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Abstract

We prove that if S is a properly embedded π_1-injective surface in a compact 3-manifold M, then $\pi_1 S$ is separable in $\pi_1 M$.

1. Introduction

A subgroup $H \subset G$ is *separable* if H equals the intersection of finite index subgroups of G containing H. Scott proved that if $G = \pi_1 M$ for a manifold M with universal cover $\widetilde{M}$, then H is separable if and only if each compact subset of $H \backslash \widetilde{M}$ embeds in an intermediate finite cover of M (see [Sco78, Lemma 1.4]). Thus, if $H = \pi_1 S$ for a compact surface $S \subset H\backslash\widetilde{M}$, then separability of H implies that S embeds in a finite cover of M. Rubinstein–Wang found a properly immersed π_1-injective surface S in a graph manifold M, with S embedded in $\pi_1 S\backslash\widetilde{M}$, such that S does not lift to an embedding in any finite cover of M. They deduced that $\pi_1 S \subset \pi_1 M$ is not separable [RW98, Example 2.6].

The objective of this paper is to prove the following theorem.

Theorem 1.1. *Let M be a compact connected 3-manifold and let $S \subset M$ be a properly embedded connected π_1-injective surface. Then $\pi_1 S$ is separable in $\pi_1 M$.*

Consequently, if $S \to M$ is a properly immersed π_1-injective surface in a compact 3-manifold M, such that S embeds in $\pi_1 S\backslash\widetilde{M}$, we have that $\pi_1 S \subset \pi_1 M$ is separable if and only if S lifts to an embedding in a finite cover of M.

The problem of separability of an embedded surface subgroup was raised for instance by Silver and Williams; see [SW09] and the references therein to their earlier works. The Silver–Williams conjecture was resolved recently by Friedl and Vidussi in [FV13], who proved that $\pi_1 S$ can be separated from some element in $[\pi_1 M, \pi_1 M] - \pi_1 S$ whenever $\pi_1 S$ is not a fiber.

We proved Theorem 1.1 when M is a graph manifold in [PW14, Theorem 1.1]. Theorem 1.1 was also proven when M is hyperbolic [Wis11]. In fact, every finitely generated subgroup of $\pi_1 M$ is separable for hyperbolic M, by [Wis11] in the case $\partial M \neq \emptyset$ and by Agol's theorem [Ago12] for M closed.

1.1 Overview

In § 2 we introduce the basic notation and reduce to studying irreducible M that is *simple* in the sense that its Seifert-fibred components are products with base surfaces of sufficient complexity. In § 3 we prove a topological result establishing separability of finite *semicovers* of M, i.e. maps required to be covers only over the interior of the blocks of the JSJ decomposition. This requires an omnipotence result for hyperbolic manifolds with boundary [Wis11, Corollary 16.15] coming from virtual specialness.

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To prove Theorem 1.1 we enhance the strategy employed in [PW14, Theorem 1.1] for graph manifolds. Its main element was [PW14, Construction 4.12] which produced S-*injective* covers of M^g, which are covers $\overline{M^g}$ to which S lifts and, among other properties, such that the intersection with S is connected for each JSJ torus or JSJ component of $\overline{M^g}$. We extend the construction of S-injective semicovers to all compact 3-manifolds in § 4. We use the double coset separability of relatively quasiconvex subgroups of π_1 of hyperbolic 3-manifolds with boundary [Wis11, Theorem 16.23] and separability of double cosets of embedded surface subgroups of π_1 of graph manifolds [PW14, Theorem 1.2].

We conclude with the proof of Theorem 1.1 in § 5.

2. Framework and reductions

2.1 Separability

We have the following finite index maneuverability: if $[H : H'] < \infty$ and $H' \subset G$ is separable, then $H \subset G$ is separable. Moreover, if $[G : G'] < \infty$, then a subgroup $H' \subset G'$ is separable if and only if $H' \subset G$ is separable. Finally, $H \subset G$ is separable if and only if for each $g \in G - H$ there is a finite quotient $\phi\colon G \to F$ with $\phi(g) \notin \phi(H)$. Thus, G is *residually finite* when $\{1_G\}$ is separable. We will freely employ these statements.

