25
Quantum chromodynamics

The primary evidence that hadrons are composed of a simpler substructure of *quarks* is the following:

- If one assumes the baryons are composed of quark triplets (qqq) and the mesons are quark–antiquark pairs ($q\bar{q}$) then, with appropriate quantum numbers for the quarks (flavors), one can describe and predict the observed supermultiplets of hadrons;
- The assumption of interaction with point-like quarks provides a marvelously simple and accurate description of electroweak currents;
- Dynamic evidence for a point-like quark–parton substructure of hadrons is obtained from deep-inelastic electron scattering $(\mathrm{e}, \mathrm{e}')$ and neutrino reactions $(\nu_\mathrm{l}, \mathrm{l}^-)$.

Quarks come in many *flavors*; the quark field can be written as

$$\psi = \begin{pmatrix} u \\ d \\ s \\ c \\ \vdots \end{pmatrix} \tag{25.1}$$

One assigns quarks an additional intrinsic degree of freedom called *color*, which takes three values $i = R, G, B$. The quark field then becomes (we focus here on the four lightest quarks)

$$\psi = \begin{pmatrix} u_R & u_G & u_B \\ d_R & d_G & d_B \\ s_R & s_G & s_B \\ c_R & c_G & c_B \end{pmatrix} = (\psi_R, \psi_G, \psi_B) \equiv \psi_i \qquad ; \, i = R, G, B \tag{25.2}$$

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It is convenient to construct a column vector from the color fields

$$\underline{\psi} \equiv \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix} \tag{25.3}$$

Matrices in this color space will be here denoted with a bar under a symbol. This is a very compact notation

- Each ψ_i has many flavors;
- Each flavor is a four-component Dirac field.

Quantum chromodynamics (QCD) is a theory of the strong interactions binding quarks into the observed hadrons. It is a Yang–Mills non-abelian gauge theory [Ya54]. It is built on the underlying color symmetry and invariance under local $SU(3)_C$.

The lagrangian density[1] for the free quark fields can be written compactly as

$$\mathscr{L} = -\underline{\bar{\psi}}\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + \underline{M}\right)\underline{\psi} \tag{25.4}$$

Here the mass term is the unit matrix with respect to color. It may be *anything* with respect to flavor, for example,

$$\underline{M} = \begin{pmatrix} m & & \\ & m & \\ & & m \end{pmatrix} \qquad m = \begin{pmatrix} m_u & & & \\ & m_d & & \\ & & m_s & \\ & & & m_c \end{pmatrix} \tag{25.5}$$

The lagrangian in Eq. (25.4) has a *global* invariance with respect to unitary transformations mixing the three internal color variables $[SU(3)_C]$. We denote the generators of this transformation by $\hat{G}^a$ with $a = 1, \ldots, 8$ and the eight parameters characterizing a three-by-three unitary, unimodular matrix by θ^a with $a = 1, \ldots, 8$. There are eight three-by-three, traceless, hermitian, Gell-Mann matrices $\underline{\lambda}_a$ — the analogs of the Pauli matrices. These matrices satisfy the Lie algebra of $SU(3)$, the same algebra as satisfied by the generators

$$[\frac{1}{2}\underline{\lambda}^a, \frac{1}{2}\underline{\lambda}^b] \quad = \quad if^{abc}\frac{1}{2}\underline{\lambda}^c \tag{25.6}$$

Here the f^{abc} are the structure constants of the group; they are antisymmetric in the indices (abc). The matrices $(\lambda^a)_{ij}$ for $a = 1, \ldots, 8$ are given in

[1] See [Fe80] for a background discussion of continuum mechanics and lagrangian densities, and [Bj65a, Fe71] for an introduction to quantum field theory.

order by

$$\begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix}\begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix}\begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix}\begin{pmatrix} & & 1 \\ & & \\ 1 & & \end{pmatrix}\begin{pmatrix} & & -i \\ & & \\ i & & \end{pmatrix}$$
$$\begin{pmatrix} & & \\ & & 1 \\ & 1 & \end{pmatrix}\begin{pmatrix} & & \\ & & -i \\ & i & \end{pmatrix}\begin{pmatrix} 1/\sqrt{3} & & \\ & 1/\sqrt{3} & \\ & & -2/\sqrt{3} \end{pmatrix} \tag{25.7}$$

