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EFFECT OF STATIC MAGNETIC FIELD ON THE HELICAL FLOW OF INCOMPRESSIBLE CHOLESTERIC LIQUID CRYSTAL BETWEEN TWO COAXIAL CIRCULAR CYLINDERS HAVING ROTATIONAL AND AXIAL VELOCITIES

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Abstract

The paper investigates the effect of a static magnetic field on the helical flow of an incompressible cholesteric liquid crystal with director of unit magnitude between two coaxial circular cylinders rotating with different angular velocities about their common axis and moving with different axial velocities. At low shear rates with a weak magnetic field in the axial direction, the axial velocity, the angular velocity and the orientation of molecules between the two cylinders have been obtained. It is found that the magnetic field has influenced the orientation of molecules while the axial velocity and the angular velocity remain uneffected by the magnetic field.

1. Introduction

Leslie [1] has derived the forms of conservation laws for incompressible cholesteric liquid crystal with director of unit magnitude and their constitutive equations in isothermal equilibrium conditions. Frank [2] has proposed a Helmholtz free energy function for this type of liquid crystals. Using these and the non-equilibrium parts of constitutive equations [3] in the absence of temperature, Sharma [4] has recently analysed the helical flow of an incompressible cholesteric liquid crystal between two coaxial circular cylinders having rotational and axial velocities about their common axis. In this paper, we shall discuss this flow in the presence of a static magnetic field acting along the common axis of cylinders and then examine the effect of a weak static magnetic field on the flow in some details.

2. Basic equations

The equations governing the motion of an incompressible cholesteric liquid crystal with director $\vec{d}$ of unit magnitude [1] are

$$v_{i,i} = 0 \tag{2.1}$$

$$\rho \frac{Dv_i}{Dt} = \rho F_i + \sigma_{ji,j} \tag{2.2}$$

$$\rho_1 \frac{D^2 d_i}{Dt^2} = \rho_1 G_i + g_i + \pi_{ji,j} \tag{2.3}$$

where $\vec{v}$, ρ, F_i, σ_{ji}, ρ_1, G_i, g_i, π_{ji} and D/Dt respectively represent the velocity vector, the uniform density, the body force per unit mass, the stress tensor, an inertial constant, the extrinsic director body force per unit mass, the intrinsic director body force per unit volume, the director stress tensor and the material time derivative.

The stress tensor σ_{ji}, the director stress tensor π_{ji} and the intrinsic director body force g_i are given by the constitutive equations [1]

$$\sigma_{ji} = -p\delta_{ij} - \rho \frac{\partial F}{\partial d_{k,j}} d_{k,i} + \alpha e_{jkp}(d_p d_i)_{,k} + \tilde{\sigma}_{ji}, \tag{2.4}$$

$$\pi_{ji} = \beta_j d_i + \rho \frac{\partial F}{\partial d_{i,j}} + \alpha e_{ijk} d_k, \tag{2.5}$$

$$g_i = \gamma d_i - \beta_j d_{i,j} - \rho \frac{\partial F}{\partial d_i} - \alpha e_{ijk} d_{k,j} + \tilde{g}_i, \tag{2.6}$$

where, p, F, α, $\vec{\beta}$, γ, $\tilde{\sigma}_{ji}$ and $\tilde{g}_i$ respectively denote the pressure, Helmholtz free energy per unit mass, a material coefficient, an arbitrary vector, the director tension, the non-equilibrium parts of extra stress and the extra intrinsic director body force. Also when the temperature T is zero, the expressions [3] for $\tilde{\sigma}_{ji}$ and $\tilde{g}_i$ take the forms

$$\tilde{\sigma}_{ji} = u_1 d_k d_p A_{kp} d_i d_j + u_2 N_i d_j + u_3 N_j d_i + u_4 A_{ij} + u_5 A_{ik} d_k d_j + u_6 A_{jk} d_k d_i, \tag{2.7}$$

$$\tilde{g}_i = \lambda_1 N_i + \lambda_2 A_{ik} d_k, \tag{2.8}$$

where

$$A_{ij} = \tfrac{1}{2}(v_{i,j} + v_{j,i}),$$

$$W_{ij} = \tfrac{1}{2}(v_{i,j} - v_{j,i}),$$

$$N_i = \frac{Dd_i}{Dt} + W_{ki} d_k, \tag{2.9}$$

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and u_i, λ_i are the material coefficients related by

$$\lambda_1 = u_2 - u_3, \qquad \lambda_2 = u_5 - u_6. \tag{2.10}$$

Helmholtz free energy F for cholesteric liquid crystal proposed by Frank [2] is

$$2\rho F = \alpha_1(d_{i,i})^2 + \alpha_2(\tau + d_i e_{ijk} d_{k,j})^2 + \alpha_3 d_i d_j d_{k,i} d_{k,j}$$
$$+ (\alpha_2 + \alpha_4)[d_{i,j}d_{j,i} - (d_{i,i})^2] \tag{2.11}$$

where α_i and τ are the material coefficients.