Note that a maximal abelian subgroup H of a residually finite group G is separable. Indeed, by maximality of H, if $g \in G - H$, then $ghg^{-1}h^{-1} \neq 1_G$ for some $h \in H$. By residual finiteness of G, there is a finite quotient $\phi\colon G \to F$ with $\phi(ghg^{-1}h^{-1}) \neq 1_F$. Since $\phi(H)$ is abelian, we obtain $\phi(g) \notin \phi(H)$.

2.2 Assumptions on M and S

Throughout this article M is a compact connected 3-manifold and might have nonempty boundary. We will make additional assumptions arising from the following reductions.

We can assume that S is not a sphere or a disc, since otherwise Theorem 1.1 follows from Hempel's residual finiteness of Haken 3-manifolds [Hem87] and Perelman's hyperbolization. By passing to a double cover we can assume that M is oriented. Furthermore, if S is not orientable, then the boundary $\widehat{S}$ of its tubular neighborhood is an oriented π_1-injective surface. As $[\pi_1 S : \pi_1 \widehat{S}] = 2$, the separability of $\pi_1\widehat{S}$ implies separability of $\pi_1 S$. Hence, we can assume that S is oriented. In the presence of our assumptions, the π_1-injectivity of S is equivalent to saying that S is *incompressible* and we will stay with this term.

2.3 Decomposition of M into blocks

An incompressible surface S in a reducible manifold can be homotoped into one of its prime factors, say M_0. Observe that there is a retraction $\pi_1 M \to \pi_1 M_0$ that kills the other factors. Consequently, if $g \in \pi_1 M_0 - \pi_1 S$, and we can separate g from $\pi_1 S$ in a finite quotient of $\pi_1 M_0$, then we can separate g from $\pi_1 S$ in a finite quotient of $\pi_1 M$. If $g \in \pi_1 M - \pi_1 M_0$, then applying [Hem87] to the factors we can find a finite cover M' of M where all of the terms of the normal form of g lie outside factor subgroups. Then the path representing g is nontrivial in the graph dual to the prime decomposition of M', and it suffices to use the residual finiteness of free groups. Hence, we can assume that M is irreducible (although possibly ∂-reducible).

We will employ the *JSJ decomposition* of M, which is the minimal collection of incompressible tori (up to isotopy) each of whose complementary components is Seifert-fibred or atoroidal. If M is a single Seifert-fibred manifold, then all finitely generated subgroups of $\pi_1 M$ are separable [Sco78], so we can assume that M is not Seifert-fibred.

By passing to a double cover we can assume that there are no π_1-injective Klein bottles in M. We can also assume that M is not a torus bundle over the circle, since then the only embedded surfaces are the fibers. Now a complementary component of JSJ tori cannot be simultaneously Seifert-fibred and algebraically atoroidal. Algebraically atoroidal components are *hyperbolic* by hyperbolization, in other words, their interior carries a geometrically finite hyperbolic structure (possibly of infinite volume if there are nontoroidal boundary components, as in a handlebody). We will call these complementary components *hyperbolic blocks*. The other complementary components are Seifert-fibred and we assemble adjacent Seifert-fibred components into *graph manifold blocks*. The JSJ tori that are adjacent to at least one hyperbolic block are called *transitional*.

We can assume that S is not a ∂-parallel annulus, since in that case separability follows easily from separability of the boundary torus group (since it is a maximal abelian subgroup) and from a variant of Lemma 3.1 with T^* in the boundary. Thus, S can be homotoped so that its intersection with each block is incompressible and not a ∂-parallel annulus. Moreover, we can assume that S intersects each Seifert-fibred component along a surface that is *horizontal*, i.e. transverse to the fibers, or *vertical*, i.e. foliated by fibers.

2.4 The m-characteristic covers and simplicity

For a manifold E let $E_{[m]}$ denote the *m-characteristic* cover of E, which is the regular cover corresponding to the intersection of all subgroups of index m in $\pi_1 E$. In particular, if T is a torus, then $T_{[m]}$ is the cover corresponding to the subgroup $m\mathbb{Z} \times m\mathbb{Z} \subset \mathbb{Z} \times \mathbb{Z} = \pi_1 T$. A Seifert-fibred manifold E is *simple* if it is the product of the circle with a surface of genus at least one that has at least two boundary components. This boundary hypothesis ensures that there is a retraction onto each boundary component. Consequently, $E_{[m]}$ restricts to m-characteristic covers on boundary tori. An irreducible 3-manifold M is *simple* if its Seifert-fibred components are simple. We will pass to a simple finite cover of M in Lemma 3.1.

