LONG PROPAGATION PERIODS OF RESIDUALS IN THE MOTION OF A COMET

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P/Halley In a previous paper (Kiang 1973), I derived a differential equation which governs the behaviour of the residual O - C in the time of perihelion passage of a comet. The work was stimulated by the discovery (Brady 1972) that a long periodicity of about 600 years seemed to be present in the residual for P/Halley. The derivation was based on the following assumptions:

1. The system consists of only the Sun, Jupiter and P/Halley, and Jupiter is assumed to move in a circular orbit with a period $P' = 11.8614$ yr.

2. P/Halley is assumed to move in a *fixed* Keplerian orbit with a period $P = (13/2)\,P'$ exactly, so that the whole system is strictly recurrent with a period $P_0 = 13\,P' = 154.198$ yr.

A fixed Keplerian orbit is, of course, not a rigorous solution of the restricted problem of three bodies, but it is a simplifying assumption that made the derivation of the differential equation possible. The result was a second-order differential equation with periodic coefficients of period P_0. It was readily reduced to the standard form of a Hill's equation (Hill 1886), the solution of which consists essentially in the evaluation of Hill's exponent c.

In the practical solution of the equation, the following should be noted:

1. The coefficients of the equation are ultimately based on the values of the partial derivatives, with respect to Jupiter's longitude λ', of the rate of change of the mean motion n and of the mean anomaly M due to the action of Jupiter, through a whole recurrence period. Because these values can change violently at certain phases, we should not use their instantaneous values, but, rather, their average values, averaged over the adopted time interval (usually about 1 or 0.5 yr).

2. Hill himself dealt with a rather special case: his "intermediary orbit" had a two-fold symmetry, hence his formulae (copied in all text books) must be modified for the general case of an "orbit" with no symmetry. The necessary modifications were given in my paper, which contains, however, the following error and misprints: the error was that a factor of 2 was left out of the right side of Expression (40) of my paper and the misprints were (i), in (39) and (40), the argument of

R. L. Duncombe (ed.), Dynamics of the Solar System, 303–306.

cot should read $\pi\sqrt{\theta_0}$ and (ii) the denominator in (39) under the summation should read $k^2 - 4\theta_0$.

The system contained one free parameter λ_0', the longitude of Jupiter at the start of each recurrence period, which I took to correspond to a perihelion passage of the comet. It was found that c was real for some values of λ_0' and imaginary for others. A real c means that the behaviour of the residual is dominated by a long-period sinusoidal variation with period $P^* = P_0/c$ (the *hyperperiod*), while an imaginary c means that a secular component is present which will eventually dominate the behaviour.

I have now made calculations for more values of λ_0' and with the error in Expression (40) removed. The new results are shown in TABLE 1.

TABLE 1 Results for P/Halley (Fixed input parameters: eccentricity e = 0.96814, inclination of orbital planes $I = 161^\circ\!.25$, longitude of ascending node of Jupiter reckoned from the direction of perihelion $\Omega = 113^\circ\!.8$, $P_0 = 2P = 13P' = 154.198$ yr)

λ_0' =	0°	12°	14°	16°	18°	20°	22°	30°
c =	$-.51\,i$	$-.67\,i$	$-.14\,i$	$-.072$	$-.128$	.170	.187	.235
P^*(yr)=	...	...	...	2160.	1203.	910.	825.	655.

	60°	90°	120°	150°	154°	158°	160°
c =	.220	.201	.228	.307	.231	$-.024$	$-.17\,i$
P^*(yr)=	702.	766.	676.	503.	668.	6400.	...

Thus it appears that just over one-sixth of possible values of λ_0' give imaginary values of c; these correspond to those configurations in which the comet have close encounters with Jupiter. The rest five-sixths give real values of c, and values of hyperperiod mostly between 500 and 800 years, except near either end of the range, when the hyperperiod becomes indefintiely large.

A Qualitative Re-statement of Result Solution of Hill's equation proceeds in the frequency domain; the result obtained are mathematically precise but not easily graspable. I shall now try to re-state qualitatively the results obtained in terms of ordinary space and time. Let us start first with just the Sun and the comet. The latter then persues a Keplerian orbit. Now let us introduce a small, constant, non-gravitational force at every perihelion passage. Then the residual defined above will simply grow and grow; in other words, the system is unstable against persistent impulses of the same sign. Now introduce the massive Jupiter whose perturbation is many hundreds times greater than the non-gravitational impulse, but it can be of either sign and is of a higher frequency. Then, apparently, provided a certain condition is satisfied, the residual due to the non-gravitational force will not grow indefinitely, or the high-frequency perturbation by Jupiter has stabilised the system against persistent impulses. The condition is that there should be no close encounters between the comet and Jupiter. It is easy to see that this condition is necessary, for if there are close encounters, then even a very small change in the circumstance of encounter, corresponding to a very small residual, will result in very large effects.