In the presence of a magnetic field $\vec{H}$, the external body forces F_i and G_i are given by [5]

$$\rho F_i = (aH_k d_k d_j + bH_j)H_{j,i}$$
$$\rho_1 G_i = ad_i H_k d_k \tag{2.12}$$

where a and b are the constant magnetic susceptibilities.

The material coefficients appearing in all the above equations have been treated as constants in the present problem.

3. Statement of problem

We consider the helical flow of an incompressible cholesteric liquid crystal with director of unit magnitude between two infinite coaxial circular cylinders in the presence of a static magnetic field acting along the common axis of cylinders. The cylinders are moving with different axial velocities and are rotating with different angular velocities about their common axis.

Now, we choose a system of cylindrical coordinates (r, θ, z) such that z-axis coincides with the common axis of cylinders. The inner and outer cylinders of radii r_1 and r_2 $(r_1 < r_2)$ have U_1 and U_2 as their axial velocities respectively and Ω_1 and Ω_2 are their respective angular velocities. Also the director $\vec{d}$ has the same orientation on the two cylinders. Since we have considered the magnetic field $\vec{H}$ in the z-direction, hence,

$$H_r = H_\theta = 0, \qquad H_z = H. \tag{3.1}$$

With the assumption that the all unknowns depend on the radius vector only, we shall determine the solutions of differential equations (2.1)–(2.3) in the following form

$$v_r = 0, \qquad v_\theta = r\omega(r), \qquad v_z = u(r),$$
$$d_r = \sin\varphi(r), \qquad d_\theta = \cos\varphi(r)\sin\psi(r), \qquad d_z = \cos\varphi(r)\cos\psi(r). \tag{3.2}$$

where u, ω, φ and ψ are to be determined.

The boundary conditions for the velocity and the orientation of molecules respectively at the two cylinders then are

$$\begin{aligned} \omega &= \Omega_1, \quad u = U_1, \quad \text{at } r = r_1 \\ \omega &= \Omega_2, \quad u = U_2, \quad \text{at } r = r_2 \end{aligned} \tag{3.3}$$

$$\begin{aligned} \varphi &= 0, \quad \psi = \psi_0, \quad \text{at } r = r_1 \\ \varphi &= 0, \quad \psi = \psi_0, \quad \text{at } r = r_2 \end{aligned} \tag{3.4}$$

where ψ_0 is a constant.

4. Formation of differential equations

The equation of continuity (2.1) is clearly satisfied by the velocity components given by (3.2).

By using (2.12) and (3.2), equations (2.2), (2.3) take the forms

$$\frac{d}{dr}(r\sigma_{rr}) - \sigma_{\theta\theta} + \rho r^2\omega^2 = 0, \tag{4.1}$$

$$\frac{d}{dr}(r\sigma_{r\theta}) + \sigma_{\theta r} = 0, \tag{4.2}$$

$$\frac{d}{dr}(r\sigma_{rz}) = 0, \tag{4.3}$$

$$\frac{1}{r}\left[\frac{d}{dr}(r\pi_{rr}) - \pi_{\theta\theta}\right] + g_r + \rho_1\omega^2\sin\varphi = 0, \tag{4.4}$$

$$\frac{1}{r}\left[\frac{d}{dr}(r\pi_{r\theta}) + \pi_{\theta r}\right] + g_\theta + \rho_1\omega^2\cos\varphi\sin\psi = 0, \tag{4.5}$$

$$\frac{1}{r}\frac{d}{dr}(r\pi_{rz}) + g_z + aH^2\cos\varphi\cos\psi = 0. \tag{4.6}$$