Finally, by separability of the JSJ tori subgroups in $\pi_1 M$, we can assume that $S \subset M$ is *straight*. This means that S does not intersect a Seifert-fibred component E of M along a vertical annulus with both boundary circles in the same boundary torus of E.

3. Extending semicovers to covers

We begin this section with the following additional simplification.

Lemma 3.1. *Let M be an irreducible 3-manifold that is not Seifert-fibred and not a Sol manifold. Then M has a finite cover M' that is simple. Moreover, given covers $\{T^*\}$ of the transitional tori $\{T\}$ in M, we can assume that all of the tori of M' covering T are isomorphic and factor through T^*.*

The notational convention is that each torus T^* in the family $\{T^*\}$ corresponds to exactly one torus T in the family $\{T\}$. A key element of the proof employs the following omnipotence result for hyperbolic 3-manifolds with boundary.

Lemma 3.2 [Wis11, Corollary 16.15]. *Let M^h be a hyperbolic 3-manifold with boundary tori $\{T\}$. There exist finite covers $\{\widehat{T}\}$ such that for any further finite covers $\{T'\}$ there exists a finite cover $M^{h'}$ of M^h that restricts on boundary tori to covers isomorphic to $\{T'\}$.*

By passing to a further cover we can assume that $M^{h'} \to M^h$ is regular.

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Proof of Lemma 3.1. Luecke and Wu proved in [LW97, Proposition 4.4] that every graph manifold block M^g of M has a finite cover $M^{g\prime}$ that is simple. Without loss of generality we can assume that $M^{g\prime} \to M^g$ is regular.

Choose m such that:

(i) for any M^g adjacent along a torus T to a hyperbolic block M^h, the cover $T'_{[m]}$ of the torus $T' \subset \partial M^{g\prime}$ covering T factors through $\widehat{T}$ of Lemma 3.2 and through T^*;

(ii) for a transitional or boundary torus $T \subset M$ adjacent to a hyperbolic block M^h but not to a graph manifold block, the cover $T_{[m]}$ factors through $\widehat{T}$ of Lemma 3.2 and through T^*, if T is transitional.

By Lemma 3.2, each hyperbolic block M^h of M has a finite regular cover $M^{h\prime}$ restricting on the boundary to $\{T'_{[m]}\}$ of part (i) or $\{T_{[m]}\}$ of part (ii). For a Seifert-fibred component E of one of the simple graph manifolds $M^{g\prime}$, as E is simple its retractive property ensures that the cover $E_{[m]}$ restricts to m-characteristic covers on its boundary tori. Gluing appropriately many copies of the various $E_{[m]}$ and $M^{h\prime}$ together provides the desired simple cover M' of M. □

Henceforth, we *always* assume that M is simple.

Definition 3.3. A *semicover* $\overline{M}$ of M with respect to transitional tori is a local embedding $\overline{M} \to M$ that restricts to a covering map over each transitional torus and over each open block. Thus, $\overline{M}$ can only fail to be a covering map at a component of $\partial\overline{M}$ that covers a transitional torus $T \subset M$. We say that $\overline{M} \to M$ is *finite* if $\overline{M}$ is compact.

We can now prove the main result of this section.

Proposition 3.4. *Any finite semicover $\overline{M}$ of M has a finite cover $\overline{M}' \to \overline{M}$ that embeds in a finite cover M' of M.*

Proof of Proposition 3.4. By Lemma 3.1, there is a finite cover $\widehat{M}$ of M such that for each transitional torus T of M all of the tori $\widehat{T} \subset \widehat{M}$ covering T are isomorphic and factor through all of the covers of T in $\overline{M}$.