The operator producing the finite color transformation is then given by

$$\hat{R} \;=\; e^{i\theta^a \hat{G}^a} \tag{25.8}$$

It has the following effect on the quark field

$$\hat{R}\underline{\psi}\hat{R}^{-1} \;=\; \underline{U}(\theta)\underline{\psi} \;=\; \left[e^{-\frac{i}{2}\underline{\lambda}^a\theta^a}\right]\underline{\psi} \tag{25.9}$$

Latin indices will now run from $1, \dots, 8$, and repeated Latin indices are summed. The transformation in Eq. (25.9) with constant, finite θ^a leaves the lagrangian in Eq. (25.4) unchanged. Here $\underline{U}(\theta)$ is a unitary, unimodular three-by-three matrix, and the quark field in Eq. (25.3) forms a basis for the fundamental representation of $SU(3)$. The symmetry is with respect to color.

One can now make this global color invariance a *local* invariance where the transformation $\theta^a(x)$ can vary from point to point in space-time by using the theory developed by Yang and Mills [Ya54, Ab73]:

1. Introduce massless vector meson fields, one for each generator

$$A_\mu^a(x) \qquad\qquad ;\; a = 1, \dots, 8 \tag{25.10}$$

These vector mesons are known as *gluons*;

2. Define the covariant derivative by

$$\frac{D}{Dx_\mu}\underline{\psi} = \left[\frac{\partial}{\partial x_\mu} - \frac{i}{2}g\underline{\lambda}^a A_\mu^a(x)\right]\underline{\psi} \tag{25.11}$$

3. Define the field tensor for the vector meson fields as

$$\mathscr{F}_{\mu\nu}^a = \frac{\partial A_\nu^a}{\partial x_\mu} - \frac{\partial A_\mu^a}{\partial x_\nu} + gf^{abc}A_\mu^b A_\nu^c \tag{25.12}$$

Here f^{abc} are the structure constants of $SU(3)$;

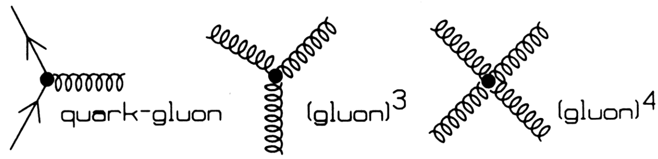

Fig. 25.1. Processes described by the interaction terms in the QCD lagrangian.

4. Under infinitesimal local gauge transformations $\theta^a \to 0$ the vector meson fields and the field tensor transform according to

$$\begin{aligned} \delta A^a_\mu &= -\frac{1}{g}\frac{\partial\theta^a}{\partial x_\mu} + f^{abc}\theta^b A^c_\mu \\ \delta\mathscr{F}^a_{\mu\nu} &= f^{abc}\theta^b\mathscr{F}^c_{\mu\nu} \qquad ; \ \theta^a \to 0 \end{aligned} \tag{25.13}$$

5. A combination of these results leads to the *lagrangian* of QCD

$$\mathscr{L}_{\text{QCD}} = -\underline{\bar{\psi}}\left\{\gamma_\mu\left[\frac{\partial}{\partial x_\mu} - \frac{i}{2}g\underline{\lambda}^a A^a_\mu(x)\right] + \underline{M}\right\}\underline{\psi} - \frac{1}{4}\mathscr{F}^a_{\mu\nu}\mathscr{F}^a_{\mu\nu} \tag{25.14}$$

The lagrangian in Eq. (25.14) can be written out explicitly in powers of the coupling constant g

$$\begin{aligned} \mathscr{L}_{\text{QCD}} &= \mathscr{L}_0 + \mathscr{L}_1 + \mathscr{L}_2 \qquad (25.15) \\ \mathscr{L}_0 &= -\underline{\bar{\psi}}\left(\gamma_\mu\frac{\partial}{\partial x_\mu} + \underline{M}\right)\underline{\psi} - \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} \\ \mathscr{L}_1 &= \frac{i}{2}g\underline{\bar{\psi}}\gamma_\mu\underline{\lambda}^a\underline{\psi}A^a_\mu(x) - \frac{g}{2}f^{abc}F^a_{\mu\nu}A^b_\mu A^c_\nu \\ \mathscr{L}_2 &= -\frac{g^2}{4}f^{abc}f^{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu \end{aligned}$$

Here

$$F^a_{\mu\nu} \equiv \frac{\partial A^a_\nu}{\partial x_\mu} - \frac{\partial A^a_\mu}{\partial x_\nu} \tag{25.16}$$

The various processes described by the interaction terms in this lagrangian are illustrated in Fig. 25.1.

