PART V

MODELS

MODELS FOR THE CIRCUMSTELLAR ENVELOPES OF Be STARS

(*Review Paper*)

J. M. MARLBOROUGH
Dept. of Astronomy, University of Western Ontario, London, Ontario, Canada

Abstract. A survey is presented of the theoretical attempts to determine the structure of the circumstellar matter around Be stars. The general equations describing the structure and dynamics of Be star envelopes are given. The complications introduced by various physical phenomena are briefly discussed and initial attempts to solve restricted problems are considered. The various *ad hoc* models proposed for Be stars are discussed and comparisons of the observations with predictions of these models are illustrated. The strengths and weaknesses of these models are evaluated and areas where progress is being or should be made are considered.

1. Introduction

Be stars were first discovered by Secchi (1867) when, from visual inspection of spectra, he noted the Hβ line in emission in γ Cas. Since that time many astronomers have contributed to the vast array of observational data pertaining to Be stars and some astronomers, notably D. B. McLaughlin and to a lesser extent P. W. Merrill, have devoted a significant fraction of their professional careers to the study of Be stars.

The first suggestion to account for the origin of the emission lines in Be stars was due to Struve (1931; also Struve and Swings, 1932). Struve proposed that the emission lines arose from matter ejected from the equatorial region of a rapidly rotating star. This ejected material was assumed to form a nebulous ring around the star. If the emitting matter had an orbital speed less than that of the star, this hypothesis could easily account for the observed fact that the width of the emission lines was less than that of the underlying photospheric features. In many respects this idea qualitatively resembled the rings of the planet Saturn, as Struve himself noted. Furthermore, by restricting the ejected matter to regions near the equatorial plane, Struve was able to account for Merrill *et al.*'s (1925) observations that all stars with double emission at Hβ had 'nebulous' absorption lines (large $v \sin i$) whereas those with single emission at Hβ had 'sharp' absorption lines (low $v \sin i$). Finally, if the orbits of individual emitters in the ring were elliptical rather than circular, the two components of a double emission line would not necessarily have the same strength. A rotation of the line of apsides of the ring might then lead to a periodic variation of the strength of the two emission components thus providing an explanation for the V/R variation, V and R being measures of the strength of the violet and red components of the emission line, respectively.

McLaughlin (1933, 1938) extended the original idea of Struve and included those of Rosseland (1926) concerning the transformation of high energy radiation to radiation of lower energy to explain the origin of emission lines in stellar spectra. McLaughlin realized that the emitting region around a Be star is large compared to

A. Slettebak (ed.), Be and Shell Stars, 335–370. *All Rights Reserved*

the dimensions of the star, in fact similar to the dimensions of a small planetary nebula, but of lower excitation. Photoionization of hydrogen atoms by Lyman continuum radiation and the subsequent recapture and cascade of electrons through various energy levels lead to the production of emission lines. In the inner part of the emitting region hydrogen would be predominantly ionized while, further out, geometrical and physical dilution will significantly deplete the stellar Lyman continuum so that hydrogen becomes predominantly neutral.

To account for some of the observed spectral variations McLaughlin proposed a rotating-pulsating model for Be stars. This suggestion was a combination of the rotational model of Struve and the idea of an expanding atmosphere introduced by Beals (1930) to explain the observations of stars of the Wolf-Rayet and P Cygni types. Assumed temperature variations of the star yield a variable radiation pressure force on the envelope material thus producing expansion and contraction. Although this idea and a similar intermittently expanding-rotating model are no longer considered relevant to the Be star phenomenon and were rejected by McLaughlin (1961) on observational considerations and by Huang (1973) on theoretical considerations, they are important for two reasons. First, McLaughlin suggested the idea that radiation pressure might be important for Be stars, i.e., some other force apart from gravitation and those arising from the pressure gradient and centrifugal effects must be considered. Secondly, McLaughlin emphasized the importance of the velocity gradient for the line optical depth. In addition, McLaughlin (1961) further extended the suggestion of Struve concerning the possibility of an elliptical ring to account for the V/R variation.

Gerasimovic (1934) objected to Struve's rotational model for a variety of reasons; Struve (1942) later showed that most of Gerasimovic's objections were no longer significant. Gerasimovic suggested that the material responsible for the emission lines is ejected from the equatorial region of a rapidly rotating star by radiation pressure, both line and continuum, aided by the rapid rotation of the star. For this type of gas flow the streamlines are spirals and the circumstellar envelope is confined between two coaxial cones with vertices near the center of the star and which extend along the rotation axis of the star in the positive and negative directions. The initial velocity of the outflowing matter is assumed to be small in the radial direction since no large line asymmetries are observed in Be star spectra. To explain some of the time variations, Gerasimovic further proposed that matter is ejected spasmodically, regulated by radiation pressure due to an inward flux of radiation scattered back toward the star from the envelope. Dissipation of the outer envelope reduces this inward flux and allows matter to escape from the star again.

Many of the above ideas can be combined to provide the following simple *schematic* picture of a Be star. Consider a rapidly rotating B star with an extended moderately dense atmosphere confined chiefly to the equatorial plane as illustrated in Figure 1. For definiteness suppose the origin of the coordinate system is at the center of the star, the z axis parallel to the rotation axis of the star and the $x-y$ plane lying in the star's equatorial plane and consider standard spherical polar coordinates r, θ, and ϕ.

In Figure 1a is shown a schematic picture of the star and circumstellar envelope. For an observer in the direction indicated and not necessarily lying in the equatorial

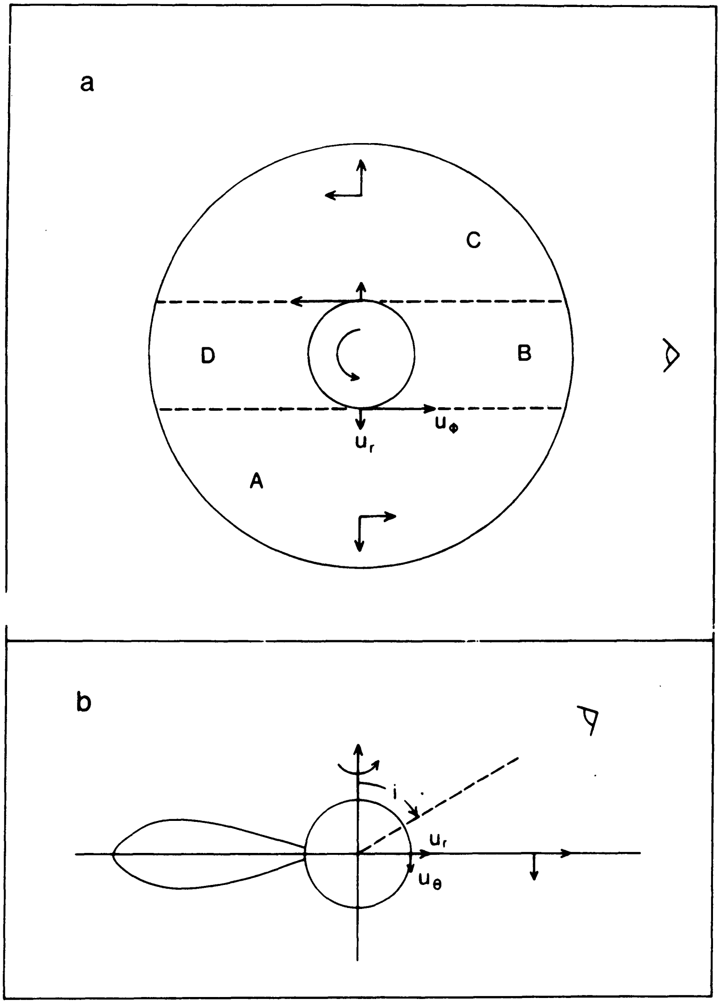


Fig. 1. A schematic representation of a Be star envelope. Part a is an equatorial plane view. The observer is to the right but not necessarily in the equatorial plane. The magnitude, direction and variation with r of the velocity components u_r and u_ϕ are illustrated schematically. Part b is a meridional plane view. The curve shown is a possible equidensity contour and the envelope is presumably symmetric about the rotation axis. The star has been drawn as spherical for simplicity. The magnitude, direction and possible variation with r of u_r and u_θ are illustrated.

plane, regions A, B, C and the part of D not occulted by the star would contribute to the emission in a given line; the part of region B projected against the star would produce the absorption component of the observed line. The observational data indicate that in the region of the envelope in which lines seen in the visible part of the spectrum are formed, $u_r \ll u_\phi$ where u_r and u_ϕ are the r and ϕ components of velocity, respectively, and this is illustrated schematically in Figure 1a. Recent evidence by Snow and Marlborough (this volume, p. 179) indicates that for some early Be stars of large $v \sin i$, u_r is an increasing function of r (see Marlborough and

Zamir, 1975, for a discussion of the inferences on $u_r(r)$ from theoretical models). This increase in $u_r(r)$ with increasing r is also illustrated schematically in Figure 1a.

In Figure 1b is shown a meridional plane view of a schematic Be star envelope. The appearance of the envelope in this case is uncertain. The curve shown may be thought of as representing an equidensity contour. Whether or not any significant amount of matter is located above the poles of the star is not known. If the model illustrated in Figure 1 is to represent both Be and shell stars, then the matter in the envelope must extend to at least one stellar radius from the equatorial plane to account for the deep shell absorption lines observed in the spectra of shell stars. Observational evidence on the magnitude of u_θ, the θ component of velocity, is uncertain. If Be stars of low $v \sin i$ are actually rapid rotators seen at small angles of inclination i, then, since the relatively sharp emission lines present in the spectra of Be stars of low $v \sin i$ are observed at their expected wavelength, the sum of the components of u_r and u_θ projected on to the line of sight cannot be large. Consequently, it is realistic to expect $u_\theta \lesssim u_r$, at least in the region of the envelope where the emission lines are formed.

A qualitative understanding of many of the observations of Be and shell stars can be obtained in the context of the schematic model illustrated in Figure 1 by allowing the density $\rho(\mathbf{r})$, the velocity $\mathbf{u}(\mathbf{r})$, the temperature $T(\mathbf{r})$ and the angle i to vary. Perhaps the strongest observational support for this model are the observations that $\Delta\lambda/\lambda$ is a constant for the emission lines in most Be stars and that many Be stars have a non-zero intrinsic linear polarization – presumed to arise from electron scattering – implying that for some values of angle i the envelope projected onto the plane of the sky, as seen by the observer, is not circularly symmetric.

Ideally one would like to predict $\rho(\mathbf{r}, t)$, $\mathbf{u}(\mathbf{r}, t)$ and $T(\mathbf{r}, t)$ as functions of position $\mathbf{r}$ and time t from a consideration of the relevant physical phenomena. In addition, one would hope to understand the origin of the cirumstellar material – does it arise from the star or a nearby companion – and its subequent fate. Furthermore, a knowledge of the mass loss rate (or mass gain rate) is of importance for determining what role the Be phenomenon may play in the evolutionary history of the star.

2. Structure and Dynamics of Be Star Envelopes

Let us now consider the general problem of determining $\rho(\mathbf{r}, t)$, $\mathbf{u}(\mathbf{r}, t)$ and $T(\mathbf{r}, t)$ for the circumstellar material around a Be star. Some initial attempts to solve for these variables have been investigated and will be discussed below.

2.1. GENERAL THEORY

We will assume that the circumstellar matter in a Be star envelope can be treated as a continuum and will use a single fluid description of this continuum. Morgan (1975b) has recently investigated both of these assumptions and concluded they are justified for Be star envelopes.

A complete hydrodynamic treatment of the structure and dynamics of Be star envelopes requires the simultaneous solution of the following equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} = 0 , \qquad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}=-\frac{1}{\rho}\nabla P_g+\mathbf{F}+\mathbf{f}\,, \tag{2}$$

$$\frac{\partial E_g}{\partial t}+\mathbf{u}\cdot\nabla E_g+P_g\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)+P_g\mathbf{u}\cdot\nabla\left(\frac{1}{\rho}\right)=G-L\,, \tag{3}$$

where Equations (1), (2), and (3) represent the equation of continuity, the equation of motion, and the equation for the conservation of energy, respectively. In these equations ρ is the density, $\mathbf{u}$ is the velocity, P_g is the gas pressure, $\mathbf{F}$ is the total body force per unit mass on a fluid element, $\mathbf{f}$ is the total boundary force per unit mass on a fluid element, E_g is the internal energy of the gas per unit mass, G is the rate of energy gain per unit mass of the gas, L is the rate of energy loss per unit mass of the gas, and all these variables are in general functions of position $\mathbf{r}$ and time t. The basic Equations (1), (2) and (3) are discussed in any book on fluid dynamics, e.g. Zel'dovich and Raizer (1966). If the gas is assumed to be a simple perfect gas of constant mean molecular weight, then $E_g=c_vT$ where T is the temperature and c_v, the specific heat at constant volume, is generally assumed to be constant if the gas is non-degenerate and non-relativistic (see Cox and Giuli, 1968, for details). For such a situation, Equations (1), (2), and (3), together with the equation of state, constitute six equations for the six dependent variables ρ, $\mathbf{u}$, P_g, and T if $\mathbf{F}$, $\mathbf{f}$, G and L do not introduce any new dependent variables. In general, however, this is not the situation.

The total body force $\mathbf{F}$ includes gravitation due to the central star and others it the star is a member of a multiple star system, self-gravitation of the envelope matter, the radiation pressure force, and the Lorentz force if a magnetic field is present. Normally, self-gravitation of the circumstellar material is neglected for Be star envelopes and this neglect seems justified based on estimates of the total mass of circumstellar matter (see Weidelt, 1970; and Biermann and Kippenhahn, 1973, for complications when self-gravitation is included).