Let $p\colon \overline{M}' \to \widehat{M}$ be the semicover that is the pullback of the semicover $\overline{M} \to M$ via the cover $\widehat{M} \to M$. Then $p^{-1}(\widehat{T}) \to \widehat{T}$ restricts to a homeomorphism on each torus of the preimage. As in [PW14, Lemma 4.11], gluing $\overline{M}'$ with appropriately many copies of the blocks of $\widehat{M}$ extends $\overline{M}'$ to a cover M' of $\widehat{M}$, and hence of M. While [PW14, Lemma 4.11] is stated for a semicover with respect to JSJ tori instead of a semicover with respect to the transitional tori, the proof is the same.

Note that $\overline{M}'$ is a cover of $\overline{M}$, since it is a pullback of the cover $\widehat{M} \to M$. □

4. Surface-injective semicovers

In this section we construct a family of semicovers of M to which a given surface $S \subset M$ lifts. We keep the assumptions from § 2.

We will use the following case of a theorem of Martínez-Pedroza.

Theorem 4.1 [MP09, Theorem 1.1]. *Let $S_0 \subset M^h$ be an incompressible geometrically finite surface properly embedded in a hyperbolic manifold M^h. Let $\partial S_0 = C_1 \sqcup \cdots \sqcup C_k$ and suppose these circles are contained in boundary tori $T_1, \ldots, T_k$ of M^h (some T_i may coincide). Then for all but finitely many cyclic covers T'_i of T_i to which C_i lift, the graph of spaces obtained by* *amalgamating S_0 with T'_i along C_i maps π_1-injectively into M^h and the image of its π_1 in $\pi_1 M^h$ is relatively quasiconvex.*

The separability of double cosets of relatively quasiconvex subgroups of π_1 of a hyperbolic 3-manifold with boundary was established in [Wis11, Theorem 16.23]. Consequently, we have the following result.

Corollary 4.2. *For all but finitely many cyclic covers T'_i described in Theorem 4.1, the group $\pi_1(S_0 \sqcup_{\{C_i\}} \{T'_i\})$ is separable in $\pi_1 M^h$.*

Corollary 4.3. *The subgroup $\pi_1 S_0$ as well as the double cosets $\pi_1 S_0 \pi_1 T_i$ are separable in $\pi_1 M^h$.*

To make sense of the double cosets $\pi_1 S_0 \pi_1 T_i$ inside $\pi_1 M^h$, pick basepoints x_i of M^h in C_i and interpret $\pi_1 S_0, \pi_1 T_i$ as subgroups of $\pi_1 M^h$ determined by loops based at x_i staying in S_0, T_i, respectively.

Definition 4.4. Let $S \subset M$ be an incompressible surface. A semicover $\overline{M} \to M$ to which S lifts is *S-injective* with respect to transitional tori if for each hyperbolic or graph manifold block $\overline{B}$ of $\overline{M}$ the intersection $S \cap \overline{B}$ is connected. We allow S itself to be disconnected.

Lemma 4.5 [PW14, Construction 4.12]. *Let $S \subset M^g$ be a possibly disconnected straight incompressible surface in a simple graph manifold. Suppose n is an integer divisible by all of the degrees of (possibly disconnected) covers $S \cap E \to F$, where $E \subset M^g$ is a Seifert-fibred component with base surface F, and $S \cap E$ is horizontal. Then there is a finite cover $\overline{M^g}$ of M^g to which S lifts such that for each torus $\overline{T} \subset \partial \overline{M^g}$ intersecting S:*

- *$S \cap \overline{T}$ is connected;*
- *$\overline{T}$ maps to a torus $T \subset \partial M^g$ with degree $n/|S \cap T|$.*

Moreover, each connected component of $\overline{M^g}$ contains exactly one connected component of S.

Here $|S \cap T|$ denotes the number of components in the intersection of the surface S with the torus T.

