotate many times through each other. However, to my knowledge, no double-barred galaxies with $\beta \approx 1$ have so far been observed.

Case B $\rightarrow \alpha < -1$. Davies & Hunter (1995a,b; see also these proceedings) have presented models of counterrotating bars within bars. They start with a single 2D disk model whose central systematic velocities are abruptly reversed. The lifetime of the counterrotating bar phase is about 750 Myr. After that the secondary bar reverses its sense of rotation, overtakes the primary bar, oscillates for some time around it and finally aligns itself with it. I have also computed

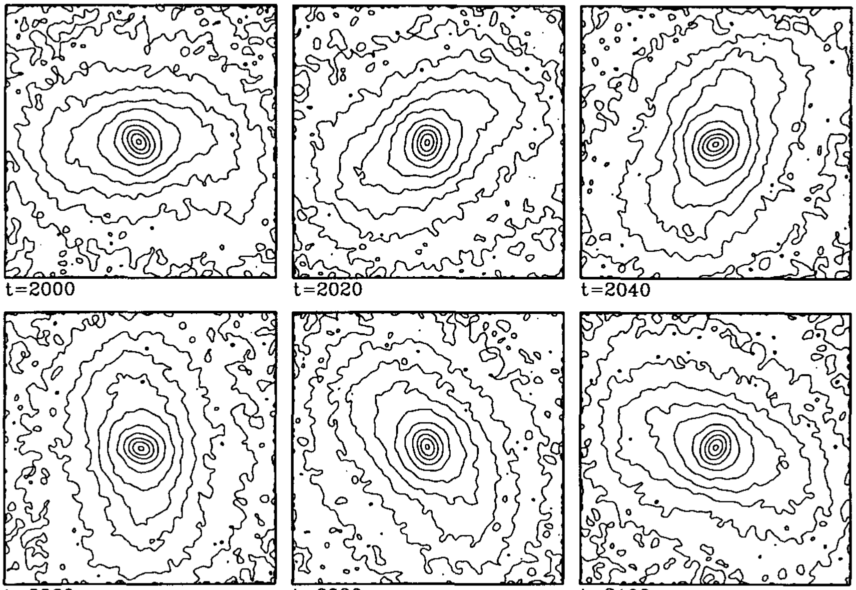

Figure 4. Time evolution of the face-on surface isodensity contours of a counterrotating bar-within-bar model. The time is indicated below each frame (in Myr). The frames are 16 kpc wide. The contour scale is logarithmic with a spacing of 0.25 dex. The secondary bar rotates clockwise whereas the primary bar rotates anti-clockwise ($\alpha \approx -1.4$).

models of 3D counterrotating, coplanar disks with different mass and scale-lengths (for details see Friedli 1995). With specific initial conditions, each disk forms its own bar and they rotate in the opposite sense (each bar rotates in the same way as the stars which compose it). I have essentially found the same evolutive behavior as Davies & Hunter (1995a,b): this evolution is mainly driven by the significant angular momentum transfer from the secondary to the primary bar. As an example, Figure 4 shows the evolution of a model with mass ratio 1/4 and scale-length ratio 1/8 which produces $\beta \approx 5.6$ and $\alpha \approx -1.4$. The kinematics of counterrotating bar-within-bar models is clearly distinct from the one of direct bar-within-bars models.

3.4. Periodic Orbits in Potentials with Bars within Bars

So far, not much work has been devoted to the search of periodic orbit families in a bar-within-bar potential. As a first step, Hasan (these proceedings) has studied the case $\alpha = 1$ with $\theta = 0°$ and $90°$. The results are similar to the ones obtained with a spherical mass concentration instead of a triaxial one (see e.g. Hasan, Pfenniger & Norman 1993). So far, nobody has tackled the case $\alpha \neq 1$ which is of course much more difficult since no single rotating reference frame can be defined. However, it can be anticipated that if $\alpha \gg 1$, the orbital structure of the secondary bar should be similar to the one of a single bar with the same pattern speed. On the contrary due to the "forbidden" center (region occupied by the secondary bar), the usual orbital structure of the primary bar should only be found for the higher values of the corresponding Jacobi constant. The most interesting question concerns possible new families common to both bars, like nearly circular orbits between the two bars which could explain the easy formation of rings there.