If the radiation pressure force makes an important contribution to $\mathbf{F}$ and/or radiative energy gains and losses occur in G and L, then the basic equations will contain terms involving the specific intensity $I_\nu(\mathbf{r}, t)$ at frequency ν, the source function $S_\nu(\mathbf{r}, t)$ and various moments of $I_\nu(\mathbf{r}, t)$, together with the mass absorption coefficient κ_ν. Since both S_ν and κ_ν can be expressed in terms of the populations of atomic energy levels contributing to the radiation at frequency ν, a set of equations determining these atomic energy level populations together with the equation of transfer must be solved simultaneously with Equations (1), (2), (3) and the equation of state. Some idea of the complexity involved even for the case of a steady-state, spherical symmetry and a very simplified treatment of the radiative transfer, can be obtained from the investigations of Cassinelli and Castor (1973) and Castor *et al.* (1975).

If a magnetic field plays an important role in the dynamics of Be star envelopes, as has been suggested by Crampin and Hoyle (1960) and Limber and Marlborough (1968) among others, then in general Equations (1) to (3) together with the equation of state must be solved simultaneously with Maxwell's equations describing the electromagnetic field. Of course, if the effects of both radiation and a magnetic field are included, the relevant equations become horrendously complex.

Whether or not viscous forces are important – i.e. whether or not $\mathbf{f}=0$ – is presently unclear. Morgan (1975b) compared the ratio of viscous to inertial terms in Equation (2) and concluded viscous effects were completely negligible. However Limber and Marlborough (1968) concluded that outward transport of angular momentum by some viscous agent – either magnetic or turbulent viscosity – was important for the dynamics of Be star envelopes. Recent investigations (see Section 2.2) seem to support both these results in that additional forces such as those arising from a magnetic field or the absorption of radiation are important in order to account for the envelope dynamics while the inclusion in Equation (2) of terms arising from the viscous stress tensor may not be necessary.

Due to differential motions in Be star envelopes, a further complication arises when the radiation field plays an important role. Equations (1) to (3) are valid in an inertial reference frame. However the equation of transfer and the equations for the population of atomic energy levels are normally expressed in a form valid only in a frame at rest with respect to the local macroscopic velocity of the matter, i.e. the fluid frame. For example, it is only in this latter frame that κ_ν is independent of the direction of the radiation being absorbed. This problem is discussed in detail in various books (see Sampson, 1965; and Pomraning, 1973). Castor (1972) has discussed the problem in detail for a spherically symmetric situation.

It should also be pointed out that it may be necessary to include non-radiative energy sources in G and L. Thomas (1970) has constantly emphasized the possible importance for stellar atmospheres – considered to be the transition zone between the stellar interior and the interstellar medium – of a supply of non-thermal kinetic energy. The recent suggestion of the existence of a chromosphere in Vega (A0V) by Praderie *et al.* (1975) may be direct evidence in support of Thomas' suggestion for relatively early type stars. Smith (1970) has shown that in rotating stellar atmospheres meridional circulation currents become unstable near the surface and generate a turbulent layer with turbulent velocities which in general are only slightly subsonic. Although the energy dissipated in the turbulent eddies is predicted to be small compared to the stellar luminosity, a non-thermal energy source of this kind may be important in the Be star envelopes.

On account of the complexity of the basic equations describing the dynamics of Be star envelopes, little progress has been made in solving the general problem for the structure and dynamics as a function of both position and time. In general, the basic equations are at least a function of two independent variables, i.e. r and θ, and thus consist of a set of non-linear partial differential equations. Such a set of equations is notoriously difficult to solve. The various attempts to date have employed a number of simplifying assumptions to reduce the general problem to one for which some solution can be found in restricted cases.

2.2. Specific solutions

In essentially all considerations so far, a steady state has been assumed. It is important to point out, however, that in many, if not all Be stars, spectroscopic and photometric features normally attributed to the circumstellar material vary on some time scale which can be anywhere from the order of minutes or less to years. Limber

(1970) has discussed the theoretical implications of these time scales. All attempts to solve the basic equations for some restrictive set of assumptions are concerned with stellar wind solutions, i.e. solutions for which $\rho(r, \theta = \pi/2)$ is non-zero for $R_* \leq r < \infty$ and for which $u_r(r, \theta = \pi/2)$ is generally an increasing function of r for $R_* \leq r < \infty$ (R_* is the stellar radius).

For the case of continuing rotationally forced ejection with gravitation and gas pressure gradient being the only forces acting, Limber (1964, 1967) has constructed steady-state solutions which are symmetric about the equatorial plane and which are limited to regions near the equatorial plane. The solutions also require the specification of the functional dependence of $u_\phi(r, \theta = \pi/2)$ on r. In these solutions, the effect of the stellar radiation field is not included explicitly but can be included by reducing the effective gravitational mass of the star. Comparisons of predictions based on these solutions with observational data are given in Section 3.

Cassinelli and Castor (1973) and Castor *et al.* (1975) have included the effects of the stellar radiation field explicitly in Equations (2) and (3) and investigated the solutions for the case of spherical symmetry in order to account for the violet displaced absorption lines observed in the far ultraviolet spectra of early type, luminous stars. Marlborough and Zamir (1975) extended the solutions of Cassinelli and Castor (1973) to the equatorial plane of rapidly rotating stars. Under the assumption that $u_\phi(r, \theta = \pi/2)$ is a known function of r and using other assumptions described in that paper Marlborough and Zamir obtained the solution for $u_r(r, \theta = \pi/2)$. If the radiation field of the star is sufficiently strong and/or the rotation rate sufficiently large, then a solution for $u_r(r, \theta = \pi/2)$ deviating considerably from hydrostatic behavior is obtained in which u_r is subsonic near the star and supersonic at larger distances. Such a solution is qualitatively similar to ones obtained by Limber (1967). Marlborough and Zamir further suggested that the solution for u_r for $\theta = 0, \pi$ would probably deviate negligibly from hydrostatic behavior. Heap (1975) discovered that all stellar lines in the ultraviolet spectrum of ζ Tau (B3IV), obtained on a rocket flight, are shifted to the violet corresponding to velocities up to 120 km s^{-1}. Analysis of *Copernicus* data by Snow and Marlborough (this volume, p. 179) indicates that several early Be stars of large $v \sin i$ show asymmetric resonance line profiles, particularly the Si IV doublet $\lambda\lambda$ 1393, 1402, indicating mass loss with typical velocities of order several hundred km s^{-1}. Other Be stars of low $v \sin i$, observed to date by *Copernicus* do not appear to show this line asymmetry. These observations are consistent with the predictions of Marlborough and Zamir. Therefore, at least for early type Be stars, the Be star phenomenon may be the result of a radiatively driven stellar wind. Massa (1975) has also concluded that the Be phenomenon is to be understood in terms of a radiation driven stellar wind. However, it should be noted that the asymmetric line profiles are also at least qualitatively consistent with any stellar wind model in which u_r is an increasing function of r in the region where the absorption lines are formed.

Two recent investigations have included the effects of a magnetic field on the dynamics of the envelope. Limber (1974) has modified the investigations of Weber and Davis (1967) concerning the angular momentum of the solar wind and extended their analysis to the region of Be star envelopes in and near the equatorial plane. Stellar wind solutions were found in which $u_r(r, \theta = \pi/2)$ is such that the radial

Alfvènic Mach number $M_A \equiv (4\pi\rho u_r^2/B_r^2)^{1/2}$ varies from <1 near the star to >1 at larger distances (B_r is the radial component of the magnetic field). This model possesses several interesting features. For essentially all values of r except very close to the star, $u_\phi(r, \theta = \pi/2) \propto r^{-1}$, indicating that centrifugal support is not important except near the star. For physically acceptable steady state solutions corresponding to realistic values for the mass of the star, its radius, the envelope temperature and the density and angular velocity at the stellar surface, B_r at the stellar surface is of order 10 to 100 gauss; the total field strength would be in the range 100 to 1000 gauss. As B_r increases from much smaller values than those considered above, the mass loss rate increases, the magnetic field transporting energy from the star to the envelope. For B_r much larger than the above values, the mass loss will most likely be suppressed due to the inability of the matter to modify the geometry of the field lines. Finally, there exists the possibility that many of the steady-state solutions are unstable leading to time dependent solutions which may account for some of the observed time variations in Be stars.

Saito (1974) has also constructed a model for a Be star envelope including the effects of a magnetic field. His analysis is restricted to the equatorial plane. In general, the solutions obtained are similar to those of Limber and the required magnetic field strengths at the stellar surface for steady state solutions are also in the range 100–1000 gauss.

It is of interest to consider briefly whether such field strengths are presently detectable. According to Landstreet (1975), there are three methods which can be used to determine magnetic field strengths. Photographic techniques are restricted to sharp line stars, those for which $v \sin i < 30$ km s^{-1}, and for a sixth magnitude B star would yield a typical standard error of a few hundred gauss. The photoelectric line scanner approach is also limited to sharp line stars. For eight B stars brighter than apparent magnitude 4.5 with $v \sin i < 30$ km s^{-1}, recent measurements of the magnetic field strength by Landstreet yield standard errors in the range 30–120 gauss. Finally, interference filters can be used to observe hydrogen lines and other strong photospheric features and should be capable of detecting magnetic fields with standard errors of 50–100 gauss in bright B stars with $v \sin i < 200$–300 km s^{-1}. It therefore seems possible that in the near future, one will be able to test these magnetohydrodynamic models by directly measuring the stellar magnetic field. Landstreet and Marlborough have recently initiated such a program for Be stars.

Morgan (1975a) has subjected Limber's isothermal, hydrostatic model (1964) to a linear stability analysis. He found that some oscillating temporal and angular variations could occur on a time scale of the order of a few minutes. Even though no consideration of the phenomena initiating these variations was investigated, the results are interesting because they provide a possible explanation for the rapid variations observed by Delplace *et al.* (1969), Kupo (1971), and Bahng (1971), among others (see also Hutchings' article in this volume, p. 13). To explain these rapid variations Limber (1970) has suggested that changes in the energy of the magnetic field due to reconnection of field lines can convert magnetic energy to kinetic energy thus producing local heating of small regions, while Hutchings (1970b) proposed short lived condensations in the envelope near the stellar surface as a possible explanation. On the other hand, Lester (1975) has suggested that photometric variability observed in EW Lac (B2pe) on a time scale of $0^{\rm d}.7$ is a result of temperature changes in the stellar photosphere and not phenomena occurring in the extended envelope. Fernie (1975) reported photometric variability of a few hundredths of a magnitude in π Aqr (B1Ve) over a time scale of hours and attributed this variation to a pulsation of the star. However North and Olofsson (1974) reported no variations greater than $0\overset{m}{.}01$ in lines or continua in the same star based on their observations in 1972. Thus a plethora of suggestions exists to explain the observed short term variation. Whether any of these is correct and what relation, if any, they have to the variations predicted by Morgan (1975a) is presently unknown.

3. *Ad Hoc* Models

Up to the present time, the solutions discussed in Section 2 have not been used to make specific predictions concerning observational quantities such as line profiles, linear polarization, etc., which can then be compared directly with observational data. The predictions concerning observable features which do exist are based on simplifications and extensions of the models discussed in Section 2 and other ideas in order to obtain the complete three dimensional structure of the envelope. In this way, many approximate models have been constructed to represent the envelopes of Be stars. Essentially all models of this type are *ad hoc* to a greater or lesser extent in that either they do not satisfy the basic Equations (1), (2), and (3) or that they have not been tested to see if they satisfy these equations. In addition, many of these models may not be self-consistent. Consequently, some caution must be exercised in using these *ad hoc* models and in evaluating the comparison of model predictions with observational data.

Some purists may argue that reliance on any *ad hoc* model is futile at best and in general may be highly misleading. However, this is not necessarily so. For example, Rybicki (1970) has emphasized that much of our present conception of extended stellar atmospheres is based to a significant extent on the formal solution of the equation of transfer employing *ad hoc* assumptions for the structure of the extended atmosphere and the source function. In the preface to his book Stellar Evolution, Struve (1950), while noting the reluctance of most scientists to rely on *ad hoc* assumptions or uncertain hypotheses, nevertheless asserts that such hypotheses or assumptions can be of value in providing guidance for future studies. Finally, to quote Chandrasekhar (1934) in the context of Wolf-Rayet stars and novae, but also applicable to Be stars at our present level of understanding: "A dynamical theory of the ejection process itself is necessary only insofar as we require the emission per unit volume and the radial velocity as functions of *r*, and consequently even without any underlying dynamical theory we can formally examine the type of band contours that could be expected from assumed hypothetical laws of velocity variation, and from a comparison with the observed contours infer something about the actual conditions obtaining in stellar atmospheres where such emission bands originate."

3.1. STEADY-STATE, STELLAR WIND MODELS

A variety of authors have constructed steady-state, stellar wind models to represent the envelopes of Be stars. Hutchings (1970a, 1971) has compared observed line profiles for the Be stars γ Cas, κ Dra and HD 142926 with theoretical ones based on his model (Hutchings, 1968) originally applied to OB supergiants. The particulars of the model are described in detail in the latter paper; only a brief summary is given here. In the radiative transfer problem electron scattering is assumed to be the dominant opacity source. Doppler broadening is taken as the dominant broadening mechanism and lines are assumed to be formed by absorption and re-emission of photospheric radiation. The star is assumed to radiate like a black body near each line of interest. Each stellar photon is assumed to undergo at most one pure scattering and one electron scattering interaction. Allowing one more scattering per photon made only a small change according to Hutchings. The atomic energy level populations were taken from Baker and Menzel (1938) modified according to a procedure described by Hutchings (1968). Each of $\rho(\mathbf{r})$, $\mathbf{u}(\mathbf{r})$, and $T(\mathbf{r})$ was written as a simple function of $\mathbf{r}$ containing some adjustable parameters and the envelope model derived empirically by modifying these adjustable parameters to reproduce

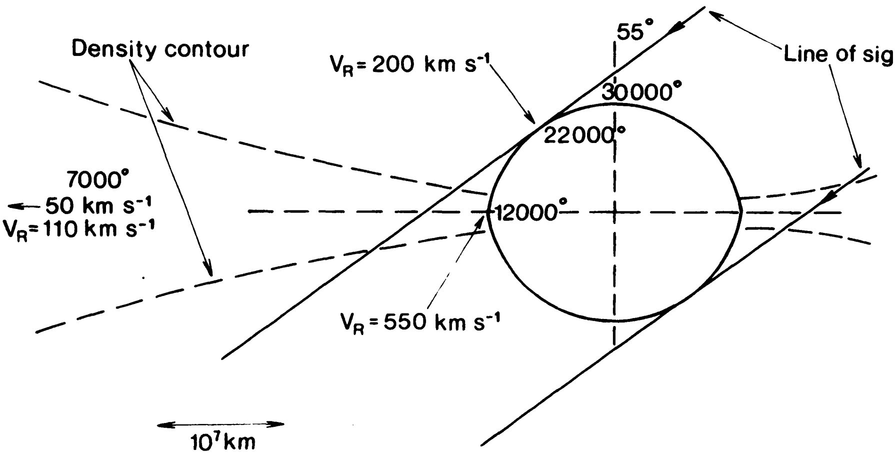


Fig. 2. Schematic model of γ Cas (Hutchings, 1970a).