Proposition 4.6. *Let $S \subset M$ be an incompressible surface. Let S_0 be a component of intersection of S with a hyperbolic or graph manifold block M_0 of M. Let T_i be the (possibly repeating) tori of ∂M_0 intersected by S_0. Let $g \in \pi_1 M_0 - \pi_1 S_0$ (respectively $g_i \in \pi_1 M_0 - \pi_1 S_0 \pi_1 T_i$ for each i). Then there is a finite S-injective semicover $\overline{M}$ with $g \notin \pi_1 \overline{M_0}$ (respectively $g_i \notin \pi_1 \overline{M_0} \pi_1 T_i$), where $\overline{M_0}$ is the block of $\overline{M}$ containing the lift of S_0.*

Proof. In the case where we assume $g \notin \pi_1 S_0$, we use that $\pi_1 S_0$ is separable in $\pi_1 M_0$. If M_0 is hyperbolic and S_0 is geometrically finite, this follows from Corollary 4.3. Otherwise, if M_0 is hyperbolic, then by covering [Thu80, Theorem 9.2.2] and tameness [Bon86] the surface S_0 is a fiber and hence $\pi_1 S_0$ is separable in $\pi_1 M_0$ as well. If M_0 is a graph manifold, we use separability of embedded surfaces in graph manifolds [PW14, Theorem 1.1]. Hence, there is a finite cover $\overline{M_0^*} \to M_0$ to which S_0 lifts with $g \notin \pi_1 \overline{M_0^*}$.

In the case where we assume $g_i \notin \pi_1 S_0 \pi_1 T_i$ for all i, we use that each double coset $\pi_1 S_0 \pi_1 T_i$ is separable in $\pi_1 M_0$. If M_0 is hyperbolic and S_0 is a fiber, then $\pi_1 S_0 \pi_1 T_i \subset \pi_1 M_0$ is a finite index subgroup, thus it is separable. Otherwise, this follows from Corollary 4.3 and [PW14, Theorem 1.2]. Hence, there exists a cover $\overline{M_0^*} \to M_0$ to which S_0 lifts with $g_i \notin \pi_1 \overline{M_0^*} \pi_1 T_i$. Let n_i be the degree of the restriction of $\overline{M_0^*} \to M_0$ to the torus intersecting (the lift of) S_0 along (the lift of) C_i.

Choose n so that it is divisible by the numbers in conditions (a)–(c) and also satisfies condition (d):

(a) every $|S \cap T|$, where T is a transitional or boundary torus;

(b) the degrees of (possibly disconnected) covers $S \cap E \to F$, where $E \subset M$ is a Seifert-fibred component with base surface F, and $S \cap E$ is horizontal;

(c) each $n_i|S \cap T_i|$ as above;

(d) we also require $n/|S \cap T|$ to be the degree of one of the covers $T' \to T$ given by Theorem 4.1 for a geometrically finite component of $S \cap M^h$ in a hyperbolic block M^h of M.

We construct the semicover $\overline{M}$ in the following way. Start with a copy $\overline{S}$ of S. Let T be a transitional or boundary torus of M. For each component of $S \cap T$ we attach along the corresponding circle in $\overline{S}$ the degree $n/|S \cap T|$ cyclic cover $\overline{T}$ of T. The value $n/|S \cap T|$ is an integer by condition (a).

For each graph manifold block M^g of M consider the finite (possibly disconnected) cover $\overline{M^g}$ from Lemma 4.5 applied to the surface $S \cap M^g$. The boundary components of $\overline{M^g}$ intersecting S coincide with the $\overline{T}$ attached to $\overline{S}$ above.

Consider now a hyperbolic block M^h of M such that $S \cap M^h$ is a union of fibers. In this case we choose $\overline{M^h}$ to be the union of $|S \cap M^h|$ copies of degree $n/|S \cap M^h|$ cyclic covers of M^h to which components of $S \cap M^h$ lift. Again, components of $\partial\overline{M^h}$ coincide with $\overline{T}$, so that we can consistently attach the $\overline{M^h}$ to $\overline{S}$.

Finally, if $S \cap M^h$ is not a union of fibers, then π_1 of each of its components is relatively quasiconvex in $\pi_1 M^h$, so by condition (d) and Corollary 4.2, there is a finite cover $\overline{M^h}$ extending $(S \cap M^h) \cup \{\overline{T}\}$, and we consistently attach the $\overline{M}^h$ to $\overline{S}$.