To obtain further insight into these results, it is useful to write the Yukawa interaction between the quarks and gluons in more detail. Recall, for example, the structure of the first two $\underline{\lambda}^a$ matrices

$$\underline{\lambda}^1 = \begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix} \qquad\qquad \underline{\lambda}^2 = \begin{pmatrix} & -i & \\ i & & \\ & & \end{pmatrix} \tag{25.17}$$

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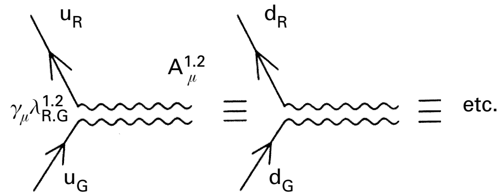

Fig. 25.2. Individual processes described by the quark–gluon Yukawa coupling in QCD.

These matrices connect the (R, G) quarks, and with explicit identification of the flavor components of the color fields, it is evident that this interaction contains the individual processes illustrated in Fig. 25.2. The quarks interact here by changing their color, which in turn is carried off by the gluons; the flavor of the quarks is unchanged and all flavors of quarks have an identical color coupling. If the gluons are represented with double lines connected to the incoming and outgoing quark lines respectively, and a color assigned to each line as indicated in this figure, then color can be viewed as running continuously through a Feynman diagram built from these components.

The Euler–Lagrange equations in continuum mechanics follow from Hamilton's principle [Fe71]

$$\delta \int \mathscr{L}\left(q, \frac{\partial q}{\partial x_\mu}\right) d^4x = 0 \tag{25.18}$$

The Euler–Lagrange equations following from the QCD lagrangian are readily derived as

$$\begin{aligned}
&\left\{\gamma_\mu \left[\frac{\partial}{\partial x_\mu} - \frac{i}{2} g \underline{\lambda}^a A^a_\mu(x)\right] + \underline{M}\right\} \underline{\psi} = 0 \\
&\underline{\bar{\psi}} \left\{\gamma_\mu \left[\frac{\overleftarrow{\partial}}{\partial x_\mu} + \frac{i}{2} g \underline{\lambda}^a A^a_\mu(x)\right] - \underline{M}\right\} = 0 \\
&\frac{\partial \mathscr{F}^a_{\mu\nu}}{\partial x_\nu} = \frac{i}{2} g \underline{\bar{\psi}} \gamma_\mu \underline{\lambda}^a \underline{\psi} + g f^{abc} \mathscr{F}^b_{\mu\nu} A^c_\nu
\end{aligned} \tag{25.19}$$

It follows from these equations of motion that currents built out of quark fields and a unit matrix with respect to color are *conserved*.

$$\begin{aligned}
&\frac{\partial}{\partial x_\mu}\left(\frac{i}{3} \underline{\bar{\psi}} \gamma_\mu \underline{\psi}\right) = 0 \qquad &&; \text{ baryon current} \\
&\frac{\partial}{\partial x_\mu}\left(i \underline{\bar{\psi}} \gamma_\mu \underline{\Sigma} \underline{\psi}\right) = 0 \qquad &&; \text{ flavor current}
\end{aligned} \tag{25.20}$$

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In the second line, $\underline{\Sigma}$ is a unit matrix with respect to color satisfying $[\underline{\Sigma}, \underline{\lambda}^a] = 0$; the flavor submatrices are *arbitrary* as long as they commute with the mass matrix

$$\underline{\Sigma} = \begin{pmatrix} \underline{\sigma} & & \\ & \underline{\sigma} & \\ & & \underline{\sigma} \end{pmatrix} \qquad ; \ [\underline{\sigma}, \underline{m}] = 0 \tag{25.21}$$

The conserved electromagnetic current for the (u, d, s, c) quarks, with charges (2/3, −1/3, −1/3, 2/3) respectively, is given by the point Dirac value

$$J_\mu^\gamma = i\underline{\bar{\psi}}\gamma_\mu \underline{Q}\,\underline{\psi} \tag{25.22}$$

$$\underline{Q} = \begin{pmatrix} \underline{q} & & \\ & \underline{q} & \\ & & \underline{q} \end{pmatrix} \quad ; \ \underline{q} = \begin{pmatrix} 2/3 & & & \\ & -1/3 & & \\ & & -1/3 & \\ & & & 2/3 \end{pmatrix}$$

The gluons are absolutely neutral to the electromagnetic interaction.