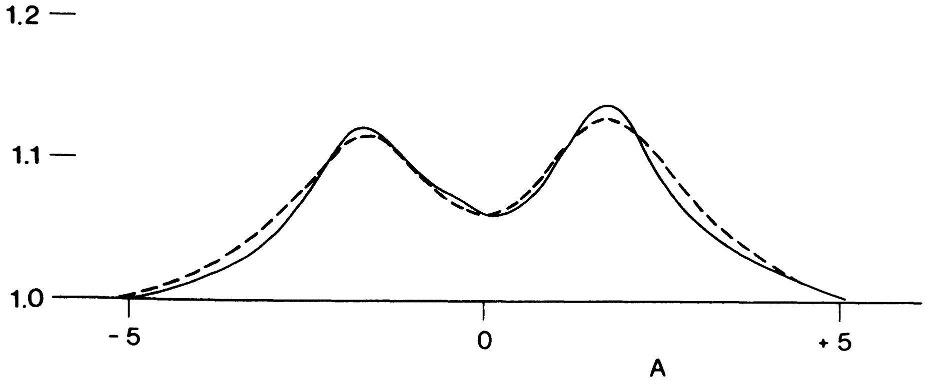


Fig. 3. Theoretical and observed Hγ line profiles for γ Cas: - - - observed; ——— theoretical. Ordinate is flux relative to the continuum (Hutchings, 1970a).

the peak separation of the two components of a double emission line, the *V/R* ratio, the central depth, the wing extent and the total peak height. The model thus derived for γ Cas (B0 IVe) is illustrated schematically in Figure 2 and the comparison between the observed and theoretical Hγ line profile for γ Cas is given in Figure 3. A similar comparison between the observed and computed Hγ profiles for κ Dra (B7p) is shown in Figure 4 and for HD 142926 (B9e) in Figure 5. The final envelope model for the latter two stars is qualitatively similar to that for γ Cas (Figure 2). The mass loss rates for each star, as given by the models, are: γ Cas, 10^{-7} $M_\odot$ yr^{-1}; κ Dra, 3.3×10^{-8} $M_\odot$ yr^{-1}; HD 142926, 2.5×10^{-8} $M_\odot$ yr^{-1}.

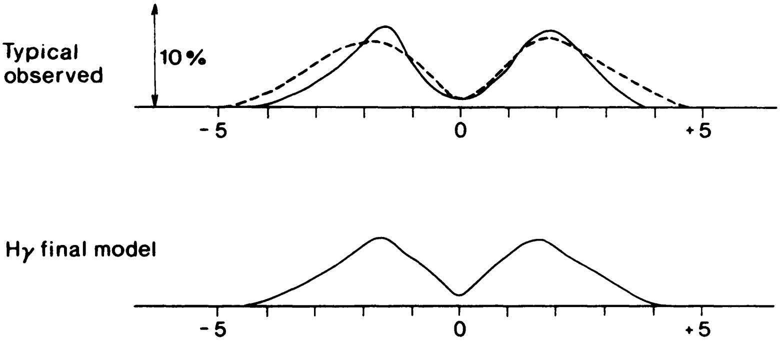


Fig. 4. Theoretical and observed Hγ line profiles for κ Dra. Ordinate same as Figure 3 (Hutchings, 1971).

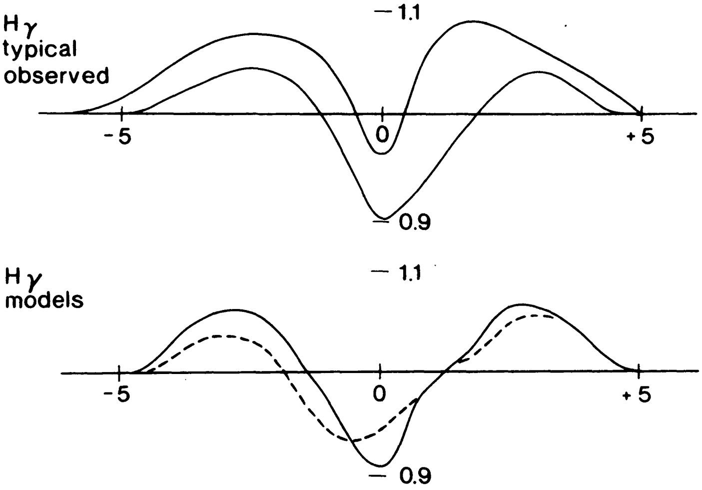


Fig. 5. Theoretical and observed Hγ line profiles for HD 142926. Ordinate same as Figure 3 (Hutchings, 1971).

Marlborough (1969, 1970) and Marlborough and Roy (1971) have discussed a steady-state stellar wind model to represent Be star envelopes. The details of the model are described in the 1969 and 1970 papers. The state of excitation and ionization of the envelope was determined in a manner similar to that employed in nebular studies. The statistical equilibrium equations included photoionization from all energy levels and collisional transitions between levels of principal quantum number differing by ±1. Terms in the statistical equilibrium equations pertaining to the radiation field contain contributions from the star and the envelope. The continuous radiation at any point in the envelope was assumed to be due solely to the star reduced by geometrical dilution and bound-free absorption between the star and the point considered; continuous radiation produced by the envelope was completely neglected. The radiation emitted by the star was taken from an appropriate model atmosphere. The contribution of the star to line radiation was neglected. The envelope contribution to line radiation in the statistical equilibrium equations was taken into account in the following approximate manner. The statistical equilibrium equations were solved for each of three cases: case I (classical nebular case A), case II (classical nebular case B) and case III for which the envelope was assumed to be optically thick in all line radiation. The final atomic energy level populations at a given point were obtained by taking weighted averages of either cases I and II or II and III, the pair to be chosen depending on the optical depths in $L\alpha$ and Hα between the point considered and the edge of the envelope in a direction parallel to the rotation axis. This direction was chosen because in general it is the one for which line photons have the greatest chance of escaping from the envelope. The details of this weighting and averaging procedure are given by Marlborough (1969). The envelope was assumed to be isothermal and $u_\phi(r, \theta = \pi/2)$ taken to vary as $r^{-1/2}$. If the meridional projections of streamlines are assumed to be straight lines originating at some arbitrary point below the star's surface, then $\rho(r, \theta = \pi/2)$ is obtained from the equation of continuity once $u_r(r, \theta = \pi/2)$ and the arbitrary point at which the streamlines originate are specified. Finally $\rho(\mathbf{r})$ is obtained from $\rho(r, \theta = \pi/2)$ by using the straight streamline assumption and by assuming Limber's hydrostatic solution (1964) is applicable at some arbitrary point $(r', \theta = \pi/2)$. Again, the details and relevant formulae are given by Marlborough (1969), as is the method used to calculate the theoretical line profiles. In Figure 6 is shown a comparison of the

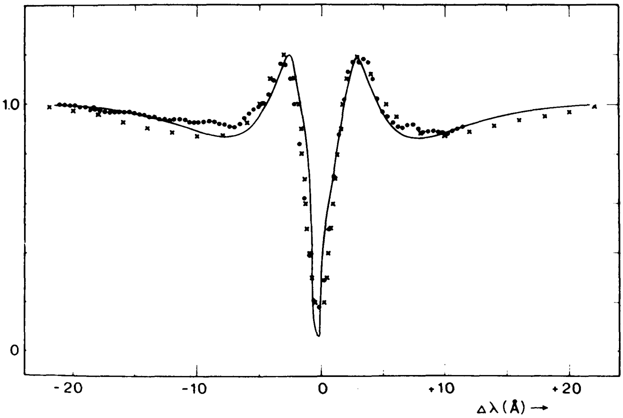


Fig. 6. Theoretical and observed Hα line profiles for 1 Del: ——— theoretical profile; . . . photoelectric observations (Gray and Marlborough, 1974); × × × × mean photographic profile (Marlborough and Cowley, 1974). Ordinate same as Figure 3.

theoretical and observed Hα line profiles for the shell star 1 Del (A0pe). The model envelope is similar to that used by Marlborough and Cowley (1974) except that the number density $N(r, \theta = \pi/2) = 1.35 \times 10^{12}$ cm^{-3} at $r = 4\ R_*$, $\theta = \pi/2$ where R_* is the radius of the star and the resulting mass loss rate is $8.8 \times 10^{-9}\ M_\odot$ yr^{-1}. In Figure 7 is shown the results of a preliminary attempt to reproduce the observed Hα line profile in the shell star HD 193182 (B7IV–Ve). The envelope model is the same as that used for 1 Del except that $N(r, \theta = \pi/2) = 2.3 \times 10^{12}$ cm^{-3} at $r = 4\ R_*$, $\theta = \pi/2$ and the corresponding mass loss rate is $1.5 \times 10^{-8}\ M_\odot$ yr^{-1}.

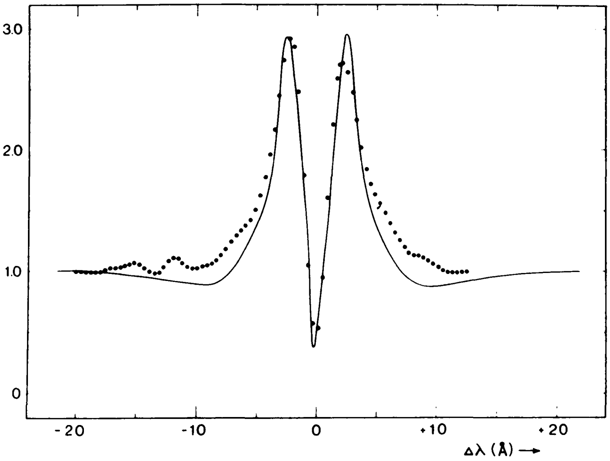


Fig. 7. Thereotical and observed Hα line profiles for HD 193182: ——— preliminary theoretical profile; . . . photoelectric observations (Gray and Marlborough, 1974). Ordinate same as Figure 3.

Doazan (1965) has constructed a stellar wind model for the Be star HD 50138 (B8e). From the analysis Doazan concluded that this star is probably intermediate in type between Be and P Cygni stars. Despite this however it is useful to consider the results of her model because in that model the radiative transfer problem is treated by Sobolev's method (1960; see also Rybicki, 1970; and Hummer's article in this volume, p. 281). For this model, calculations were confined to the equatorial plane. It was assumed that $N_e(r, \theta = \pi/2) \propto r^{-2}$, $u_\phi(r, \theta = \pi/2) \propto r^{-1}$, $u_r(r, \theta = \pi/2) \propto r^s$ and $u_r(R_*, \theta = \pi/2)/u_\phi(R_*, \theta = \pi/2) = k$ where s and k are adjustable parameters and N_e is the electron density. The observed and calculated Hβ profiles are illustrated in Figure 8 for $s = -1.4$ and $k = 2$. In Figure 9 is shown $u_r(r, \theta = \pi/2)$ for the model illustrated in Figure 8 and also a modified form in which $u_r(r, \theta = \pi/2)$ initially increases with r. The Hβ profile for this second model is shown in Figure 10. The resulting mass loss rate is approximately $5 \times 10^{-7}\ M_\odot$ yr^{-1}.

For the stellar wind models considered above, the mass loss rates inferred from the comparison of observed and theoretical line profiles lie in the range 10^{-7} to $10^{-9}\ M_\odot$ yr^{-1}. The main sequence lifetimes for B stars vary from approximately 10^7 yr for B0V to 5×10^8 yr for A0V. If the Be phase occupies a fraction $f(\leq 1)$ of the main sequence lifetime where f includes the transient nature of the presence of circumstellar matter as deduced from the appearance and disppearance of emission

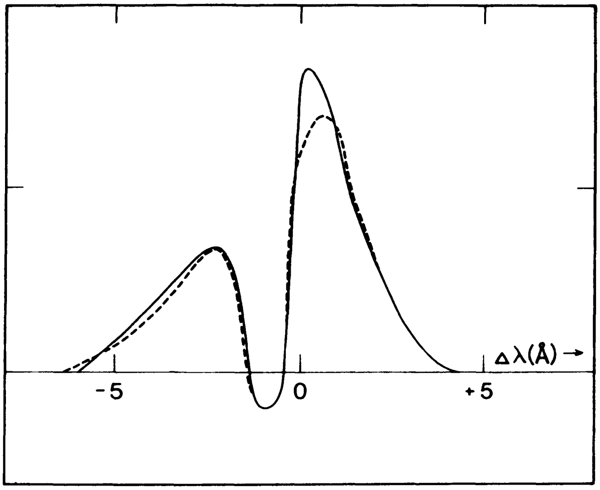


Fig. 8. Theoretical and observed Hβ line profiles for HD 50138: ——— theoretical profile for $s = -1.4$, $k = 2$; - - - observed profile from plate W 1783. Ordinate same as Figure 3 (Doazan, 1965).

