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BOOK REVIEWS

PEDERSEN, G. K., (ed) *C^*-algebras and their automorphism groups* (London Mathematical Society Monographs No. 14, Academic Press, 1979) ix + 416 pp. £26.

It is less than half a century since Murray and von Neumann began the study of operator algebras, but in that time, especially in the last twenty years, the subject has expanded at a rate which has been remarkable even in a period of rapid growth for mathematics in general. The modern worker can no longer be expected to be active in the whole of the field—he will usually specialise either in C^*-algebras or in von Neumann algebras, while keeping an eye open on developments in the other half of the subject. But it is possible to classify most operator algebraists much more precisely—for example, are they more comfortable working on Hilbert spaces or abstractly—are they interested in applications to mathematical physics—do operator algebras appear most naturally to them as ordered vector spaces, as spaces of affine functions on their state spaces, or as non-commutative analogues of spaces of continuous functions?

Although Murray and von Neumann were motivated partly by quantum mechanics, it is only subsequently that the full significance of operator algebras, especially C^*-algebras, as mathematical models of quantum systems has been appreciated, and it is this which has usually been used to justify the mass industry in the subject which has arisen. No-one would claim that this branch of mathematics has yet had any direct practical effect on common man, but there is at least some prospect, however tenuous, of its doing so through the link with physics.

The spread of interest in operator algebras has only recently been accompanied by a sudden and overdue increase in the number of books available on the subject. This particular work is merely one of several published almost simultaneously, and fortunately each of these adopts a very different approach. The author is undoubtedly an abstract C^*-algebraist, and this is reflected very strongly in the book, which is based on his own work. Hilbert spaces and von Neumann algebras appear only when their omission would be technically impossible or strikingly unnatural. Connections with quantum mechanics are mentioned only very incidentally, even though several of the topics included were originally motivated entirely by the physical models. Instead the approach involves a study of the order-structure of C^*-algebras, combined with some non-commutative measure theory. Thus for example, the original Murray-von Neumann type theory is developed as far as possible for Borel $*$-algebras before specialising to von Neumann algebras.

The book is not an introductory text, although it assumes very little specific knowledge beyond that which any graduate student of functional analysis might be expected to have. A great deal of material is covered, nearly all of it in the maximum possible generality, almost the only unavoidable concession being the occasional assumption of separability. Consequently certain proofs are so abbreviated that they require considerable amplification by the reader. The early chapters are organised in such a way as to lead as rapidly as possible to the deep results on decomposition and type theory (both of which are developed according to the author's own personal style, the latter being perhaps more successful than the former) in the middle of the book and the selection of recent developments covered in the last two chapters. Even the expert may initially be surprised by some of the devices used, whose significance is ingenuously revealed only much later. The only light relief comes in the author's occasional use of colloquial English in expounding his mathematical philosophy.

The topics of current research covered in the last part of the book, and mostly available in book form for the first time, are all in the general field of locally compact groups of $*$-automorphisms. They include crossed products, Arveson's spectral theory, Tomita–Takesaki theory (delayed as long as possible, and using the real Hilbert space approach due to Rieffel and van Deale), and Connes' classification of type III factors.

The book has been expertly produced, misprints and mathematical errors being very rare, and the notation and layout commendably well chosen.

In summary, anyone new to operator algebras might be advised to use some other text as an introduction. However as a work of reference for the specialist, this book will be invaluable.

C. J. K. BATTY

HUTSON V. and PYM, J. S. *Applications of Functional Analysis and Operator Theory* (Mathematics in Science and Engineering, Volume 146, Academic Press, London, 1980), xii + 390 pp.

The title of this book could mislead the potential reader. It might suggest that a working knowledge of the basics of functional analysis was assumed and that the text was intended to give reasonably extensive introductions to a number of modern applications of the standard theory. This, however, is not the case as the authors present a considerable amount of theory along with more modest introductions to the modern applications. Indeed, of the fourteen chapters in the book, the first nine could be said to be theoretical in nature, even if there is a plentiful supply of illustrative examples.

In their introduction, the authors claim that the abstract functional analysis required in a wide range of applications is not too great. Their aim is to substantiate this claim by making a careful choice of what they regard as the essential abstract techniques and by then putting these techniques into practice, using "solution of equations" as a unifying theme. The book is intended as the basis for a postgraduate course for applied mathematicians, physicists and engineers or others wishing to become acquainted with the powerful techniques of functional analysis. The prerequisites mentioned in the book are real analysis and linear algebra but others could be added to the list, such as a knowledge of Green's functions which appear regularly in the examples.

The book begins with a chapter on Banach and Hilbert spaces with many of the standard function spaces used as illustrations. Next comes a brief survey of those results in Lebesgue integration required for the L^p spaces, proofs usually being omitted. Chapter 3 introduces the foundations of linear operator theory including the Open Mapping Theorem, the Principle of Uniform Boundedness, elementary spectral theory and closed operators. Chapter 4 provides an introduction to non-linear operators covering the Contraction Mapping Theorem, Fréchet derivatives, the Implicit Function Theorem and Newton's Method, with examples involving non-linear integral equations. After a short chapter on compactness in Banach spaces, Chapter 6 introduces the concept of an adjoint operator, first in a Banach space and then in a Hilbert space. The spectral theory of bounded self-adjoint operators in Hilbert space is given and the adjoint of an unbounded operator defined. Chapter 7 deals with the spectral theory of linear compact operators and the theory is illustrated in the context of the numerical solution of linear integral equations. In Chapter 8, non-linear compact operators are discussed and the Schauder Fixed Point Theorem proved. Positive and monotone operators in partially ordered Banach spaces are examined and applications given to some non-linear differential and integral equations. Chapter 9 is devoted to the Spectral Theorem for both bounded and unbounded self-adjoint operators in Hilbert space.

Chapter 10 makes the transition from the theory to the applications with a discussion of generalised eigenfunction expansions associated with ordinary differential equations, including a treatment of self-adjoint extensions of symmetric operators and deficiency indices. In Chapter 11, we enter the world of partial differential equations with weak derivatives, Sobolev spaces and the generalised Dirichlet problem for linear elliptic equations. The next application is a brief discussion of the finite element method in the context of the generalised Dirichlet problem for the operator $-\nabla^2+p$. Chapter 13 contains an introduction to the Leray–Schauder degree with frequent use of homotopy invariance and an application to a problem in radiative transfer. Degree theory also appears in the last chapter which discusses bifurcation theory, both local and global, and includes examples involving buckling of a compressed rod and periodic wave trains in deep water.