At this point we have constructed a finite S-injective semicover $\overline{M}$, without yet separating g (respectively g_i). Now we replace the block $\overline{M_0}$ with its fiber product with $\overline{M_0^*}$. (Algebraically π_1 of the *fiber product* is $\pi_1\overline{M_0} \cap \pi_1\overline{M_0^*} \subset \pi_1 M_0$.) This is possible by condition (c) which guarantees that the fiber product agrees with $\overline{M_0}$ on its boundary components intersecting S_0. After this replacement, $\overline{M}$ satisfies the requirement on g (respectively g_i), by definition of $\overline{M_0^*}$. □

5. Separability

In § 2 and Lemma 3.1 we reduced Theorem 1.1 to the following.

Theorem 5.1. *Let M be a compact connected oriented simple 3-manifold. Let $S \subset M$ be a properly embedded straight incompressible surface. Then $\pi_1 S$ is separable in $\pi_1 M$.*

Proof. Choose a basepoint of M in S outside all JSJ and boundary tori. Let $f \in \pi_1 M - \pi_1 S$. Consider the based cover M^S of M with fundamental group $\pi_1 S$. Let γ^S be a path in M^S starting at the basepoint and representing f. Then γ^S does not terminate on S. Assume that γ^S is chosen so that its image in M intersects the transitional tori a minimal number of times.

First, consider the case where γ^S terminates in a block $M_0^S \subset M^S$ that intersects the lift of S. Denote $S_0 = S \cap M_0^S$ and let $M_0 \subset M$ be the block covered by M_0^S. In the case where S_0 contains the basepoint, let $g \in \pi_1 M_0$ be an element represented by a path in M_0^S from the basepoint to the endpoint of γ^S.

By Proposition 4.6 there is a finite S-injective semicover $\overline{M}$ of M with $g \notin \pi_1\overline{M_0}$. Thus, γ^S projects to a path $\overline{\gamma}$ in $\overline{M}$ that ends in $\overline{M}_0$ outside the lift of S_0. By Proposition 3.4 the semicover $\overline{M}$ has a finite cover $\overline{M}'$ that extends to a finite cover M' of M. Since the endpoint of

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the lift of $\overline{\gamma}$ to M', which lies in $\overline{M}'$, does not terminate on the based connected component of the preimage of S, we have $f \notin \pi_1 M' \pi_1 S$, as desired.

Second, consider the case where γ^S terminates in a block of M^S disjoint from the lift of S. Let $T^S \subset M^S$ be then the first connected component of the preimage of a transitional torus $T \subset M$ crossed by γ^S and disjoint from S. Let M_0^S be the last block that γ^S travels through before it hits T^S. Let $S_0 = S \cap M_0^S$ and let $M_0 \subset M$ be the block covered by M_0^S. If T coincides with one of the tori $T_i \subset M_0$ crossed by S_0 along C_i, then let $x_i \in C_i$ be a basepoint for M_0. Let x_i' be a lift of x_i in T^S. We keep the notation x_i for the lift of x_i to $S_0 \subset M_0^S$. Let $g_i \in \pi_1 M_0$ be an element represented by a path in M_0^S from x_i to x_i'.

Since T^S is disjoint from S_0, we have $g_i \notin \pi_1 S_0 \pi_1 T_i$. By Proposition 4.6 there is a finite S-injective semicover $\overline{M}$ of M with $g_i \notin \pi_1 \overline{M_0} \pi_1 T_i$ for all i. In other words, $\overline{\gamma}$ leaves $\overline{M_0}$ through a torus disjoint from S_0.

By Proposition 3.4 the semicover $\overline{M}$ has a finite cover $\overline{M}'$ that extends to a finite cover M' of M. By separability of the transitional tori groups (since they are maximal abelian) and residual finiteness of the free group (dual to transitional tori), by replacing M' with a further cover we can assume that the lift of $\overline{\gamma}$ to M' does not pass twice through the same transitional torus.

Let $T' \subset M'$ be the projection of T^S. Consider the double cover M'' obtained by taking two copies of M', cutting along T', and regluing. Then the based connected component of the preimage of S lies in one copy of (the cut) M' in M'', while the endpoint of the lift of $\overline{\gamma}$ lies in the other copy. Hence, $f \notin \pi_1 M'' \pi_1 S$, as desired. □

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