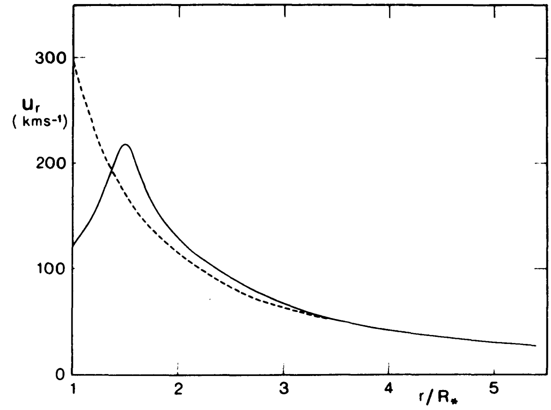


Fig. 9. $u_r(r, \theta = \pi/2)$ for theoretical profiles in Figures 8 and 10: - - - u_r for Figure 8; ——— u_r for Figure 10 (Doazan, 1965).

and/or shell lines and if the above estimates of mass loss are realistic, it would seem that the mass loss during the Be phase will not have any significant effect on the star's future evolution, at least as far as the total mass is concerned. Whether the corresponding angular momentum loss will significantly reduce the initial total angular momentum of the star is presently unclear for these stellar wind models, although for Limber's hydromagnetic model (1974 and Section 2.2), the angular momentum loss over a period of 10^7 yr is predicted to be about one tenth of the total initial angular momentum.

In all of the stellar wind models the circumstellar envelope extends to large distances from the star. If the envelope is sufficiently extensive, some Be stars may be

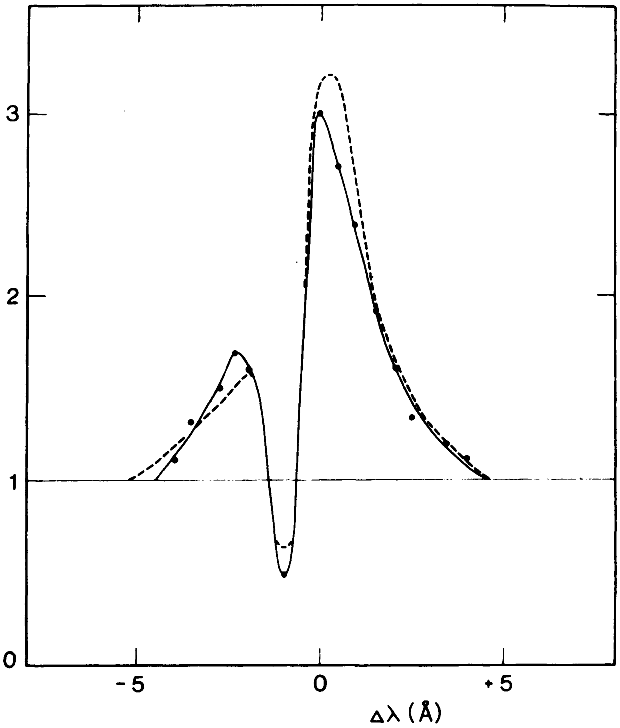


Fig. 10. Theoretical and observed Hβ line profiles for HD 50138: ——— theoretical profile; --- observed profile from plate W 576. Ordinate same as Figure 3 (Doazan, 1965).

detectable as thermal radio sources. Trasco *et al.* (1970) have reported the detection of κ Dra (B7p) at $\lambda = 6$ cm at a flux level of 0.020 Jy (see the article by Purton, this volume, p. 157, for a discussion of the observations). No predictions however have as yet been made of the expected radio continuum emission for any of the models discussed in Section 3.

It would be useful to be able to decide which of the above models best represents the envelopes of Be stars. Unfortunately, this does not appear possible at the present time and is partly due to the fact that the comparison of observational data with theoretical line profiles is made for different stars. In the absence of a comparison of predicted line profiles from different models for the same star one might perhaps decide the best model based on a criterion such as the smallest number of *ad hoc* assumptions. Even this is not completely satisfactory, however, because the arbitrariness of a particular assumption is at least partly, if not greatly, a subjective decision. The fact that each of the above models uses a different approach to determine the excitation and ionization state of the envelope and to solve the radiative transfer problem and at the same time can satisfactorily reproduce the observed line profiles may simply be an indication that each model possesses a sufficient number of adjustable parameters. More likely, however, the agreement is an indication that to lowest order, the gross structure of line profiles is dominated by the geometry and velocity field and less by the radiative transfer effects. When the effect of Doppler broadening is large, as it is in Be stars – at least for moderate to large values of angle i – the intrinsic shape of the line will not have great significance in determining the observed profile. Nevertheless, better treatments of the line transfer problem are necessary if only for the fact that the methods used in the above three models, particularly the first two, are primitive at best.

A glance at Figures 3 to 10 indicates that the stellar wind models discussed above can satisfactorily reproduce observed line profiles for which $V = R$ and for which $R > V$. It is by no means apparent, however, whether models in which $u_r(\mathbf{r})$ is everywhere positive can generate a line profile for which $V > R$. At present this is the severest limitation of the stellar wind models because they appear incapable, without the addition of further *ad hoc* assumptions, of accounting for the observed V/R variation, particularly those cases in which V/R becomes significantly greater than unity. Alternately, this discrepancy may simply be a reflection of the elementary radiative transfer solutions employed. Magnan (1970, 1972) has demonstrated that complex line profiles can arise in differentially rotating slabs when a more realistic treatment of the radiative transfer is employed.

3.2. TIME DEPENDENT, STELLAR WIND MODELS

Limber (1969) introduced a simple time dependent, stellar wind model for the structure and evolution of the envelope to explain Pleione's (B8p) shell episode between the years 1938 and 1954. In this model Limber predicted the profile of the Balmer line H25 by assuming that the population of the atomic energy level of principal quantum number $n = 2$, at any point r in the envelope, was proportional to some power of the number density of hydrogen atoms there and by neglecting re-emission in the envelope. The model is one in which $u_\phi(r, \theta = \pi/2) \propto r^{-1/2}$, the hydrostatic solutions (Limber, 1964) are used for the envelope structure perpendicular to the equatorial plane and $u_r(r, \theta = \pi/2)$ increases from small values for $r \simeq R_*$ to larger values at greater r and is independent of time. The time dependence in this model is introduced by the variation with time of the mass loss rate as illustrated in Figure 11. A comparison of the predictions of the model and the observations is also seen in Figure 11. As is evident, Limber's model accounts very well for Merrill's observations (1952) of the approximate strength of the shell lines and for the radial velocity of H25. No agreement could be obtained for situations in which $u_r(r, \theta = \pi/2)$ was a decreasing function of r.

Marlborough (1971), for the same model, determined the time variation of the line profiles of Hα, H15, H25 and H35, the latter three being included to possibly account for the observed Balmer progression (Merrill, 1952). Re-emission effects were neglected for H15, H25 and H35. The predicted line profile changes are shown in Figures 12, 13, 14, 15, 16 and 17, and in Figure 18 the upper Balmer lines are superimposed to show the shift to shorter wavelength with time after 1945. Limber's model reproduces the observed equality of R and V for Hα for most of the shell phase, the change to $R > V$ during the later stages, and a qualitative similarity of the predicted Hα profile in 1945 to a tracing of the observed Hα profile given by Merrill (1952). However there are also some important disagreements. Specifically, according to the model the emission at Hα does not begin early enough – emission at Hα

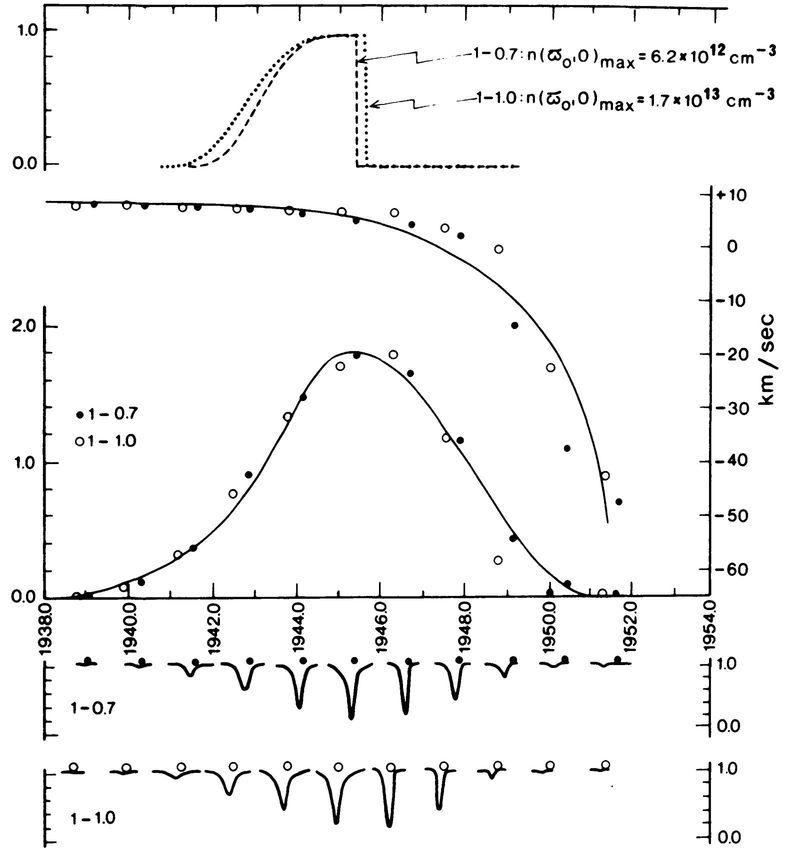


Fig. 11. Comparison of Limber's model with observations of Pleione. Dashed and dotted curves in the upper part give the rate of mass loss with time for two models. The upper solid curve is the smoothed version of the observed radial velocity of H25. The lower solid curve represents Merrill's observations of the time variation of the strength of the shell lines. The filled and open circles are the predicted variation for each of the two models. At the bottom are the line profiles for the two models and the filled and open circles represent the position of the line center with respect to the center of the star. Both models are for the case that $u_r(r, \theta = \pi/2)$ generally is an increasing function of r (Limber, 1969).

being reported in October, 1938, by McLaughlin (1938) and Mohler (1938) – and the emission at Hα disappears too rapidly near the end of the shell phase – Slettebak (1954) reported emission at Hβ in 1954 (see also Burd, 1954, for a description of spectral changes after 1947). In addition, no apparent Balmer progression is evident in Figure 18.

Marlborough and Gredley (1972) included reemission from the envelope in the upper Balmer lines to see if this would produce better agreement between the observed and theoretical Balmer progression. No significant change was produced by the inclusion of envelope reemission in these lines. In summary, Limber's model accounts very well for a number of observational results pertaining to Pleione's shell phase, but it is unsatisfactory in accounting for the early appearance of emission at

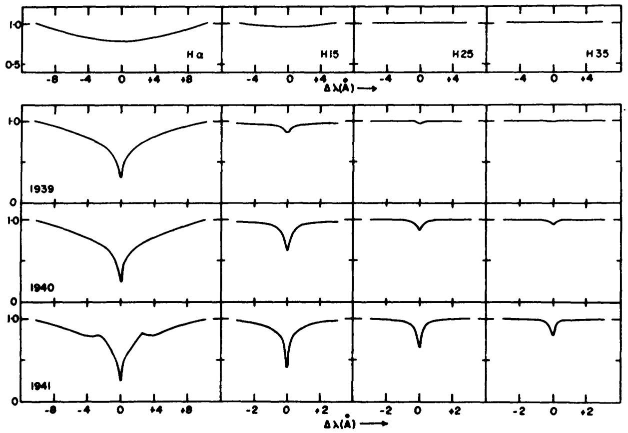


Fig. 12. Theoretical Balmer line profiles for Limber's model for Pleione for 1939–1941. The first row is the rotationally broadened stellar lines. Ordinate same as Figure 3 (Marlborough, 1971).

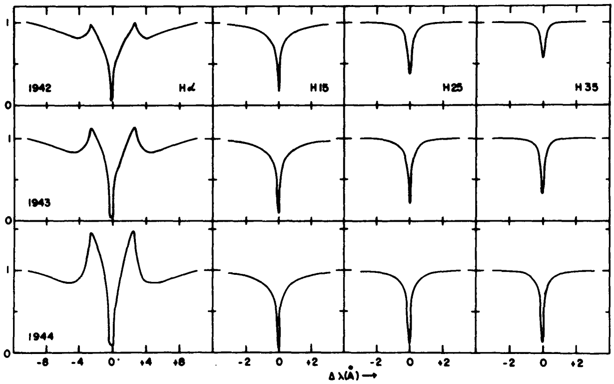


Fig. 13. Theoretical Balmer line profiles for 1942–1944 (Marlborough, 1971).

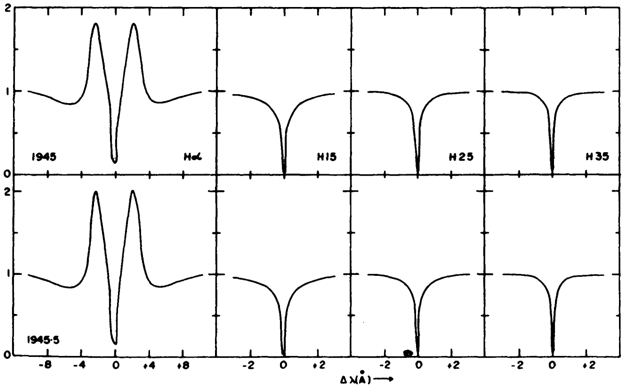


Fig. 14. Theoretical Balmer line profiles for 1945 and 1945.5 (Marlborough, 1971).

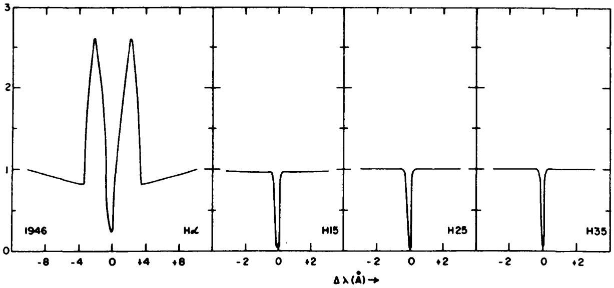


Fig. 15. Theoretical Balmer line profiles for 1946 (Marlborough, 1971).