3.5. Dissipation and Gas Flows in Potentials with Bars within Bars

The fate of dissipative test particles in bar-within-bar potentials has been investigated by Pfenniger & Norman (1990). They have chosen $\beta=\gamma=10$ and $\alpha=5.86$ in order to have $\mathrm{CR}_s \approx \mathrm{ILR}_p$. In addition to fixed points (high dissipation rate), limit cycles (intermediate dissipation rate) and chaotic attractors (intermediate and low dissipation rate) have been found by these authors as expected for such dissipative and time-dependent systems. Unfortunately, dissipative test particles cannot account for collective effects associated with a viscous fluid (like shocks). Detailed gas flow studies similar to the single bar case (see e.g. Athanassoula 1992) should be performed and would be most instructive.

4. Summary

The number of observed disk galaxies having secondary bars or triaxial bulges within primary bars is quickly increasing. The present surveys are still too poor and biased to put forward a reliable percentage of the occurrence of these systems but this percentage could certainly be high especially among early-type SB's. So, it is not yet clear if a limited number of disk galaxies are double-barred galaxies for a long period of time (i.e. many Gyr) or if each disk galaxy is evolving through a bar-within-bar phase for a short period of time (i.e. less than 1 Gyr). Dynamical secular evolution of galaxies is likely to play a significant role in the evolution of galaxies (see Martinet 1995 for a thorough review of this problem) and double-barred systems could represent one of the signatures of such a process.

The observed secondary bars are misaligned with respect to primary bars without preferential orientation; in particular, secondary bars can both lead or trail primary bars (with regard to its sense of rotation). Models with time-dependent misalignments are most likely and, at least in some of the galaxies, the secondary bar is expected to rotate faster than the primary one. Numerical simulations have demonstrated the viability of such systems, and both direct and counterrotating bar-within-bar models can be computed. In these models, the secondary bar ends at its corotation radius which is coincident with the ILR of the primary bar. Circumnuclear rings of star formation are frequently hosted by galaxies with bars within bars which is consistent with the easy formation of gaseous rings between the two bars in numerical simulations. Secondary bars could also play a significant role in fueling AGN's whose frequency appears to be high in double-barred galaxies.

5. Conclusions

Clearly, disk galaxies with *two misaligned nested stellar bars* with *two different pattern speeds* do exist both inside computers and in nature. By accumulating the problems of non-axisymmetric, non-linear, time-dependent, and dissipative systems, they open new theoretical prospects and challenges like to find time-dependent solutions of the collisionless Boltzmann equation or to deal with limit cycles and strange attractors. Many decades have been necessary to begin to slowly understand the dynamics of a single bar. I anticipate that many years will also be consumed before the complex secrets of galaxies with multiple triaxial components and pattern speeds will fully be unveiled.

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Discussion

E. Athanassoula: Have you tried a simulation with a different viscosity?

D. Friedli: No. With the SPH technique, you cannot easily play with the artificial viscosity parameters and control what you are physically doing.

R. Miller: You seem to have analyzed these patterns under the assumption that the features are coplanar. Is there any evidence for this?

D. Friedli: In the simulations, the dynamical center and the rotation axis of both bars coincide as well as the center of density which can however be different from the center of mass of the galaxy. In the observations, the photometric centers of secondary and primary bars seem to be identical.

A. Zasov: If you had some mechanism of accretion of gas onto the center, could you form a secondary bar without a primary one?

D. Friedli: Yes. If a lot of dynamically cold stars are formed in the center, one would expect a bar instability there. Also, if the rotation curve is steep, a very short primary bar will be produced (i.e. a nuclear bar).

P. Teuben: How robust are α and the double-bar structure in your simulations?

D. Friedli: The lifetime of the double-barred structure seems to depend on the level of central dissipation and thus on the code viscosity, but typically 0.5–1 Gyr. The indicated values for α are time-averaged over the lifetime although in fact α is slightly time-dependent in a complicated way.

J. Garcia-Barreto: In order to have a secondary bar, do you need different Q parameters for inner and outer regions?

D. Friedli: No. In the simulations Q is not a basic parameter and $Q = Q(R) \approx 1.2 - 1.5$. If Q is very low near the center, due to the short dynamical time-scale, you could form first the secondary bar but it will heat so much the outer regions that it is not obvious that the primary bar will then still form!