Now using (2.4) in (4.1) and then integrating, we get

$$p = p_0 + \int_{r_1}^{r}\left[\rho r\omega^2 + \frac{d\hat{\sigma}_{rr}}{dr} + \frac{(\hat{\sigma}_{rr} - \hat{\sigma}_{\theta\theta})}{r}\right]dr \tag{4.7}$$

where p_0 is the pressure on the inner cylinder and

$$\hat{\sigma}_{rr} = p + \sigma_{rr}, \qquad \hat{\sigma}_{\theta\theta} = p + \sigma_{\theta\theta}.$$

Now, substituting the values of $\sigma_{r\theta}$ and $\sigma_{\theta r}$ from (2.4), (2.7), (2.11) in (4.2), we get

$$G_1(\varphi, \psi, \xi, \eta) + \frac{1}{r}\frac{d}{dr}[r^2H_2(\varphi)\sin\psi\cos\psi\,\xi + r^2\{H_1(\varphi) + H_2(\varphi)\sin^2\psi\}\eta] = 0. \tag{4.8}$$

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Here,

$$\xi = \frac{1}{2}\frac{du}{dr}, \qquad \eta = \frac{1}{2}r\frac{d\omega}{dr},$$

$$H_1(\varphi) = u_4 + (u_5 - u_2)\sin^2\varphi,$$

$$H_2(\varphi) = (2u_1\sin^2\varphi + u_3 + u_6)\cos^2\varphi,$$

$$F_1(\varphi) = \alpha_1\cos^2\varphi + \alpha_3\sin^2\varphi,$$

$$F_2(\varphi) = (\alpha_2\cos^2\varphi + \alpha_3\sin^2\varphi)\cos^2\varphi,$$

$$\begin{aligned}
G_1(\varphi, \psi, \xi, \eta)
&= \sin\psi\Big[F_1(\varphi)\frac{d^2\varphi}{dr^2} + \frac{1}{2}\frac{d}{d\varphi}F_1(\varphi)\left(\frac{d\varphi}{dr}\right)^2 - \frac{1}{2}\frac{d}{d\varphi}F_2(\varphi)\left(\frac{d\psi}{dr}\right)^2 \\
&\quad + \frac{1}{r}F_1(\varphi)\frac{d\varphi}{dr} - \frac{1}{r}\sin\psi\cos\psi\frac{d}{d\varphi}F_2(\varphi)\frac{d\psi}{dr} \\
&\quad - \frac{2\alpha_3}{r}\sin\varphi\cos\varphi\sin\psi\cos\psi\frac{d\psi}{dr} + \frac{1}{2r^2}\frac{d}{d\varphi}F_1(\varphi) \\
&\quad - \frac{1}{4r^2}\cos^2\psi\frac{d}{d\varphi}F_2(\varphi) - \frac{\alpha_3}{2r^2}\sin\varphi\cos\varphi\cos^2\psi \\
&\quad + 2\alpha_2\tau\sin\varphi\cos\varphi\frac{d\psi}{dr}\Big] \\
&\quad - \tan\varphi\cos\psi\Big[F_2(\varphi)\frac{d^2\psi}{dr^2} + \frac{1}{r}\sin\psi\cos\psi\frac{d}{d\varphi}F_2(\varphi)\frac{d\varphi}{dr} \\
&\quad + \frac{2\alpha_3}{r}\sin\varphi\cos\varphi\sin\psi\cos\psi\frac{d\varphi}{dr} + \frac{1}{r}F_2(\varphi)\frac{d\psi}{dr} \\
&\quad + \frac{d}{d\varphi}F_2(\varphi)\frac{d\varphi}{dr}\frac{d\psi}{dr} - 2\alpha_2\tau\sin\varphi\cos\varphi\frac{d\varphi}{dr}\Big] \\
&\quad + (\lambda_1 + \lambda_2)(\xi\cos\psi + \eta\sin\psi)\cos^2\varphi\sin\psi \\
&\quad + (\lambda_1 - \lambda_2)\eta\sin^2\varphi.
\end{aligned} \tag{4.9}$$