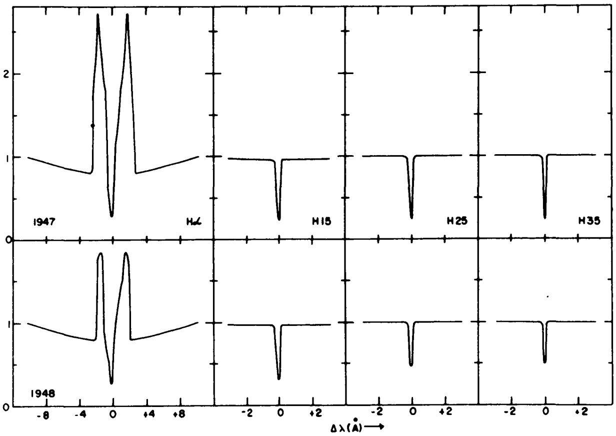


Fig. 16. Theoretical Balmer line profiles for 1947 and 1948 (Marlborough, 1971).

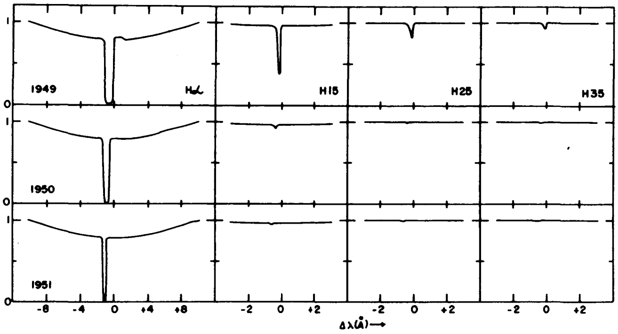


Fig. 17. Theoretical Balmer line profiles for 1949–1951 (Marlborough, 1971).

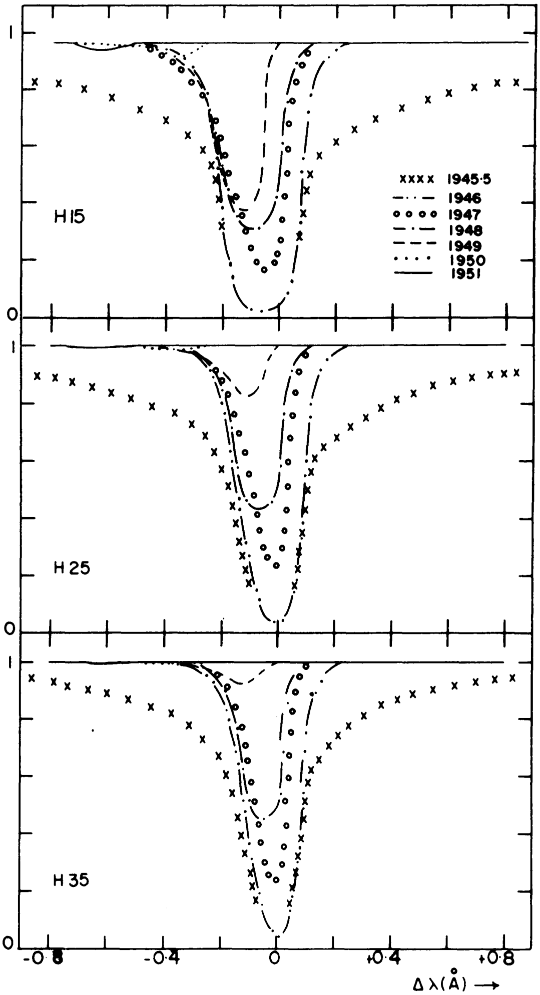


Fig. 18. Theoretical profiles for upper Balmer lines for 1945.5–1951. Ordinate same as Figure 3 (Marlborough, 1971).

Hα, the persistence of emission at Hα when the shell lines have essentially vanished and the Balmer progression especially during the later stages.

3.3. ELLIPTICAL RING MODEL

Huang (1972, 1973) and Albert and Huang (1974) have reconsidered the elliptical ring model originally suggested by Struve and elaborated on by McLaughlin. A qualitative summary of the major ideas is given by Huang (1975). According to this picture the emitting and absorbing atoms in a Be star envelope are strongly concentrated to the equatorial plane and move on individual orbits according to Keplerian motion in the same way that the particles comprising the ring of Saturn revolve around the planet. In each of the papers detailed line profiles are not computed because the radiative transfer problem in the ring is not considered. Instead what is done is to calculate the broadening function $B(t)$ which relates the observed line profile $F_{\text{obs}}(\Delta\lambda)$ to $F(\Delta\lambda)$, the line profile in an individual radiating element of the system seen in a frame at rest with respect to the local macroscopic velocity of the matter. Apart from possible normalization factors, the relationship between $F_{\text{obs}}(\Delta\lambda)$ and $F(\Delta\lambda)$ is given by

$$F_{\text{obs}}(\Delta\lambda)=\int_{-\infty}^{+\infty} B(t)F(\Delta\lambda - t)\,\mathrm{d}t. \tag{4}$$

In the situation that the intrinsic line profile is extremely narrow and/or the Doppler broadening large, the observed line profile is then approximately given by the broadening function. In practice, the observed profile will be wider than the broadening function due to other broadening mechanisms such as thermal motions.

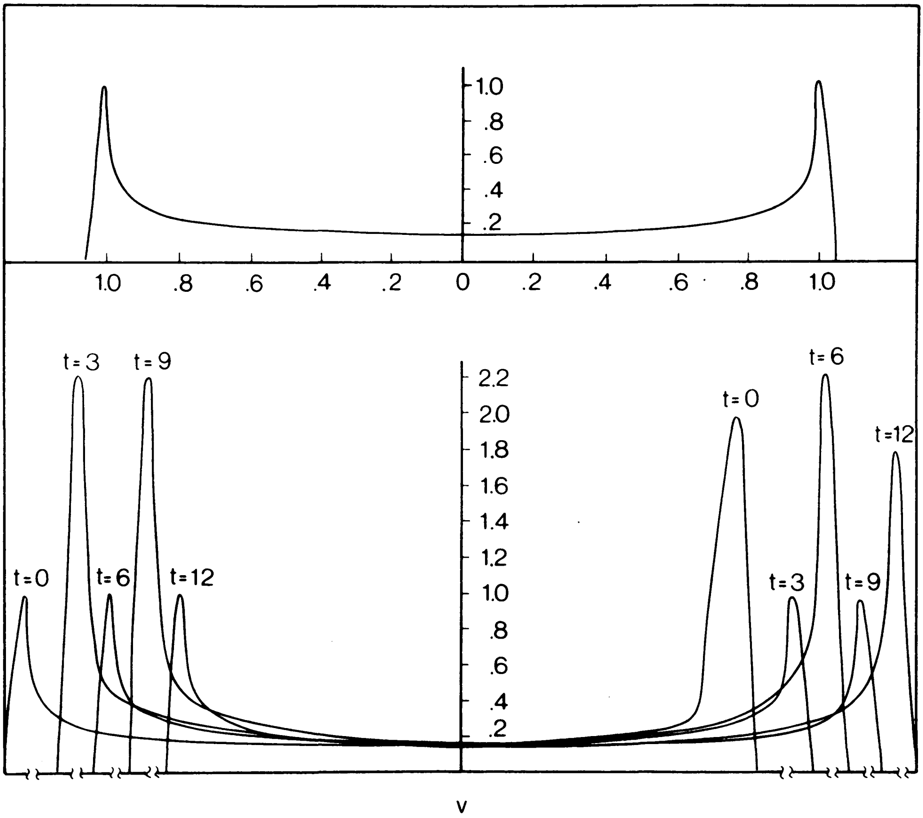


Fig. 19. The profile of the broadening function normalized so that its maximum value is unity for a circular ring for an axisymmetric and a non-axisymmetric distribution of matter. Abscissa scale is the radial velocity expressed in units of the Keplerian orbital speed of a particle on a reference circle taken to be the outer boundary of the circular ring and is the same for both profiles. The top profile is for the axisymmetric distribution; the bottom profiles are for a non-axisymmetric distribution given by Equation (5) for $\rho_1 = 2\rho_0$ normalized in such a way that the peak emission is unity when $\Delta\rho = 0$ (Huang, 1972).

In the first paper Huang (1972) computed the broadening function for a circular ring for both a symmetric and an asymmetric distribution of atoms in the ring. Due to differential revolution the asymmetric distribution predicts profiles which change continuously with time, the time scale being of the order of the revolution period of an atom in the ring, i.e. hours or days. A typical result for the simple case

$$\rho(r, \phi, t) = \rho_0(r) + \Delta\rho(r, \phi, t)\,, \tag{5}$$

with

$$\begin{aligned} \Delta\rho(r, \phi, 0) &= \rho_1 = \text{constant}, \qquad r_1 \le r \le r_2, \quad 0 \le \phi \le \alpha \\ &= 0 \text{ elsewhere}\,, \end{aligned}$$

is shown in Figure 19.

In the second paper Huang (1973) considers an elliptical ring. The essence of this model is illustrated schematically in Figure 20. The major result of the model is that the revolution of the major axis of the elliptical ring, assumed to arise due to the oblateness of the central star (Johnson, 1958), produces a V/R variation. In Figure 21 is shown the change of the broadening function for an elliptical ring of eccentricity 0.4 and width 0.093 in units of the semi-major axis of the outer edge. Note

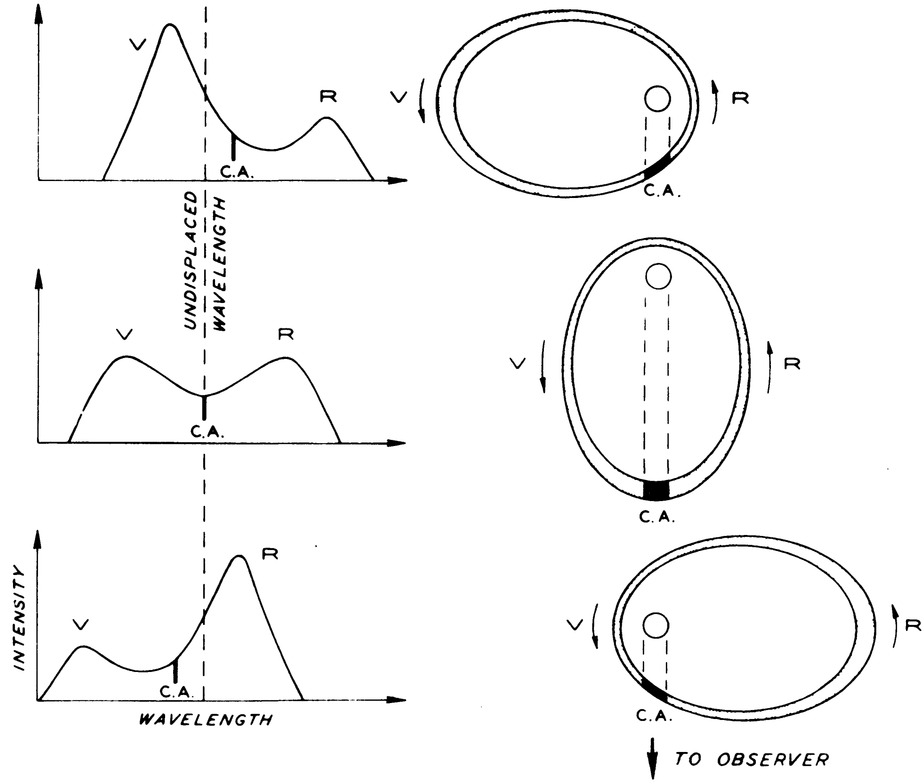


Fig. 20. Schematic representation of the elliptical ring model showing V/R variation (Huang, 1975).

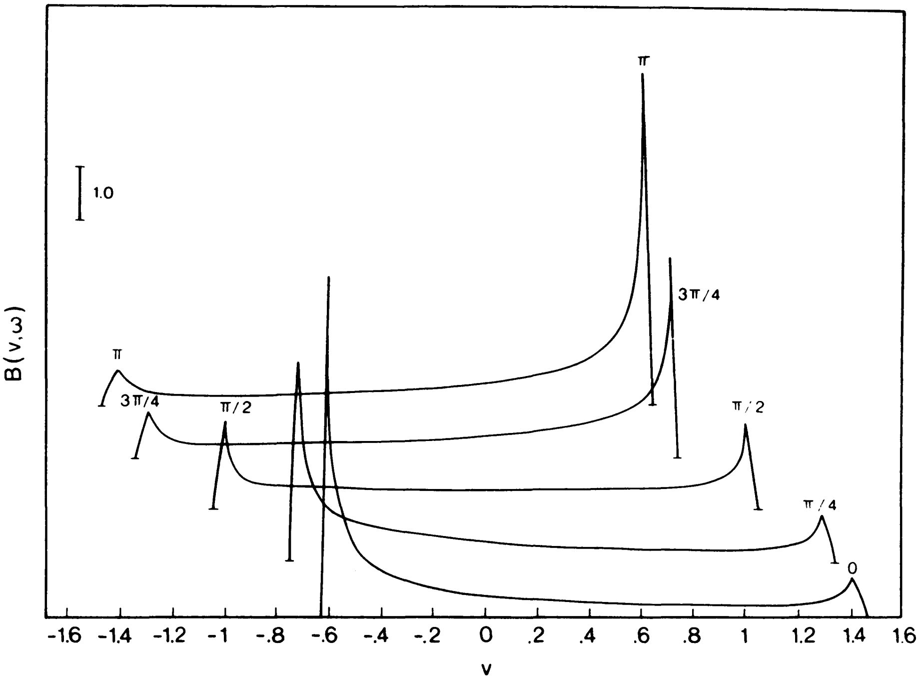


Fig. 21. The profile of the broadening function for an elliptical ring. Abscissa scale is the radial velocity in units of the Keplerian orbital velocity of a particle in an elliptical orbit at the outer boundary of the ring. The profiles are for a ring of eccentricity 0.4 and $a_1 = 1$, $a_2 = 0.907$ where a_1 and a_2 are the semi-major axes of the outer and inner edges, respectively. Each profile is labelled by ω, the longitude of periastron. Only the cycle for $\omega = 0$ to $\omega = \pi$ is shown, the other half being given by the condition $B(v, \omega) = B(-v, \pi \pm \omega)$. If the elliptical ring is undergoing apsidal motion, ω measures time. Note that the profile changes shape as well as position with time (Huang, 1973).

specifically that during the cycle, the shape of the line, the position of the line and the V/R ratio all change. A comparison of McLaughlin's observations (1966) of the V/R ratio and the radial velocities of the red and violet emission edges and the central absorption for the Be star 105 Tau (B2Vp) and the predicted variations for a narrow elliptical ring with eccentricity 0.2 and a revolution period for the major axis of 11.5 yr is illustrated in Figure 22. Albert and Huang (1974) have been able to account for similar observational data for the Be stars HD 20336 (B2Ve), 25 Ori (B1Vpe) and β^1 Mon (B3Ve). Satisfactory agreement with the predictions of the elliptical ring model is obtained. In each case the semi-major axis of the outer edge of the ring is about 3–4 R_* and $0.2 \lesssim e \lesssim 0.3$, where e is the eccentricity.

