ATMOSPHERIC TURBULENCE EFFECTS ON ASTROMETRIC DETERMINATIONS

V.I.Ivanov
Siberian Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, Irkutsk, USSR

1. INTRODUCTION

It is well known that the turbulent state of the atmosphere gives rise to fluctuations of the coefficient of light refraction. In astrometric observations these fluctuations produce the phenomenon of image motion which serves as the source of accidental errors of observations. The need for the study of the effect of atmospheric turbulence is documented in (G.Teleki, 1967) and others.
The crucial point of the problem in question is the dependence of the accidental error on the length of observation. In searching for this dependence different methods have been applied: empirical data of various authors (K.Lambeck, 1968), autocorrelation functions of image motion (I.G.Kolchinsky, 1973), and power spectra (E.Høg, 1968). In the last paper for the external and internal accidental mean errors the following expressions have been obtained:

$$m^2_{ext.} = m^2_\tau + (m^2_t - m^2_{\Delta t})/n \tag{1}$$

$$m^2_{int.} = m^2_t - (n \cdot m^2_\tau - m^2_{\Delta t})/(n-1) \tag{2}$$

where t is the length of one coordinate count, $t=\tau/n$ is the time between counts, τ is the general length of observations, n is the number of counts.
All errors in (1) and (2) are found from data on power spectrum of image motion W(f), for example:

$$m^2_\tau = \int_0^\infty W_\tau(f)\cdot df = \int_0^\infty W(f)\cdot\left(\frac{\sin \pi f\tau}{\pi f\tau}\right)^2 \cdot df \tag{3}$$

The method of spectral analysis of the motion process has also been used in our investigations and in order to study the general properties of spectra data on normalized spectral density have been obtained. Then equation (3) will be of the form:

E. Tengström and G. Teleki (eds.), Refractional Influences in Astrometry and Geodesy, 67–72.

$$\frac{m_\tau^2}{\sigma_\infty^2} = \int_0^\infty \frac{W(f)}{\sigma_\infty^2} \cdot \left(\frac{\sin \pi f \tau}{\pi f \tau} \right)^2 \cdot df \qquad (3a)$$

where σ_∞^2 means the dispersion of motion in the frequency interval from 0 to ∞.
From formulas (1)-(3a) it follows that for the estimation of accidental errors of observations one must know the amplitude (or dispersion) of image motion σ. The peculiar feature of this paper is that the calculation of σ and consequently of the value m_τ is carried out for specific conditions of observations and instruments.

2. PROCEDURE OF CALCULATING THE AMPLITUDE OF IMAGE MOTION

The statistical theory of wave propagation in a turbulent medium (V.I.Tatarsky, 1967) provides for σ the expression that is valid in the case of a steady-state random process. In astrometric determinations due to differing times of observation τ the stationarity is not realized, therefore the dependence σ on τ should be taken into account. Denoting it through $I(\tau)$ for σ^2 we shall have the expression:

$$\sigma^2 = 2{,}84 \cdot D^{-1/3} \cdot \sec Z \cdot I(\tau) \cdot \int_0^\infty C_n^2(h) \cdot dh \qquad (4)$$

where D is the objective diameter, z is the zenithal distance, C_n is the structural constant of pulsations of the refraction coefficient, h is the height above Earth surface. In order to employ practically formula (4) one should verify theoretical dependences σ on D and Z, find the function $I(\tau)$ and define the influence of meteorological conditions characterized by the integral of C_n^2. For this purpose, observations have been carried out of motion of stars and the artificial light source (mire) in the ground layer of the atmosphere with the AZT-14 telescope (D=48 cm, F=7.7 m), as well as that of the Sun and Moon with the ACU-5 telescope (D=44 cm, F=17 m). Observations were accomplished during a year at the Sayan Observatory of SibIZMIR, Siberian Department, USSR Academy of Sciences (H=2000 m). Photographic and photoelectric methods of recording the image motion were used.
Verification of the dependences of σ on D and Z has shown that on the average they satisfy well the theory and data of other authors (I.G.Kolchinsky, 1967). For the apertures 3-48 cm, $\sigma_D \sim D^{-1/6}$. The power index at sec Z obtained from analysis of short and long realizations of motions of stars became equal to 0.49±0.10. The amplitude of motion in the zenith for our point equals $\sigma_0 = \pm 0''.33$.
To find the function $I(\tau)$ we made use of our spectra of image motions of point and extended light sources and $I(\tau)$

is expressed in the form:

$$I(\tau) = \int_{1/\tau}^{f_0} \frac{W(f)}{\sigma^2} \cdot df \qquad (5)$$

The spectral density $W(f)/\sigma^2$ is defined over a broad frequency range from 10^{-5} to 400 Hz. Table 1 lists the values of $I(\tau)$.