Also substituting the values of π_{rr}, $\pi_{\theta\theta}$, $\pi_{r\theta}$, $\pi_{\theta r}$, g_r, g_θ, g_z from (2.5), (2.6), (2.8), (2.11) in the equations (4.4) to (4.6) and then eliminating γ between them, we obtain

$$G_1(\varphi, \psi, \xi, \eta) = 0, \tag{4.10}$$

$$G_2(\varphi, \psi, \xi, \eta) + \rho_1\omega^2\sin\varphi\cos\varphi\cos\psi = 0, \tag{4.11}$$

where

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$$\bar{H} = aH^2,$$

$$\begin{aligned}
G_2(\varphi, \psi, \xi, \eta) \\
= \cos\psi\Big[& F_1(\varphi)\frac{d^2\varphi}{dr^2} + \frac{1}{2}\frac{d}{d\varphi}F_1(\varphi)\left(\frac{d\varphi}{dr}\right)^2 - \frac{1}{2}\frac{d}{d\varphi}F_2(\varphi)\left(\frac{d\psi}{dr}\right)^2 \\
& + \frac{1}{r}F_1(\varphi)\frac{d\varphi}{dr} - \frac{1}{r}\sin\psi\cos\psi\frac{d}{d\varphi}F_2(\varphi)\frac{d\psi}{dr} \\
& - \frac{2\alpha_3}{r}\sin\varphi\cos\varphi\sin\psi\cos\psi\frac{d\psi}{dr} + \frac{1}{2r^2}\frac{d}{d\varphi}F_1(\varphi) \\
& - \frac{1}{4r^2}\sin^2\psi\frac{d}{d\varphi}F_2(\varphi) - \frac{\alpha_3}{r^2}\sin\varphi\cos\varphi \\
& - \frac{\alpha_3}{2r^2}\sin\varphi\cos\varphi\sin^2\psi + 2\alpha_2\tau\sin\varphi\cos\varphi\frac{d\psi}{dr}\Big] \\
+ \tan\varphi\sin\psi\Big[& F_2(\varphi)\frac{d^2\psi}{dr^2} + \frac{d}{d\varphi}F_2(\varphi)\frac{d\varphi}{dr}\frac{d\psi}{dr} \\
& + \frac{1}{r}\sin\psi\cos\psi\frac{d}{d\varphi}F_2(\varphi)\frac{d\varphi}{dr} + \frac{2\alpha_3}{r}\sin\varphi\cos\varphi\sin\psi\cos\psi\frac{d\varphi}{dr} \\
& + \frac{1}{r}F_2(\varphi)\frac{d\psi}{dr} - 2\alpha_2\tau\sin\varphi\cos\varphi\frac{d\varphi}{dr} + \frac{2\alpha_2\tau}{r}\cos^2\varphi\Big] \\
+ & (\lambda_1 + \lambda_2)(\xi\cos\psi + \eta\sin\psi)\cos^2\varphi\cos\psi \\
+ & (\lambda_1 - \lambda_2)\xi\sin^2\varphi - \bar{H}\sin\varphi\cos\varphi\cos\psi. \qquad (4.12)
\end{aligned}$$

Using (4.10) in (4.8) and then integrating, we get

$$H_2(\varphi)\sin\psi\cos\psi\ \xi + \{H_1(\varphi) + H_2(\varphi)\sin^2\psi\}\eta = \frac{l}{r^2}, \qquad (4.13)$$

where l is the constant of integration.

Integrating (4.3) and then substituting the value of σ_{rz} from (2.4), (2.7), (2.11), we get

$$\{H_1(\varphi) + H_2(\varphi)\cos^2\psi\}\xi + H_2(\varphi)\sin\psi\cos\psi\ \eta = \frac{k}{r}, \qquad (4.14)$$

where k is a constant of integration.

Now, solving the equations (4.13), (4.14) for ξ and η, we get

$$\xi = \frac{1}{\Delta}[kr\{H_1(\varphi) + H_2(\varphi)\sin^2\psi\} - lH_2(\varphi)\sin\psi\cos\psi], \qquad (4.15)$$

$$\eta = \frac{1}{\Delta}[l\{H_1(\varphi) + H_2(\varphi)\cos^2\psi\} - krH_2(\varphi)\sin\psi\cos\psi], \qquad (4.16)$$