Despite this satisfactory agreement between predictions of the elliptical ring model and the observational data for the stars studied, the model has some drawbacks. The ring must be narrow, according to Huang (1973), i.e. $(a_2 - a_1) \ll a_2$, where a_2 and a_1 are the semi-major axes of the outer and inner edges, respectively, in order to preserve the elliptical shape for long times to explain the V/R variation. However, it is difficult to see how a narrow ring confined chiefly to the equatorial plane of the star can produce the central absorption features seen in Be stars. For example, for $i \approx 90°$, which is thought to correspond to the case of shell stars, it seems

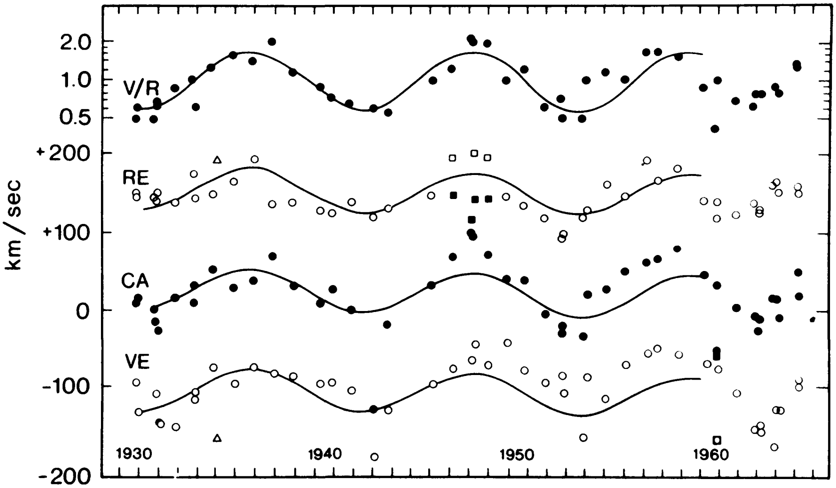


Fig. 22. Comparison of observations and predictions of the elliptical ring model for 105 Tau. Upper curve is V/R variation on a log scale. Lower curves are radial velocity of red emission edge (RE), central absorption (CA), and violet emission edge (VE). For the lower three curves: open circles are emission edges (mean of Hγ and Hβ); filled circles are central absorption; open squares are Hγ emission edges; filled squares are Hβ emission edges; triangles are Hα emission edges. The solid curves are derived from a narrow elliptical ring with eccentricity 0.2, assumed period of V/R variation of 11.5 yr, and velocity amplitude of 132 km s^{-1} (Huang, 1973).

unlikely that the elliptical ring can produce the deep metallic shell lines. The well known shell star 48 Lib (B3p) has shown conspicuous V/R variations and the self reversals have approached zero intensity (see Underhill, 1960, p. 145 and Figure 2). It would therefore appear that in order for the elliptical ring model to account for the observations of 48 Lib a large absorbing region, presumably lying outside the elliptical ring and extending to at least one stellar radius from the equatorial plane, would have to be added. Where this matter comes from and how it is related to Huang's picture of the formation and evolution of the ring is unclear. Perhaps it represents the remains from the dissipation of an earlier ring or rings.

For smaller values of the angle i, other difficulties arise. Let us assume that the central absorption in the hydrogen lines occurs due to the fact that the ring is seen projected against the stellar disk in the observer's line of sight. For a spherical star of radius R_* and an elliptical ring of eccentricity e, semi-major axis of the inner edge a_1 and situated in the equatorial plane, the minimum angle, $i_{\min}$, for which the observer's line of sight is tangent to both the inner edge of the ring at periastron and the star's surface is given by

$$i_{\min} = \cos^{-1}\left[\frac{R_*}{a_1(1-e)}\right]. \tag{6}$$

If the ring extends to a height h perpendicular to the equatorial plane, then $i_{\min}$ for which the observer's line of sight is tangent to both the inner edge of the ring at height h at periastron and the star's surface is related to h, a, and e by

$$(h/R_*) \sin i_{\min} = [a_1(1-e)/R_*] \cos (i_{\min}) - 1\,. \tag{7}$$

In Table I are the data pertaining to the elliptical ring model for the stars discussed by Albert and Huang (1974). Columns 2 and 3 are the eccentricity e and semi-major axis a_1 for the inner edge of the ring, in units of R_*, obtained from fitting the data. Column 4 is the inclination i deduced by Huang from $v \sin i$ and the equatorial rotation speed of the star. Column 5 is the angle $i_{\min}$ from Equation (6) determined for each star using the values of e and a_1/R_* in columns 2 and 3, respectively, for a ring in the equatorial plane. It is only for angles $i > i_{\min}$ that the specific ring considered would be projected against the stellar disk in the observer's line of sight at periastron. In column 6 are the values of the extent of the ring above the equatorial plane, in units of R_*, if the inner edge of the ring is to be projected against the stellar disk at periastron for an observer for whom the angle of inclination i is given by the values in column 4. Again, the numbers in column 6 represent the minimum extent of the ring from the equatorial plane for given a_1/R_*, e, and i if the central absorption is due to the ring seen projected against the stellar disk.

TABLE I

Elliptical ring model

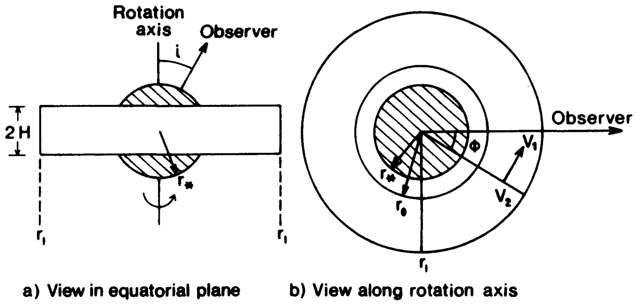
Star	e	a_1/R_*	i	$i_{\min}$	h/R_*
105 Tau	0.20	3.4	26°	68°	3.3
HD 20336	0.25	3.2	40°	65°	1.3
25 Ori	0.26	3.9	35°	70°	2.4
β^1 Mon	0.32	3.8	43°	67°	1.3

A comparison of columns 4 and 5 leads to the immediate conclusion that it is geometrically impossible for the rings considered to be projected against the stars for the values of i deduced by Huang if the ring lies predominantly in the equatorial plane. The star 105 Tau possesses a conspicuous absorption in Hγ and presumably Hβ as well (see Burbidge and Burbidge, 1953, Figure 12). Of course, as Huang (1973) noted, considerable uncertainty exists in the values of i in column 4. Nevertheless the discrepancy between columns 4 and 5 is disturbing. If the ring is not confined to the equatorial plane, then it must extend to ~1–3 R_* for the stars considered if Huang's values of i are approximately correct. It is not immediately apparent how such a structure could survive for a period of time long enough to produce several cycles of the observed V/R variation. If the central absorption does not arise from direct absorption of stellar radiation then the above arguments are not applicable. The central absorption in the elliptical ring model would then arise from self absorption along the line of sight through the ring. The limited geometrical extent assumed for the ring by Huang makes this possibility rather unlikely. In either case, it would be useful in assessing the relevance of the elliptical ring model if some line profile calculations based on this general model were available in order to determine whether or not the above discussion represents a major source of difficulty.

3.4. DISK AND DISK-LIKE MODELS

A variety of disk and/or disk-like models have been considered. In most of these models the circumstellar matter is assumed to lie within two coaxial cylinders whose axis coincides with the star's rotation axis, the extent of the cylinder along the rotation axis and the inner and outer radii being free parameters. One of the earliest disk-like or lenticular models was employed by Burbidge and Burbidge (1953) in their analysis of the outer atmospheres of some Be stars.

Kogure (1969) has discussed a disk model to interpret the observations of several pole-on Be stars. The model is illustrated schematically in Figure 23. The envelope was assumed to be isothermal, V_2 and V_3 were assumed to be zero where V_2 is



a) View in equatorial plane b) View along rotation axis

Fig. 23. Schematic representation for a Be star envelope (Kogure, 1971).

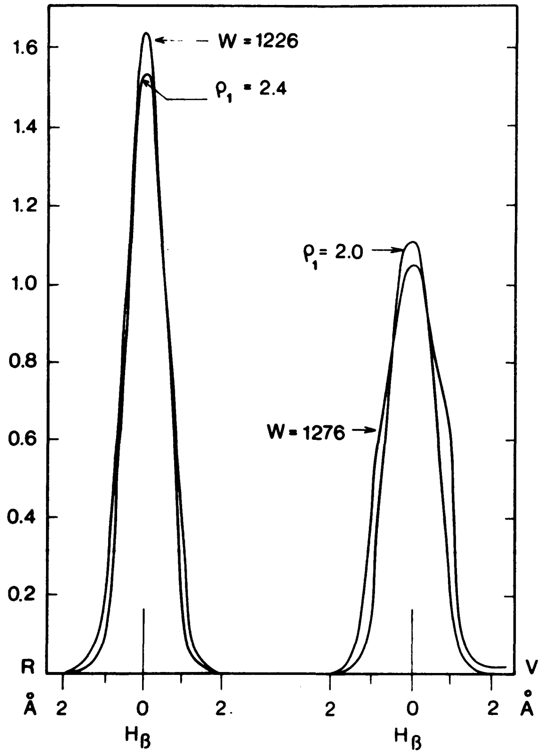


Fig. 24. Observed and theoretical Hβ profiles for HD 58343. ρ_1 is the ratio of the outer radius of the envelope to the stellar radius. Ordinate same as Figure 3 (Kogure, 1971).

$u_r(r, \theta = \pi/2)$ in the above notation, V_3 represents the velocity component perpendicular to the equatorial plane and $V_1 \propto r^{-1}$ where V_1 is $u_\phi(r, \theta = \pi/2)$. The radiative transfer solution used is that based on the work of Miyamoto and Kogure for a static plane parallel atmosphere for which the optical depth in the Lyman continuum is taken to be approximately unity and for which a variety of assumptions are employed for optical depths in other continua and discrete lines (see Kogure, 1967, and references contained therein for details). The Eddington approximation is used in the solution for the radiation field. Caution should be used in applying radiative transfer solutions for static plane parallel models to physical situations where the extent of the atmosphere is large so that curvature terms in the transfer equation are important and where velocity gradients exist (see Böhm, 1973; and Hummer's article in this volume, p. 281). In the case being considered where the model is applied to pole-on stars, the differences between the static and dynamic atmosphere predictions may not be too great because the velocity gradient for small angles i may actually be small and the effective emitting and absorbing part of the extended atmosphere may be strongly concentrated to the equatorial plane. For all cases the total envelope extent parallel to the rotation axis was assumed to be one stellar radius. The predicted and observed Hβ profiles for the stars HD 58343 (B3Ve) and HD 212076 (B2Ve) are shown in Figures 24 and 25, respectively. The model satisfactorily reproduces the observed Hβ profiles.

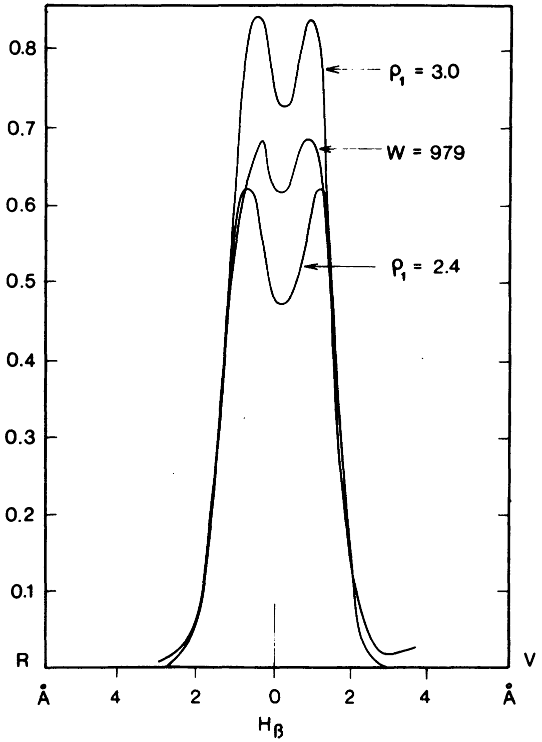


Fig. 25. Observed and theoretical Hβ profile for HD 212076. ρ_1 is same as Figure 24 and ordinate same as Figure 3 (Kogure, 1971).

Observations of the continuous infrared emission for Be stars have been published by various authors (see Gehrz *et al.*, 1974, and Swings' article in this volume, p. 219). For none of the models discussed in Sections 3.1, 3.2 and 3.3 have predictions been made concerning the expected infrared continuum radiation. Predicted infrared emission has been made for thin disk models for which all physical parameters are treated as constants for simplicity. The infrared observations of ϕ Per (B1III–Vpe) and βCMi (B7Ve) are compared with predictions for a free-free continuum in Figures 26 and 27, respectively. There does not appear to be any serious discrepancy in accounting for the infrared flux as arising from free-free emission in a disk-like circumstellar envelope.

