BERRICK, A. J., *An approach to algebraic K-theory* (Research Notes in Mathematics 56, Pitman, 1982), 108 pp. £7.95.

SILVESTER, J. R., *Introduction to algebraic K-theory* (Chapman and Hall, 1981), xi+255 pp. £15 (hardback) or £6.95 (paperback).

These two books are of very different characters, as their titles suggest. Silvester aims to treat the subject at the most elementary level possible. His book has undergraduate algebra as its only prerequisite, and its theme is the extension of linear algebra from fields to more general rings. The material consists of the easiest parts of the subject. Algebraic K-theory is the study of a sequence of functors K_q (q an integer) from rings to abelian groups. In general their definitions need topological ideas, but K_0, K_1 and K_2 have purely algebraic definitions. Silvester's book studies K_0, K_1 and K_2, giving their algebraic definitions, their basic properties, and some computations.

Berrick's book is aimed at algebraists, topologists and algebraic number-theorists. It is intended to "complement, not reproduce, established references", and so hopes to interest veterans as well as novices. Its interests are structural rather than computational. The bulk of the book is designed to give a natural definition of K_q for any integer q, and thus explain the properties of the algebraically defined K_0, K_1 and K_2. When q is positive K_qA (A a ring) is in fact the qth homotopy group of a topological space X depending on A. The construction of X involves a long excursion through the byways of elementary, but subtle, homotopy theory; in particular there is a detailed study of Quillen's plus-construction. There are a few original results.

How well do the books succeed? Silvester's book really is written at an elementary level; it gives its arguments carefully and in detail. As with all books, it helps to have more than the bare prerequisites; for this book, a little knowledge of categories, modules and group presentations would be useful. The material, particularly given the few prerequisites, is well-chosen: the functors K_q with $q=0$, 1 or 2 are the ones that usually appear in applications. On the other hand there is very little guidance for the reader; definitions, theorems and proofs follow directly on one another, and the proofs tend to be computational rather than explanatory. The scope of the book is similar to that of J. Milnor's *Introduction to algebraic K-theory* (Annals of Mathematics Studies 72, Princeton, 1971). Milnor's book is written at a more advanced level, but gives more guidance, and I think people who can follow it will prefer it. Silvester's book would be suitable for a taught course.

Berrick's book has an interesting combination of aims. It gives a summary of the main results of algebraic K-theory, many without proof, and this should be useful to anybody. It also gives a detailed study of certain parts of the theory. This is worth skimming in any case, but a thorough reading needs a background in algebraic topology. The book is written in a lively and humorous style, and the structures of the arguments are made clear. The author goes to some trouble to find slick proofs. In places the style is allusive, as though the book were really aimed at a veteran rather than a novice. A good example is the opening sentence: "The beginning is predictable enough..." Unfortunately there are several errors, both typographical ("For $q \leqq 2$" instead of "For $q \geqq 2$" in statement (12.1)) and mathematical (the alleged ring CA defined on page 22 is not closed under multiplication). To sum up, much of the book is for a veteran with some knowledge of topology; the novice will enjoy it, but should be careful.

Both books are reproduced from typescript, and it is interesting to compare their appearances. Silvester's has type of uniform thickness and symbols in italics, while Berrick's has type of variable thickness and symbols in roman. Berrick's has larger type and more space. I found Berrick's easier to read, and I deduce that it is not worth putting symbols in italics. I hope this conclusion will be agreeable to typists.

RICHARD STEINER

KASCH, F. *Modules and rings* (translated by D. A. R. Wallace) (Academic Press, London 1982), xiii +372 pp. £33.80.

This book has grown out of lectures and seminars given over the years by Professor Kasch. It only assumes a very basic knowledge of the elementary concepts of ring and module theory, and from there develops the standard results of the subject as well as delving more deeply in some topics to which Professor Kasch has made important contributions. The original book in German was very well received, and so, no doubt, will be this very good translation by Professor Wallace.

The author believes "that the concepts of projective and injective modules are among the most important fundamental concepts of the theory of rings and modules" and accordingly starts with a chapter on the fundamentals of category theory. But this is kept within bounds and only the material needed later is developed. Categories are used as a tool here and the introduction is compact and informative without getting bogged down in "abstract nonsense". The next chapter introduces modules, submodules and factor modules; then there is a treatment of homorphisms of modules and rings. Chapter 4 is on direct products, sums and free modules. The next topics treated are: injective and projective modules, artinian and noetherian modules, local rings and the Krull–Remak–Schmidt theorem, semisimple modules and rings and the radical and socle. This covers what might be called the standard theory. Use is made of categorical ideas, in particular the idea of generators and cogenerators are used in many places.

The last third of the book develops the material of a more specialized nature. This consists of four chapters: the first on the tensor product, flat modules and regular rings, the next on semiperfect modules and perfect rings, then rings with perfect duality and finally quasi-frobenius rings. Many of the ideas studied here arise from the imposition of various finiteness conditions on the generators or cogenerators or both. This leads to a number of very satisfying characterisation results. Each chapter in the book has a collection of exercises at the end, of varying difficulty, which help to understand and apply further the ideas from the preceding pages.

This is a useful addition to the literature. It is quite pleasant to read and presents its material well. The first part of the book could be used as an optional final year undergraduate course on ring theory, and the last part as a series of seminars for postgraduate students. The specialised material presented is not too esoteric and presents a coherent and well-developed theory. So in terms of content this is a very satisfactory book. The standard of printing and production is high, but the number of misprints is rather on the high side. Fortunately none should cause real trouble, though some will cause the reader a few puzzled moments. The price, at nearly 10p/page, seems high or is that wishful thinking for a book of this nature?

J. D. P. MELDRUM

ARROWSMITH, D. K. and PLACE, C. M., *Ordinary differential equations: a qualitative approach with applications* (Chapman and Hall, 1982), 250 pp., cloth £18, paper £7.95.

This undergraduate textbook develops the qualitative theory of differential equations and applications in mathematical modelling. It is intended for approximately the third year level (Scottish pattern) and assumes a knowledge of basic calculus with several variables and linear algebra. Naturally, great emphasis is placed on geometrical aspects and there is a very effective use of diagrams (well over half the text pages contain a diagram). The special orientation leads to a particular selection of material and the omission of certain important topics in differential equations. Consequently, the book may be more appropriate for a second course, following a more traditional one.

In the introduction, first order equations and two-dimensional systems are employed to introduce the fundamental geometrical notions of phase portrait, qualitative equivalence, flow and evolution. Existence and uniqueness theorems are stated. The geometrical emphasis means that an example using isoclines is worked out in the text, whereas techniques of separation, homogeneous equations, exact equations and integrating factors are reserved for exercises.

The second chapter, on linear systems, contains a classification of autonomous, homogeneous systems by the real, canonical form of their matrices ("algebraic type") and by the idea of qualitative equivalence ("qualitative type"). The evolution operator is defined and used in the solution of inhomogeneous (affine) systems.

The third chapter concerns non-linear systems in the plane and discusses local and global phase portraits. Local theory is represented by theorems on linearisation and stability; global theory by