where

$$\Delta = r^2 H_1(\varphi)\{H_1(\varphi) + H_2(\varphi)\}.$$

Some trigonometrical simplifications between (4.10) and (4.11) yields

$$\begin{aligned}
&2F_1(\varphi)\left\{\frac{d^2\varphi}{dr^2}+\frac{1}{r}\frac{d\varphi}{dr}\right\}+\frac{d}{d\varphi}F_1(\varphi)\left\{\left(\frac{d\varphi}{dr}\right)^2+\frac{1}{r^2}\right\}\\
&\quad-\frac{d}{d\varphi}F_2(\varphi)\left\{\frac{d\psi}{dr}+\frac{1}{r}\sin\psi\cos\psi\right\}^2\\
&\quad+\sin 2\varphi\left[\left(2\alpha_2\tau-\frac{\alpha_3}{r}\sin 2\psi\right)\frac{d\psi}{dr}+\frac{1}{r}\left\{\alpha_2\tau\sin 2\psi-\frac{\alpha_3}{r}(1+\sin^2\psi)\cos^2\psi\right\}\right]\\
&\quad+2(\lambda_1+\lambda_2\cos 2\varphi)(\xi\cos\psi+\eta\sin\psi)\\
&\quad-2\bar{H}\sin\varphi\cos\varphi\cos^2\psi+2\rho_1\omega^2\sin\varphi\cos\varphi\cos^2\psi=0,
\end{aligned}\tag{4.17}$$

$$\begin{aligned}
&F_2(\varphi)\left\{\frac{d^2\psi}{dr^2}+\frac{1}{r}\frac{d\psi}{dr}\right\}+\frac{d}{d\varphi}F_2(\varphi)\frac{d\varphi}{dr}\frac{d\psi}{dr}\\
&\quad+\frac{1}{r}\sin\psi\cos\psi\left(\frac{d\varphi}{dr}+\frac{1}{4r}\cot\varphi\cos 2\psi\right)\left\{\frac{d}{d\varphi}F_2(\varphi)+2\alpha_3\sin\varphi\cos\varphi\right\}\\
&\quad+\cos\varphi\left\{\frac{1}{r}\cos\varphi\sin\psi\left(2\alpha_2\tau\sin\psi-\frac{\alpha_3}{r}\cos\psi\right)-2\alpha_2\tau\sin\varphi\frac{d\varphi}{dr}\right\}\\
&\quad+(\lambda_1-\lambda_2)(\xi\sin\psi-\eta\cos\psi)\sin\varphi\cos\varphi\\
&\quad-\bar{H}\cos^2\varphi\sin\psi\cos\psi+\rho_1\omega^2\cos^2\varphi\sin\psi\cos\psi=0.
\end{aligned}\tag{4.18}$$

5. Solutions of differential equations for low shear rates with a weak magnetic field

In this section, we shall examine the solutions of differential equations (4.15) to (4.18) for low shear rates with a weak magnetic field in the axial direction by omitting the inertial terms in (4.17) and (4.18).

In this case, the solutions of (4.15) to (4.18) are of the form

$$u(r) = U_1 + ku_1(r) + lu_2(r) + O(k^2, l^2, kl), \tag{5.1}$$

$$\omega(r) = \Omega_1 + k\omega_1(r) + l\omega_2(r) + O(k^2, l^2, kl), \tag{5.2}$$

$$\varphi(r) = k\varphi_1(r) + l\varphi_2(r) + \bar{H}\varphi_3(r) + O(k^2, \cdots, kl, \cdots), \tag{5.3}$$

$$\begin{aligned}\psi(r) &= \psi_0 + f(r)\\ &= \psi_0 + g(r) + k\psi_1(r) + l\psi_2(r) + \bar{H}\psi_3(r) + O(k^2, \cdots, kl, \cdots),\end{aligned}\tag{5.4}$$

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where u_i, ω_i $(i = 1,2)$, φ_j, ψ_j $(j = 1,2,3)$, k, l and g are to be determined by using the boundary conditions (3.3) and (3.4).

Thus, it can be shown that

$$u = U_1 + \left(\frac{U_2 - U_1}{AC - B^2}\right)\left[AC\frac{\log(r/r_1)}{\log(r_2/r_1)} - B^2\frac{r_2}{r}\left(\frac{r - r_1}{r_2 - r_1}\right)\right]$$
$$+ (\Omega_2 - \Omega_1)\left(\frac{AB}{AC - B^2}\right)\left[\frac{\log(r/r_1)}{\log(r_2/r_1)} - \frac{r_2}{r}\left(\frac{r - r_1}{r_2 - r_1}\right)\right], \tag{5.5}$$