It follows from the four-divergence of the third of Eqs. (25.19) and the antisymmetry of $\mathscr{F}^a_{\mu\nu} = -\mathscr{F}^a_{\nu\mu}$ that the color current, the source of the color field, is also conserved.

$$\frac{\partial}{\partial x_\mu}\left(\frac{i}{2} g \underline{\bar{\psi}}\gamma_\mu \underline{\lambda}^a \underline{\psi} + g f^{abc} \mathscr{F}^b_{\mu\nu} A^c_\nu\right) = 0 \tag{25.23}$$

The theory of QCD can again be characterized by a set of *Feynman rules*. Here we give the Feynman rules for the Green's functions, which characterize the quantum field theory [Fe71]. The quark Green's function in the vacuum sector is defined by

$$\begin{aligned} iG_{\alpha\beta}(\mathbf{x}_1 t_1, \mathbf{x}_2 t_2) &\equiv \langle 0 | P[\hat{\psi}_\alpha(\mathbf{x}_1 t_1), \hat{\bar{\psi}}_\beta(\mathbf{x}_2 t_2)] | 0 \rangle \\ &\equiv \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x_1 - x_2)} iG_{\alpha\beta}(k) \end{aligned} \tag{25.24}$$

The Feynman rules for $iG(k)$ are derived in [Qu83, Ch84, Ai89, Wa91]; they are as follows:[2]

1. Draw all topologically distinct, connected diagrams;
2. Include the following factors for the quark, gluon, and *ghost* lines, respectively (Fig. 25.3):[3]

[2] See Ref. [Ch84] for a much more extensive discussion, including Feynman rules with other choices of gauge.

[3] All quark indices are now explicit: $i, j = R, G, B$ for color; $l, m = u, d, s, c, \cdots$ for flavor.

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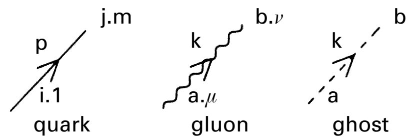

Fig. 25.3. Propagators in QCD.

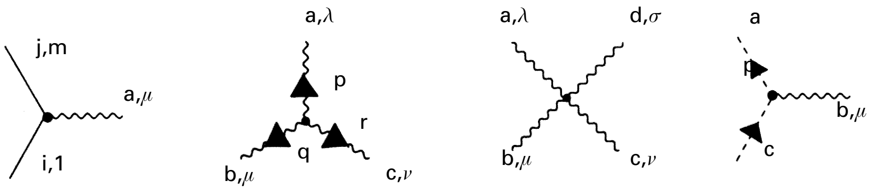

Fig. 25.4. Vertices in QCD.

$$\frac{1}{i}\frac{1}{i\gamma_\mu p_\mu}\delta_{ij}\delta_{lm} \qquad ; \text{ quark (massless)}$$

$$\frac{1}{i}\delta^{ab}\frac{1}{k^2}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right) \qquad ; \text{ gluon (Landau gauge)}$$

$$\frac{1}{i}\delta^{ab}\frac{1}{k^2} \qquad ; \text{ ghost} \tag{25.25}$$

The ghost is an internal element, coupled to gluons, that is required to generate the correct S-matrix in a non-abelian gauge theory;

3. Include the following factors for the vertices indicated in Fig. 25.4:

$$-g\frac{1}{2}\lambda^a_{ji}\delta_{lm}\gamma_\mu \qquad ; \text{ (quark)}^2\text{—gluon}$$

$$gf^{abc}[(q-r)_\lambda\delta_{\mu\nu}+(p-q)_\nu\delta_{\lambda\mu}+(r-p)_\mu\delta_{\lambda\nu}] \qquad ; \text{ (gluon)}^3$$

$$\begin{aligned}-ig^2[&f^{abe}f^{cde}(\delta_{\lambda\nu}\delta_{\sigma\mu}-\delta_{\lambda\sigma}\delta_{\mu\nu})+f^{ace}f^{bde}(\delta_{\lambda\mu}\delta_{\sigma\nu}-\delta_{\lambda\sigma}\delta_{\mu\nu})\\ &+f^{ade}f^{cbe}(\delta_{\lambda\nu}\delta_{\sigma\mu}-\delta_{\sigma\nu}\delta_{\lambda\mu})] \qquad ; \text{ (gluon)}^4\end{aligned}$$

$$-gf^{abc}p_\mu \qquad ; \text{ (ghost)}^2\text{—gluon} \tag{25.26}$$

4. Take the Dirac matrix product along fermion lines;
5. Conserve four-momentum at each vertex;
6. Include a factor $\int d^4q/(2\pi)^4$ for each independent internal line;
7. Include a factor of $(-1)^{F+G}$ where F is the number of closed fermion loops and G is the number of closed ghost loops;