Disk or disk-like models have also been employed in connection with the observations of linear polarization in Be stars. For a discussion of the observations see the article by Coyne in this volume, p. 233. To explain the polarization observations of ζ Tau (B2IVep), Capps *et al.* (1973) considered a homogeneous isothermal thin disk of completely ionized hydrogen illuminated by a point source star and included both free-bound and free-free radiation from the disk. The

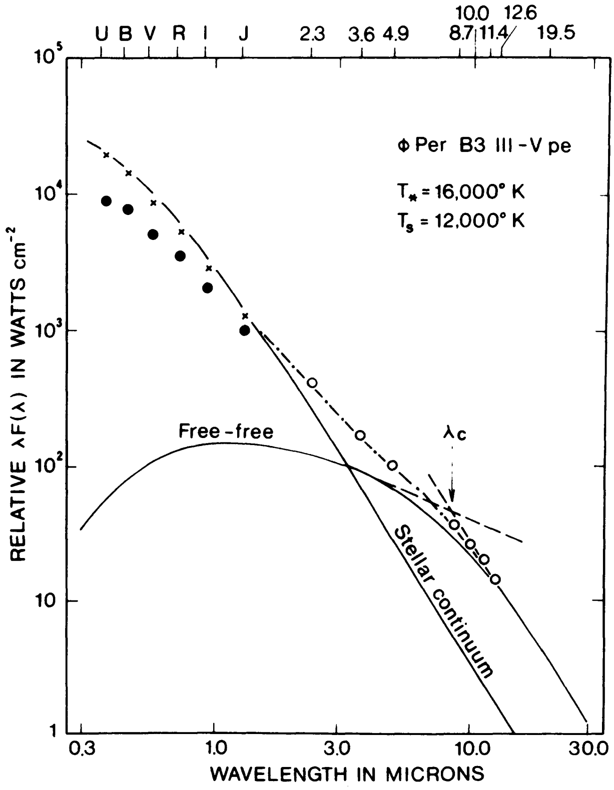


Fig. 26. Observed and theoretical infrared emission for ϕ Per for a free-free shell which is optically thick at large λ. Filled circles are *UBVRIJ* data and crosses represent reddening corrections. Open circles are infrared measures of Gehrz *et al.* λ_c is wavelength at which $\tau(\lambda_c) \equiv 1$ for intersection of optically thin and optically thick segments of shell spectrum (Gehrz *et al.*, 1974).

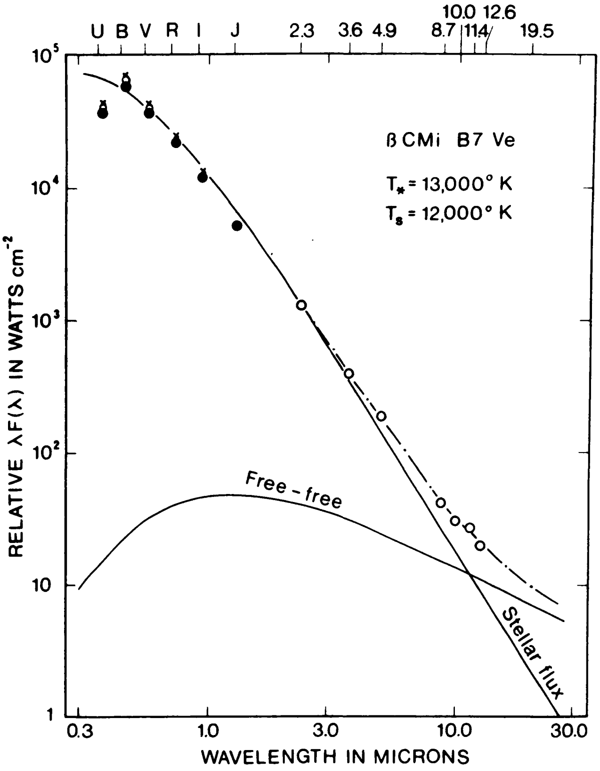


Fig. 27. Same as Figure 26 for β CMi for an optically thin free-free shell (Gehrz *et al.*, 1974).

observations and predicted polarization are shown in Figure 28. More elaborate calculations have recently been performed by Haisch (1975, and the paper in this volume, p. 375). These involve a more rigorous treatment of the radiative transfer problem for more realistic models for the extended atmosphere than those provided by isothermal homogeneous disks. R. Poeckert is presently considering the predictions of linear polarization for the stellar wind model considered by Marlborough (Section 3.1) in connection with detailed linear polarization observations as a function of wavelength for the Be Stars EW Lac (B2pe) and γ Cas (B0IVe).

3.5. OTHER MODELS

Numerous other models have been considered. In general, however, they have either not been investigated quantitatively or have not been subjected to as detailed a comparison with observations as have those considered above.

Nariai (1970) has proposed a qualitative model which combines circulatory motion of gas in the extended envelope together with precession of the primary star and applied this model to the shell star ζ Tau. In a meridional plane matter in the circumstellar envelope moves outward in and near the equatorial plane and returns to the vicinity of the stellar surface from above and below the equatorial plane.

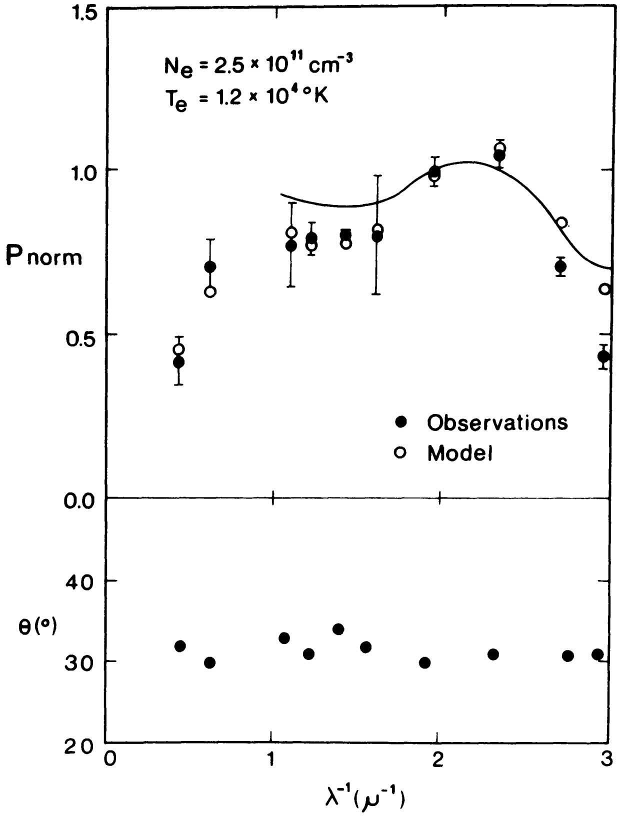


Fig. 28. Comparison of observed polarization for ζ Tau normalized so that $P = 1$ at $\lambda^{-1} = 2\mu^{-1}$ and theoretical predictions for a homogeneous isothermal completely ionized disk. Solid curve is mean relation for high latitude Be stars (Capps *et al.*, 1973).

Because of precession, the observer sometimes sees the outward moving part of the circulation pattern while at other times he sees the inward motion. This model does not appear to have been investigated further.

Hazlehurst (1967) suggested a simple hydromagnetic model to account for the loss of matter from the circumstellar envelopes of Be stars and applied it to Pleione's shell phase from 1938 to 1954. In this model the most important effect of the magnetic field is to transfer angular momentum outward from the star to the shell, ultimately allowing the shell material to escape. The model specifically predicts that $u_r(r, \theta = \pi/2)$ should be a decreasing function of r. However, Limber's study of Pleione (Section 3.2) has conclusively demonstrated that no agreement with observations could be obtained for cases in which $u_r(r, \theta = \pi/2)$ decreases with increasing r. It would therefore appear that Hazlehurst's model is not relevant to the Be star problem.

Henriksen (1969) has attempted to explain Pleione's behavior with a hydromagnetic model initially suggested by Crampin and Hoyle (1960). In this picture the radial component of the magnetic field plays a dominant role. Differential rotation of the matter in Keplerian orbits near the star leads to the production of an azimuthal component of the magnetic field. The resultant amplification of this component ultimately leads to large radial forces acting and subsequent explosive dissipation of the circumstellar ring. This model predicts an increase of $u_r(r, \theta = \pi/2)$ with r and is qualitatively similar with Merrill's observations (1952) for Pleione. Beginning with the outer part of the envelope successive rings of material are dissipated by this process until the entire shell has disappeared. This model has some interesting features in that it does contain an explanation for the dissipation of the shell. For this reason it would be useful to compute line profiles for several upper Balmer lines to compare with observations. One must keep in mind, however, that this model is not complete. As mentioned in Section 3.2, the entire envelope could not have disappeared about 1951 since emission lines were still visible at that time and also later on.

3.6. Binary star models

All the models discussed to this point have explicitly or implicitly assumed that the Be star phenomenon involves a single star. If a specific Be star happens to be a member of a binary system, the companion may exert considerable influence upon the dynamics and state of ionization and excitation of the circumstellar envelope, but it is not assumed to be the origin of the circumstellar envelope around the Be star.

On the other hand, various authors have advanced the suggestion that some Be stars are members of binary systems in which the circumstellar envelope arises due to mass transfer from a companion filling its inner Lagrangian surface. In particular, Kriz and Harmanec (1975) have tentatively suggested that all Be stars are members of mass-exchanging binary systems. A schematic picture of this process is shown in Figure 29.

Over the past few years binary stars have again become a subject of considerable interest. This renewed interest arose primarily because of the discovery of x-ray sources in binary systems and also because of the possible discovery of black holes. A variety of disk or disk-like models are available for the disk surrounding the accreting star. Most of these disks, however, do not necessarily represent rigorous solutions of the basic equations. The hydrodynamic problem for the transfer of matter in binary systems is complicated. Prendergast and Taam (1974) have investigated a numerical solution of the gas flow in a semi-detached binary system to account for the observations of the binary system U Cep. Lubow and Shu (1975) have recently concluded a detailed study of the gas dynamics in semi-detached binaries. Neither of these studies however presents detailed models for the disk-like region around the accreting star so that a detailed comparison of observations and theoretical predictions can be made. An initial attempt to study line profiles in a binary system undergoing mass exchange is due to Sima (1973).

Qualitative models involving disks and gas streams have been advanced to account for the observations of several Be stars. Many of these are discussed by Plavec (1973) and by Plavec and Harmanec in this volume and for this reason will not be repeated here.

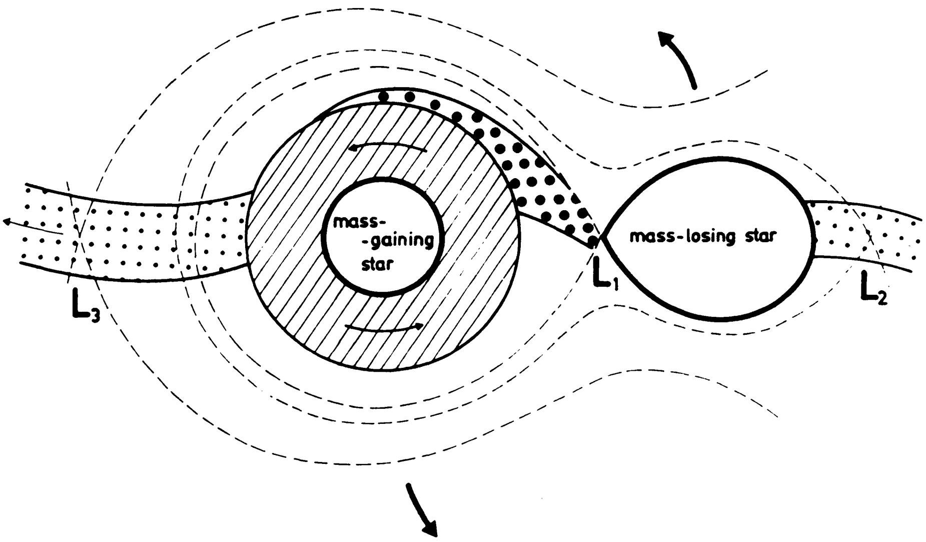


Fig. 29. A schematic picture of the distribution of circumstellar matter in a close binary. The dashed lines represent Roche equipotentials. The region with large dots represents matter flowing from L_1 to the disk (hatched area). The regions with small dots near L_2 and L_3 represent possible escape of matter from the system (Kriz and Harmanec, 1975).

4. Conclusions

In the above discussion we have considered the general problem for the solution of the basic equations pertaining to the structure of Be star envelopes and described some of the attempts to solve restricted problems as rigorously as possible. The observations obtained in the far ultraviolet from rockets and satellites provide some support for stellar wind solutions but do not provide a preference for the radiation driven solutions compared to those involving a magnetic field. The required magnetic field strengths are not beyond experimental detection especially if they lie toward the upper end of the expected range and future measurements can be expected to provide direct evidence for or against the hydromagnetic models.

The various *ad hoc* models account satisfactorily for some observations. The far ultraviolet observations appear to provide some support for the *ad hoc* stellar wind models, particularly those for which $u_r(r, \theta = \pi/2)$ is an increasing function of r. However it is difficult to see how the stellar wind models in their present form can account for the V/R variation, particularly when $V > R$. On the other hand the elliptical ring model can satisfactorily account for the V/R variation, but it may encounter difficulties in matching line profiles, especially the absorption components.

Clearly what is needed is to subject the predictions of each of the models to more observational tests. For example the stellar wind models should be used to predict the infrared continua, linear polarization as a function of wavelength, etc. Likewise the elliptical ring model should be employed to predict line profiles, linear polarization, etc., in order to see whether or not one or both approaches is completely inadequate.