$$\omega = \Omega_1 + (U_2 - U_1)\left(\frac{BC}{AC - B^2}\right)\left(\frac{r_1 r_2}{r^2}\right)\left(\frac{r - r_1}{r_2 - r_1}\right)\left(\frac{r_2 - r}{r_2 + r_1}\right)$$
$$+ \left(\frac{\Omega_2 - \Omega_1}{AC - B^2}\right)\left(\frac{r_2}{r}\right)\left(\frac{r - r_1}{r_2 - r_1}\right)\left[AC\frac{r_2}{r}\left(\frac{r + r_1}{r_2 + r_1}\right) - B^2\right], \tag{5.6}$$

$$\varphi = D\Big[(U_2 - U_1)\Big\{c\cos\psi_0[(r_2 - r)\log r_1 + (r - r_1)\log r_2$$
$$- (r_2 - r_1)\log r] + \frac{B}{2}\sin\psi_0\log\left(\frac{r_1}{r}\right)\log\left(\frac{r}{r_2}\right)\log\left(\frac{r_1}{r_2}\right)\Big\}$$
$$+ (\Omega_2 - \Omega_1)\Big\{B\cos\psi_0[(r_2 - r)\log r_1 + (r - r_1)\log r_2$$
$$- (r_2 - r_1)\log r] + \frac{A}{2}\sin\psi_0\log\left(\frac{r_1}{r}\right)\log\left(\frac{r}{r_2}\right)\log\left(\frac{r_1}{r_2}\right)\Big\}\Big], \tag{5.7}$$

$$\psi = \psi_0 + E\Big[aH^2\cos\psi_0\{(r_2^2 - r^2)\log r_1 + (r^2 - r_1^2)\log r_2$$
$$- (r_2^2 - r_1^2)\log r\} - 8\alpha_2\tau\sin\psi_0\{(r_2 - r)\log r_1$$
$$+ (r - r_1)\log r_2 - (r_2 - r_1)\log r\}$$
$$- 2\alpha_3\cos\psi_0\log\left(\frac{r_1}{r}\right)\log\left(\frac{r}{r_2}\right)\log\left(\frac{r_2}{r_1}\right)\Big]. \tag{5.8}$$

Here,

$$A = \left[\frac{2\{u_4 + (u_3 + u_6)\sin^2\psi_0\}}{u_4(u_3 + u_4 + u_6)}\right]\log\left(\frac{r_2}{r_1}\right),$$

$$B = \left[\frac{(u_3 + u_6)\sin^2\psi_0}{u_4(u_3 + u_4 + u_6)}\right]\left(\frac{r_2 - r_1}{r_1 r_2}\right),$$

$$C = \left[\frac{\{u_4 + (u_3 + u_6)\cos^2\psi_0\}}{u_4(u_3 + u_4 + u_6)}\right]\left(\frac{r_2^2 - r_1^2}{r_1^2 r_2^2}\right),$$

$$D = \frac{(\lambda_1 + \lambda_2)}{\alpha_1(u_3 + u_4 + u_6)(AC - B^2)\log(r_1/r_2)},$$

$$E = \frac{\sin\psi_0}{4\alpha_2\log(r_2/r_1)}. \tag{5.9}$$

6. Absence of magnetic field

The problem of helical flow of incompressible cholesteric liquid crystal between two coaxial circular cylinders having rotational and axial velocities has been solved elsewhere [4] when there is no magnetic field. The results obtained there are particular cases of the results obtained in this paper when $H = 0$. We note that the expressions for u, ω and φ given by (5.5), (5.6) and (5.7) do not contain H and hence they hold good irrespective of whether there is magnetic field or not. However, ψ given by (5.8) contains a term involving H, and hence putting $H = 0$, we get the expression for ψ in the absence of magnetic field as

$$\psi = \psi_0 - 2E\Big[4\alpha_2\tau\sin\psi_0\{(r_2 - r)\log r_1 + (r - r_1)\log r_2 - (r_2 - r_1)\log r\} + \alpha_3\cos\psi_0\log\left(\frac{r_1}{r}\right)\log\left(\frac{r}{r_2}\right)\log\left(\frac{r_2}{r_1}\right)\Big]. \tag{6.1}$$

7. Conclusion

At low shear rates with a weak magnetic field in the axial direction, the axial velocity u and the angular velocity ω given by (5.5) and (5.6) remain unaffected by the magnetic field. The angle φ in (5.7) determining the orientation d_i remains also unaffected by the magnetic field. But the angle ψ given by (5.8) is affected by the magnetic field in having a term which varies as the square of the magnetic field. The space variation of this term depends upon the squares and logarithms of the radial distance.

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