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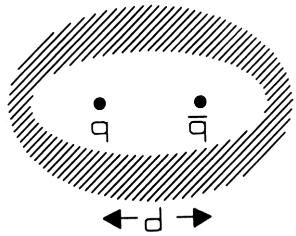

Fig. 25.5. Confinement in QCD. Lattice gauge theory calculations indicate that the separation energy grows linearly with d.

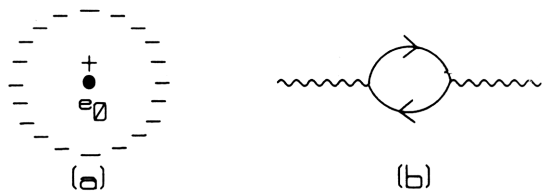

Fig. 25.6. (a) Shielding of point charge by (b) vacuum polarization in QED.

QCD has two absolutely remarkable properties, confinement and asymptotic freedom.

Colored quarks and gluons, the basic underlying degrees of freedom in the strong interactions, are evidently never observed as free asymptotic scattering states in the laboratory; you cannot hold an isolated quark or gluon in your hand. Quarks and gluons are confined to the interior of hadrons. There are strong indications from lattice gauge theory calculations [Wi74], that *confinement* is indeed a dynamic property of QCD arising from the strong, nonlinear gluon couplings in the lagrangian. One can show in these calculations, for example, that the energy of a static $(q\bar{q})$ pair grows linearly with the distance d separating the pair (see Fig. 25.5). What actually happens as the $(q\bar{q})$ pair is separated is that another $(q\bar{q})$ pair is formed, completely shielding the individual color charges of the first pair, and producing two mesons from one.

The second remarkable property is *asymptotic freedom*. Recall from QED that vacuum polarization shields a point electric charge e_0 as indicated in Fig. 25.6 (a). The renormalized charge e_2^2 changes with the distance scale, or momentum transfer λ^2, at which one measures the interior charge. The mathematical statement of this fact is the renormalization group equation of Gell-Mann and Low [Ge54]

$$\frac{de_2^2}{d\ln(\lambda^2/M^2)} = \psi(e_2^2) \tag{25.27}$$

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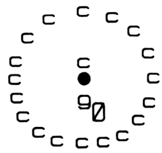

Fig. 25.7. Anti-shielding of color charge in QCD by strong vacuum polarization.

The lowest order modification of the charge in QED arises from the vacuum polarization graph indicated in Fig. 25.6 (b). The renormalization group equations can be used to sum the leading logarithmic corrections to the renormalized charge to all orders. The result is that the renormalized charge measured at large $\lambda^2 \gg M^2$ is related to the usual value of the total charge e_1^2 by

$$e_2^2 \approx \frac{e_1^2}{1 - (e_1^2/12\pi^2)\ln(\lambda^2/M^2)} \tag{25.28}$$

The first term in the expansion of the denominator arises from the graph in Fig. 25.6 (b). The renormalized electric charge in QED is evidently *shielded* by vacuum polarization; the measured charge *increases* as one goes to shorter and shorter distances, or higher and higher λ^2.

Similar, although somewhat more complicated, arguments can be made in QCD. An isolated color charge g_0 is modified by strong vacuum polarization and surrounded with a corresponding cloud of color charge as indicated schematically in Fig. 25.7. In this case, the renormalization group equations lead to a sum of the leading ln corrections for $\lambda^2 \gg \lambda_1^2$ of the form [Gr73a, Gr73b, Po73, Po74]

$$g_2^2 \approx \frac{g_1^2}{1 + (g_1^2/16\pi^2)(33/3 - 2N_f/3)\ln(\lambda^2/\lambda_1^2)} \tag{25.29}$$

Here N_f is the number of quark flavors.[4] An expansion of the denominator again gives the result obtained by combining the lowest-order perturbation theory corrections to the quark and gluon propagators and quark vertex. The plus sign in the denominator in this expression is crucial. One now draws the conclusion that there is *anti-shielding*; the charge *decreases* at shorter distances, or with larger λ^2.[5] The implications are enormous, for

[4] With $N_f = 1$, no gluon contribution of 33/3, and the observation $\mathrm{tr}(\frac{1}{2}\lambda^a\frac{1}{2}\lambda^b) = \frac{N_f}{2}\delta^{ab}$, one recovers the result in Eq. (25.28). It is the gluon contribution that changes the sign in the denominator.