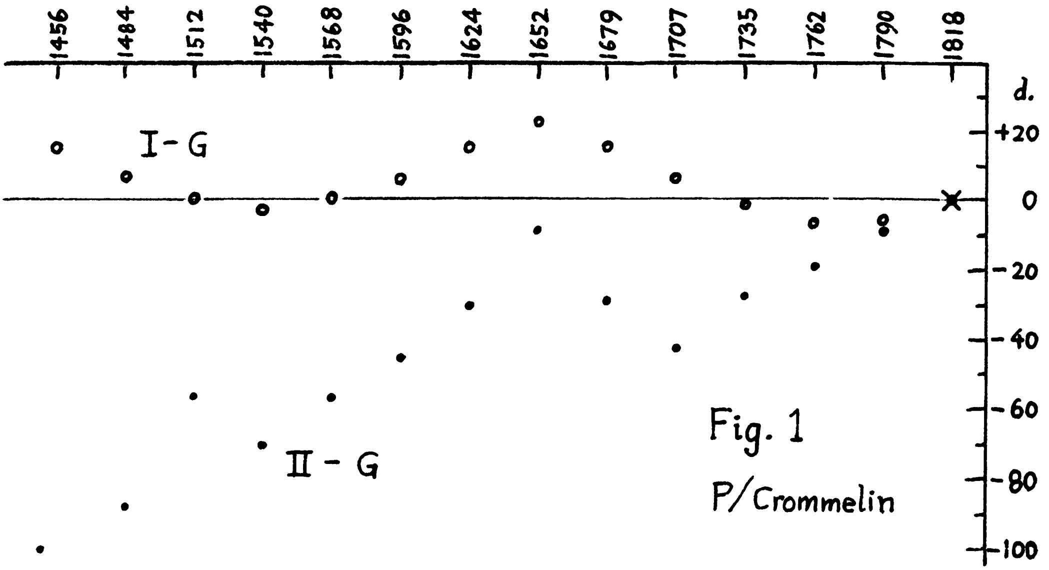

Fig. 1

P/Crommelin

P/Crommelin Dr. B. G. Marsden (1973) kindly supplied me 3 sets of perihelion passage times of this comet, 1 calculated for a purely gravitational model (G) and 2 non-gravitaional models (I: $A_1 = +1$, $A_2 = +0.1$; II: $A_1 = +1$, $A_2 = -0.1$). Fig. 1 shows the two runs of residuals I - G and II - G. There is a hint of a period of about 230 yr in the first while the second appears to be non-periodic. After having idealised the comet to have a period of $P = (7/3)\,P'$ exactly, the results in TABLE 2 were obtained.

TABLE 2 Results for P/Crommelin ($e = 0.919$, $I = 30°$, $\Omega = 335.°3$, $P_0 = 3P = 7P' = 83.030$ yr)

$\lambda_0{}'$ =	0°	15°	30°	45°	60°	75°	90°	105°
c =	$-.50\,i$	.1654	.1086	.1458	.1610	.1932	.0675	$-.23\,i$
P^*(yr) =	...	501.	764.	570.	515.	429.	1229.	...

It appears that no value of $\lambda_0{}'$ can give a hyperperiod as low as 230 yr, while about one-fourth of the initial conditions will give a non-periodic variation. This suggests that the non-gravitational model II may be closer to truth than model I.

Leonids and P/Tempel-Tuttle In a compilation of Chinese historical records of shower meteors, recently become available in English, Zhuang Tian-shan (1977) pointed out that there may be a 300-yr period in the visibility of the Leonid Meteor Stream. This stream is known to be associated with the comet P/Tempel-Tuttle, which has a period of 33yr. I therefore made calculations of c and P^* for an ideal comet with $P = (11/4)\,P' = 32.6189$ yr and the same eccentricity and orientation as P/Tempel-Tuttle. The results are shown in TABLE 3.

TABLE 3 Results for P/Tempel-Tuttle ($e = 0.9044$, $I = 163^\circ.6$, $\Omega = 169^\circ.3$
$P_0 = 4P = 11P' = 130.475$ yr)

λ_0' =	0°	15°	30°	45°	60°	75°
c =	$-.29\,i$	$-.09\,i$	.2210	.2426	.2406	$-.17\,i$
P^*(yr) =	...	...	590.	538.	541.	...

It appears that the "observed" period of 300 yr must be due to factors other than a residual in the time of perihelion passage. The visibility of metoers is, of course, critically dependent on the position of the node in relation to Earth, and the present theory does not apply to any oscillation in the node.

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DISCUSSION

Comment by Schubart: May I call your attention to the big display of Leonids seen in Europe in 1366. There are records from Portugal and from a monestary in Bohemia; compare Alexander von Humboldt's collection of references in "Cosmos." This shower was perhaps not visible in China.

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