At the same time, many aspects of these *ad hoc* models can be improved. One immediate step is a more realistic treatment of the radiative transfer problem. An important step in this direction has been initiated by Kriz (1974), and the paper in this volume. It would also be useful to include better representations of $\rho(\mathbf{r})$, $\mathbf{u}(\mathbf{r})$ and $T(\mathbf{r})$, perhaps using those obtained from solutions described in Section 2. In this way the number of adjustable parameters characterizing these models might be reduced.

Finally the reader is wise if he does not take any of these models too seriously. Nature is undoubtedly much more subtle that we can imagine but also, perhaps, simpler than we think.

Acknowledgements

I wish to thank R. L. Poeckert for his comments on the manuscript and J. D. Landstreet for the discussion relating to the measurement of the magnetic field in B stars. The work on this paper was supported by the National Research Council of Canada.

References

Albert, E. and Huang, S. S.: 1974, *Astrophys. J.* **189**, 479.

Bahng, J. D. R.: 1971, *Astrophys. J. Letters* **168**, L75.

Baker, J. B. and Menzel, D. H.: 1938, *Astrophys. J.* **88**, 52.

Beals, C. S.: 1930, *Publ. Dominion Astrophys. Obs.* **4**, 294 and 297.

Biermann, P. and Kippenhahn, R.: 1973, *Astron. Astrophys.* **25**, 63.

Böhm, K. H.: 1973, in A. H. Batten (ed.), 'Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems', *IAU Symp.* **51**, 148.

Burbidge, G. R. and Burbidge, E. M.: 1953, *Astrophys. J.* **117**, 407.

Burd, S.: 1954, *Publ. Astron. Soc. Pacific* **66**, 208.

Capps, R. W., Coyne, G. V., and Dyck, H. M.: 1973, *Astrophys. J.* **184**, 173.

Cassinelli, J. P. and Castor, J. I.: 1973, *Astrophys. J.* **179**, 189.

Castor, J. I.: 1972, *Astrophys. J.* **178**, 779.

Castor, J. I., Abbott, D. C., and Klein, R. I.: 1975, *Astrophys. J.* **195**, 157.

Chandrasekhar, S.: 1934, *Monthly Notices Roy. Astron. Soc.* **94**, 522.

Cox, J. P. and Giuli, R. T.: 1968, *Principles of Stellar Structure*, Vol. 1, Gordon and Breach, New York, Ch. 9.

Crampin, J. and Hoyle, F.: 1960, *Monthly Notices Roy. Astron. Soc.* **120**, 33.

Delplace, A. M., Herman, R., and Peton, A.: 1969, in L. Detre (ed.), *Non-Periodic Phenomena in Variable Stars*, D. Reidel, Dordrecht-Holland, p. 223.

Doazan, V.: 1965, *Ann. Astrophys.* **28**, 1.

Fernie, J. D.: 1975, *Astrophys. J.* **201**, 179.

Gehrz, R. D., Hackwell, J. A., and Jones, T. W.: 1974, *Astrophys. J.* **191**, 675.

Gerasimovic, B. P.: 1934, *Monthly Notices Roy. Astron. Soc.* **94**, 737.

Gray, D. F. and Marlborough, J. M.: 1974, *Astrophys. J. Suppl.* **27**, 121.

Haisch, B. M.: 1975, Ph.D. Thesis, Univ. Of Wisconsin, Madison (unpublished).

Hazlehurst, J.: 1967, *Z. Astrophys.* **65**, 311.

Heap, S. R.: 1975, *Phil. Trans. Roy. Soc. London* **279**, 371.

Henriksen, R. N.: 1969, *Astron. Astrophys.* **1**, 457.

Huang, S. S.: 1972, *Astrophys. J.* **171**, 549.

Huang, S. S.: 1973, *Astrophys. J.* **183**, 541.

Huang, S. S.: 1973, in A. H. Batten (ed.), 'Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems', *IAU Symp.* **51**, 22.

Huang, S. S.: 1975, *Sky Telesc.* **49**, 359.
Hutchings, J. B.: 1968, *Monthly Notices Roy. Astron. Soc.* **141**, 329.
Hutchings, J. B.: 1970a, *Monthly Notices Roy Astron. Soc.* **150**, 55.
Hutchings, J. B.: 1970b, in A. Slettebak (ed.), *Stellar Rotation*, Gordon and Breach, New York, p. 283.
Hutchings, J. B.: 1971, *Monthly Notices Roy. Astron. Soc.* **152**, 109.
Johnson, M.: 1958, in *Etoiles a raies d'emission*, Institute d'Astrophysique, Liège, p. 219.
Kogure, T.: 1967, *Publ. Astron. Soc. Japan* **19**, 30.
Kogure, T.: 1969, *Astron. Astrophys.* **1**, 253.
Kriz, S.: 1974, *Bull. Astron. Inst. Czech.* **25**, 143.
Kriz, S. and Harmanec, P.: 1975, *Bull. Astron. Inst. Czech*, **26**, 65.
Kupo, I. D.: 1971, *Alma Ata Publications* **16**, 165.
Landstreet, J. D.: 1975, private communication.
Lester, D. F.: 1975, *Publ. Astron. Soc. Pacific* **87**, 177.
Limber, D. N.: 1964, *Astrophys. J.* **140**, 1391.
Limber, D. N.: 1967, *Astrophys. J.* **148**, 141.
Limber, D. N.: 1969, *Astrophys. J.* **157**, 785.
Limber, D. N.: 1970, in A. Slettebak (ed.), *Stellar Rotation*, Gordon and Breach, New York, p. 274.
Limber, D. N.: 1974, *Astrophys. J.* **192**, 429.
Limber, D. N. and Marlborough, J. M.: 1968, *Astrophys. J.* **152**, 181.
Lubow, S. H. and Shu, F. H.: 1975, *Astrophys. J.* **198**, 383.
McLaughlin, D. B.: 1933, *Proc. Nat. Ac. Sci.* **19**, 44.
McLaughlin, D. B.: 1938, *Pop. Astron.* **46**, 361.
McLaughlin, D. B.: 1938, *Astrophys. J.* **88**, 622.
McLaughlin, D. B.: 1961, *J. Roy. Astron. Soc. Can.* **55**, 13 and 73.
McLaughlin, D. B.: 1966, *Astrophys. J.* **143**, 285.
Magnan, C.: 1970, *J. Quant. Spectrosc. Radiat. Transfer* **10**, 1.
Magnan, C.: 1972, *Astron. Astrophys.* **21**, 361.
Marlborough, J. M.: 1969, *Astrophys. J.* **156**, 135.
Marlborough, J. M.: 1970, *Astrophys. J.* **159**, 575.
Marlborough, J. M.: 1971, *Astrophys. J.* **163**, 525.
Marlborough, J. M. and Roy, J. R.: 1971, *Astrophys. J.* **169**, 327.
Marlborough, J. M. and Gredley, P. R.: 1972, *Astrophys. J.* **178**, 477.
Marlborough, J. M. and Cowley, A. P.: 1974, *Astrophys. J.* **187**, 99.
Marlborough, J. M. and Zamir, M.: 1975, *Astrophys. J.* **195**, 145.
Massa, D.: 1975, *Publ. Astron. Soc. Pacific* **87**, 777.
Merrill, P. W.: 1952, *Astrophys. J.* **115**, 145.
Merrill, P. W., Humason, M. and Burwell, C. G.: 1925, *Astrophys. J.* **61**, 389.
Mohler, O. C.: 1938, *Astrophys. J.* **88**, 623.
Morgan, T. H.: 1975a, *Astrophys. J.* **195**, 391.
Morgan, T. H.: 1975b, *Astrophys. Space Sci.* **33**, 99.
Nariai, K.: 1970, *Publ. Astron. Soc. Japan* **22**, 313.
Nordh, H. L. and Olofsson, S. G.: 1974, *Astron. Astrophys.* **31**, 343.
Plavec, M.: 1973, in A. H. Batten (ed.). 'Extended Atmospheres and Circumstellar Matter in Spectroscopic Binary Systems', *IAU Symp.* **51**, 216.
Pomraning, G. C.: 1973, *The Equations of Radiation Hydrodynamics*, Pergamon, Oxford.
Praderie, F., Simonneau, E. and Snow, T. P.: 1975, *Bull. Am. Astron. Soc.* **7**, 359.
Prendergast, K. H. and Taam, R. E.: 1974, *Astrophys. J.* **189**, 125.
Rosseland, S.: 1926, *Astrophys. J.* **63**, 218.
Rybicki, G. B.: 1970, in H. G. Groth and P. Wellmann (eds.), *Spectrum Formation in Stars with Steady-State Extended Atmospheres*, NBS Spec. Publ. 332, p. 87.
Saito, M.: 1974, *Publ. Astron. Soc. Japan* **26**, 103.
Sampson, D. H.: 1965, *Radiative Contributions to Energy and Momentum Transport in a Gas*, Interscience, New York, Ch. 3 and App. B.
Secchi, A.: 1867, *Astron. Nachr.* **68**, 63.
Sima, Z.: 1973, *Astrophys. Space Sci.* **24**, 421.
Slettebak, A.: 1954, *Astrophys. J.* **119**, 460.
Smith, R. C.: 1970, *Monthly Notices Roy. Astron. Soc.* **148**, 275.
Sobolev, V. V.: 1960, *Moving Envelopes of Stars* (Engl. Transl.), Harvard Univ. Press, Cambridge.
Struve, O.: 1931, *Astrophys. J.* **73**, 94.
Struve, O.: 1942, *Astrophys. J.* **95**, 134.
Struve, O.: 1950, *Stellar Evolution*, Princeton Univ. Press, Princeton, p. ix.

Struve, O. and Swings, P.: 1932, *Astrophys. J.* **75**, 161.

Thomas, R. N.: 1970, in H. G. Groth and P. Wellman (eds.), *Spectrum Formation in Stars with Steady-State Extended Atmospheres*, NBS Spec. Publ. 332, p. 259.

Trasco, J. D., Wood, H. J., and Roberts, M. S.: 1970, *Astrophys. J. Letters* **161**, L129.

Underhill, A. B.: 1960, in J. L. Greenstein (ed.), *Stellar Atmospheres*, Univ. of Chicago Press, Chicago, Ch. 10.

Weber, E. J. and Davis, L. D.: 1967, *Astrophys. J.* **148**, 217.

Weidelt, R. D.: 1970, *Astrophys. Space Sci.* **6**, 205.

Zel'dovich, Ya. B. and Raizer, Yu. P.: 1966, *Physics of Shock Waves and High Temperature Hydrodynamic Phenomena*, Academic Press, New York, Ch. 1.

DISCUSSION

Slettebak: You discussed radiation driven stellar wind models (for example, Hutchings' calculations for γ Cas) in which gravity darkening was taken into account and an equatorial temperature of 12 000° was predicted. But what about a late B-type shell star like Pleione: a B8 star? When you include gravity darkening surely the equator must be very cool. Do you have enough radiation then to drive material into the shell?

Marlborough: I don't really know whether one could account for the phenomenon in Pleione or not, in terms of a radiation driven stellar wind. Perhaps radiation from the hotter, polar regions might be sufficient to generate a stellar wind from the outer regions of a disk in the equatorial plane. However, the problem remains as to how to form the disk or extended atmosphere.

Heap: What observations lead you to conclude that the shell surrounding a Be star is concentrated toward the equatorial plane?

Marlborough: I would argue that the distribution of electrons (linear polarization is assumed to arise from electron scattering) could not be spherically symmetric and produce a non-zero intrinsic linear polarization, so there has to be flattening in some sense. But I think that is the only strong evidence we have concerning the overall structure of the circumstellar envelope. In the case of shell stars the absorbing matter must extend to at least one stellar radius from the equatorial plane to account for the very low flux in the cores of strong shell lines.

Hutchings: I would like to make a comment which essentially agrees with what you have just said. I have computed a number of models of the type you have described, *ad hoc* models with quite a range in geometries, in order to see if one could find a dependence on i or on the geometries, or any other rather obvious parameters. It turned out, as far as I could make out, that nothing was very sensitive to anything. You could have very nearly spherically symmetrical envelopes and still reproduce profiles fairly satisfactorily.

Marlborough: If you consider only one piece of observational data like one line profile, for example, then generally these *ad hoc* models can account for the observational evidence considered because the models contain many free parameters. You can always adjust the parameters within realistic bounds to reproduce one line profile. If one considers several line profiles, one may be able to restrict the models a little more, but this has not yet been done.

Doazan: How do you interpret the change in the observed Balmer progression as a function of the velocity law?

Marlborough: I do not know how at the present time.

Delplace: How can you explain the variation of the period of the radical velocities which is observed in some shell stars like ζ Tau and 48 Lib?

Marlborough: In this type of picture, i.e. a stellar wind model, I cannot. This is one difficulty with any of these *ad hoc* models: they explain some observations but there are many things they cannot explain. And as all the observations which have been presented over the past few days indicate, the amount of data that the models can explain relative to what they cannot explain seems to be smaller and smaller. Maybe we should be very pessimistic, throw out all these models, and suggest that there be no new theories as far as Be stars are concerned for 10 years. Or alternatively, no new observations for 10 years.

Peters: There are a number of 'equator-on' Be stars which show featureless or only slightly structured Hα profiles. These objects typically show Fe II emission and O I λ 7774 emission. In your opinion, what physical parameters and/or geometries characterize the envelopes of these objects?

Marlborough: In one of his review articles McLaughlin comments that Be stars with strong hydrogen emission generally have Fe II in emission also. In the context of stellar wind models, this fact suggests a higher density envelope. Perhaps O I λ 7774 emission can be explained in the same way. Qualitatively one might account for a featureless Hα by having a region of moderate density and small extent in the equatorial plane just above the surface of the star to explain the Hα, Fe II and O I emission. If the outer edge expanded very rapidly, the density in the wind might be low enough so no strong absorption component to Hα occurred.

Hummer: I would like to stress the necessity of testing models for a number of lines, covering all parts of the model. In order to do this, it is essential that observers obtain, as near to one time as possible, and publish profiles for many lines, including photospheric lines, for a few typical objects. Concentration on only one or two 'interesting' features in an object gives very little assistance in inferring its structure in any reasonably unique way.

Marlborough: I strongly support these comments.