[5] The vacuum in QCD thus acts like a *paramagnetic* medium, where a moment surrounds itself with like moments, rather than the *dielectric* medium of QED where a charge surrounds itself with opposite charges.

one now concludes that it is consistent to do *perturbation theory* at very short distances, or high momentum transfer. The renormalization group equations then provide a tool for summing the leading ln's of perturbation theory. The powerful result of asymptotic freedom in QCD is due to Gross and Wilczek [Gr73a, Gr73b] and Politzer [Po73, Po74]; see also Fritzsch and Gell-Mann [Fr72a, Fr73a]. References [Ma78, Re81, Wi82, Do93] contain good background material on QCD.

While a multitude of QCD-inspired models exist [Wa95], the most ambitious attempt to solve the QCD field equations relies on lattice gauge theory where the theory is put on a finite space-time lattice [Wi74].[6] Low-energy applications can be found in terms of *effective field theory*, where hadronic degrees of freedom are the generalized coordinates of choice, and an effective lagrangian constructed which reflects the symmetry properties of QCD [Do93, Se97]. While we will not give an extensive discussion of effective field theory here, it is possible to capture the spirit of these efforts.

Consider the nuclear domain of massless (u, d) quarks. The kinetic energy term as rewritten in Eq. (26.2), and hence $\mathscr{L}_{QCD}$, is invariant under the chiral $SU(2)_L \otimes SU(2)_R$ transformation $\psi_L \rightarrow \underline{L}\psi_L$, $\psi_R \rightarrow \underline{R}\psi_R$ where $\underline{L}$ and $\underline{R}$ are global $SU(2)$ matrices. Consider the pion sector of the hadronic theory and represent the pion field $\boldsymbol{\pi}$ through the $SU(2)$ matrix

$$\underline{U} = \exp\left(i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f_\pi\right) \tag{25.30}$$

Here f_π reflects the mass scale (say m_p) at which this chiral symmetry, as manifest in nature, is spontaneously broken. The leading term in an effective lagrangian can be constructed as follows

$$\mathscr{L}_2 + \mathscr{L}_{\text{csb}} = -\frac{f_\pi^2}{4}\text{tr}\left(\frac{\partial \underline{U}^\dagger}{\partial x_\lambda}\right)\left(\frac{\partial \underline{U}}{\partial x_\lambda}\right) + \frac{f_\pi^2 m_\pi^2}{4}\text{tr}\left(\underline{U} + \underline{U}^\dagger - 2\right) \tag{25.31}$$

If $m_\pi = 0$, this lagrangian is invariant under the chiral transformation $\underline{U} \rightarrow \underline{LUR}^\dagger$ (the pion mass term reflects chiral symmetry-breaking at the lagrangian level through u, d quark masses). This effective lagrangian should be applicable in the low energy domain where $q/f_\pi \ll 1$. Higher order terms in the effective lagrangian can now be similarly constructed in terms of $\underline{U}, \partial\underline{U}/\partial x_\lambda$. An expansion of the exponential then leads to

$$\begin{aligned}\mathscr{L}_2 + \mathscr{L}_{\text{csb}} \;&=\; -\frac{1}{2}\left(\frac{\partial \boldsymbol{\pi}}{\partial x_\lambda}\right)^2 - \frac{m_\pi^2}{2}\boldsymbol{\pi}^2 \\ &\quad -\frac{1}{6f_\pi^2}\left[\left(\boldsymbol{\pi}\cdot\frac{\partial \boldsymbol{\pi}}{\partial x_\lambda}\right)^2 - \boldsymbol{\pi}^2\left(\frac{\partial \boldsymbol{\pi}}{\partial x_\lambda}\right)^2\right] + \frac{m_\pi^2}{24 f_\pi^2}\boldsymbol{\pi}^4 + \cdots\end{aligned} \tag{25.32}$$

and $\boldsymbol{\pi}$–$\boldsymbol{\pi}$ scattering to $O(1/f_\pi^2)$ can now be calculated from this result.

[6] An extensive introduction to lattice gauge theory can be found in [Wa95].

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