Table 1

f, Hz	τ, s	$I(\tau)$
0.00001	100000	4.0850
0.0001	10000	2.2890
0.001	1000	1.8917
0.01	100	1.6975
0.1	10	1.3342
1	1	0.7485
10	0.1	0.3545
100	0.01	0.0167
400	0.0025	0.0000

If the frequency f_u is the upper and f_l the lower boundary of the range, then for the reduction of the measured σ for a given frequency interval it would be sufficient to determine the subtractions $\Delta I(\tau) = I(\tau_u) - I(\tau_l)$. They allow σ to be reduced to the frequency interval for which $\Delta I=1$.
In order to determine the influence of meteorological conditions one should know the profile $C_n^2(h)$. Since it is unknown we have used the characteristics being local or common criteria of the turbulent state of the atmosphere. For them we have taken Richardson's gradient number Ri (S.S.Zilitinkevich, 1970), Monin's stratification parameter μ (S.S.Zilitinkevich, 1970) and the thickness Δh of the turbulent layer. For comparison with them we have searched for reduced dispersions:

$$\sigma_{01}^2 = \frac{\sigma^2 \cdot D^{-1/3}}{\sec z \cdot \Delta I(\tau)} \qquad (6)$$

Then, by approximating, e.g., the dependence of σ_{01}^2 on Ri, we have

$$\sigma_{01}^2 = 0.045 + 0.0018/Ri \qquad (7)$$

Similar expressions have been derived also for the characteristics of μ and Δh.
Taking into account the above data, the procedure of calculating the amplitude of motion is as follows. Using the differences of temperature and wind speed at the two levels 1 and 4 m - $\Delta T(1-4)$ and $\Delta V(1-4)$ characteristics of turbulent behaviour of Ri, μ or Δh are determined. Then from

formulas (7) and (6), the amplitude of motion for the set aperture of the telescope, zenithal distance, frequency range and conditions of observation, are found. The procedure of the calculation of σ has been tested using observations of stars. The correlation coefficient of the measured and calculated values of σ made up $r = 0.84 \pm 0.03$.

3. DEPENDENCE OF THE RANDOM ERROR ON THE LENGTH OF OBSERVATION

Applying numerical methods and data on normalized spectrum from formula (3a) were found the values of m_τ/σ_∞ which are listed in Table 2 for the broad frequency range. In order to obtain directly the value m_τ one should know σ_∞. It is obtained from the formula:

$$\sigma_\infty^2 = \Delta I_\infty(\tau) \cdot \frac{\sigma_0^2}{\Delta I_0(\tau)} \tag{8}$$

where $\Delta I_0(\tau)$ and $\Delta I_\infty(\tau)$ are subtractions of $I(\tau)$ functions corresponding to the amplitudes σ_0 and σ_∞.
It has been found that for the calculation of σ_∞ one should practically take only the frequency range from 10^{-3} to 400 Hz. At more high frequencies the amplitude of motion is very small, lower frequencies belong to the region of refraction motions which in the main are taken into account in astrometric determinations from meteorological data.

Table 2

τ, sec	m_τ/σ_∞	m_τ'' at $\sigma_\infty = 0''.45$ ($\sigma_0 = 0''.33$)
0.003	1.366	0''.615
0.01	1.342	0.604
0.1	1.173	0.528
1	0.957	0.431
10	0.633	0.285
100	0.349	0.157
1000	0.196	0.088
10000	0.110	0.050
100000	0.064	0.029

Since $\sigma_0 = 0''.33$, and from Table 1 $\Delta I_0(\tau) \approx 1.0$, $\Delta I_\infty(\tau) = 1.89$, we obtain $\sigma_\infty = 0''.45$. Table 2 by using this quantity lists the values of m_τ as a function of observation time.
Thus having the amplitude of motion of σ_0 measured or calculated by the above procedure, one can obtain the value m_τ and from formulas (1) and (2) the random errors of astrometric determinations applied to either program of observation.

The scheme just considered has been applied for several works. Thus for the Pulkovo photographic vertical circle (PVC) the estimate of determination of the zenithal distance was $\pm 0''.161$, while its real value from 1975-76 observations is $\pm 0''.18$ (for zenith).

4. CONCLUSIONS

Let us emphasize the main results of essential importance for astrometry.

a) From the dependence of the amplitude of motion on the behaviour of the aperture of the telescope it follows that D=20 cm should be considered as the optimum objective diameter of astrometric instruments. With further increase in D, the value σ decreases little. This conclusion confirms the correctness of choice of the value D for present instruments (e.g. PVC).

b) The optimum length of astrometric determinations (see Table 2) on the average, is the 100 s interval (of about 2 min). Continuous averaging of readings should be considered as the best variant of measurements. In discrete readings, it is more advantageous to cover a greater time interval of observations rather than to maximize the number of readings.

c) It has been found that the normalized spectral density of motions is some universal function of frequency since it practically is independent of D, Z and the conditions of observations. This implies that the character of the dependence of the random error of astrometric determinations on the time of observations will be a function but the value of error itself will be a function of astroclimatic conditions. One should search for places with minimum amplitude of image motion.

d) The developed procedure for calculating the amplitude of motion allows one to do estimation of the random error for specific instruments and conditions of observations. It has been found that the error due to atmospheric turbulence makes up about 37% contribution to the dispersion of the random error of a single measurement of coordinates with PVC and almost fully accounts for the random error of determination of clock correction for Pulkovo transit instruments.

The above results should be utilized in evaluating the length and the program of astrometric observations, in developing new instruments and methods of coordinate determinations and their automation.

References

Høg, E.: 1968, Zeitschrift für Astroph., 69, pp. 313-325.
Kolchinsky, I.G.: 1967, Optical instability of the Earth atmo-

sphere from observations of stars. Kiev,"Naukova dumka".
Kolchinsky,I.G.:1973,Astrometriya i astrofizika, 20, pp.19-39.
Lambeck,K.:1968,SAO special report, 269,pp.1-27.
Tatarsky,V.I.:1967,Propagation of waves in a turbulent atmosphere. M.,"Nauka".
Teleki,G.:1967,Proceedings of the 17 Astron. Conf. USSR. L.,"Nauka", pp.101-106.
Zilitinkevich,S.S.:1970,Atmosphere boundary layer dynamics. L.,Gidrometeoizdat.