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THESIS ABSTRACTS

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Madeline G. Barnicle, ***Uniform Properties of Ideals in Rings of Restricted Power Series***, University of California, Los Angeles, USA, 2019. Supervised by Matthias Aschenbrenner. MSC: 11D88, 13L05. Keywords: model theory, p-adic numbers, definability, prime ideals, radical ideals, Weierstrass Division.

Abstract

When is an ideal of a ring radical or prime? By examining its generators, one may in many cases definably and uniformly test the ideal's properties. We seek to establish such definable formulas in rings of p-adic power series, such as $\mathbb{Q}_p\langle X\rangle$, $\mathbb{Z}_p\langle X\rangle$, and related rings of power series over more general valuation rings and their fraction fields. We obtain a definable, uniform test for radicality, and, in the one-dimensional case, for primality. This builds upon the techniques stemming from the proof of the quantifier elimination results for the analytic theory of the p-adic integers by Denef and van den Dries, and the linear algebra methods of Hermann and Seidenberg.

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Osvaldo Guzmán González, ***P-points, MAD families and Cardinal Invariants***, Centro de Ciencias Matemáticas, UNAM, Morelia, Michoacan, Mexico, 2017. MSC: Primary 03E17, 03E35. Secondary 03E65. Keywords: P-points, MAD families, +-Ramsey, Silver model, cardinal invariants.

Abstract

The main topics of this thesis are cardinal invariants, P-points and MAD families. *Cardinal invariants of the continuum*are cardinal numbers that are bigger than $\aleph_0$ and smaller or equal than $\mathfrak{c}$. Of course, they are only interesting when they have some combinatorial or topological definition. An *almost disjoint family*is a family of infinite subsets of ω such that the intersection of any two of its elements is finite. A MAD family is a maximal almost disjoint family. An ultrafilter $\mathcal{U}$ on ω is called a *P-point*if every countable $\mathcal{B} \subseteq \mathcal{U}$ there is $X \in \mathcal{U}$ such that $X \setminus B$ is finite for every $B \in \mathcal{B}$. This kind of ultrafilters has been extensively studied, however there is still a large number of open questions about them.

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In the preliminaries we recall the principal properties of filters, ultrafilters, ideals, MAD families and cardinal invariants of the continuum. We present the construction of Shelah, Mildenberger, Raghavan, and Steprāns of a completely separable MAD family under $\mathfrak{s} \leq \mathfrak{a}$. None of the results in this chapter are due to the author.

The second chapter is dedicated to a principle of Sierpiński. The *principle*$(*)$ *of Sierpiński* is the following statement: There is a family of functions $\{\varphi_n : \omega_1 \longrightarrow \omega_1 \mid n \in \omega\}$ such that for every $I \in [\omega_1]^{\omega_1}$ there is $n \in \omega$ for which $\varphi_n[I] = \omega_1$. This principle was recently studied by Arnie Miller. He showed that this principle is equivalent to the following statement: There is a set $X = \{f_\alpha \mid \alpha < \omega_1\} \subseteq \omega^\omega$ such that for every $g : \omega \longrightarrow \omega$ there is α such that if $\beta > \alpha$ then $f_\beta \cap g$ is infinite (sets with that property are referred to as *$\mathcal{IE}$-Luzin sets*). Miller showed that the principle of Sierpiński implies that $\mathsf{non}(\mathcal{M}) = \omega_1$. He asked if the converse was true, i.e., does $\mathsf{non}(\mathcal{M}) = \omega_1$ imply the principle$(*)$of Sierpiński? We answer his question affirmatively. In other words, we show that $\mathsf{non}(\mathcal{M}) = \omega_1$ is enough to construct an $\mathcal{IE}$-Luzin set. It is not hard to see that the $\mathcal{IE}$-Luzin set we constructed is meager. This is no coincidence, because with the aid of an inaccessible cardinal, we construct a model where $\mathsf{non}(\mathcal{M}) = \omega_1$ and every $\mathcal{IE}$-Luzin set is meager.

The third chapter is dedicated to a conjecture of Hrušák. Michael Hrušák conjectured the following: Every Borel cardinal invariant is either at most $\mathsf{non}(\mathcal{M})$ or at least $\mathsf{cov}(\mathcal{M})$ (it is known that the definability is an important requirement, otherwise $\mathfrak{a}$ would be a counterexample). Although the veracity of this conjecture is still an open problem, we were able to obtain some partial results: The conjecture is false for "Borel invariants of ω_1^ω" nevertheless, it is true for a large class of definable invariants. This is part of a joint work with Michael Hrušák and Jindřich Zapletal.

In the fourth chapter we present a survey on destructibility of ideals and MAD families. We prove several classic theorems, but we also prove some new results. For example, we show that every almost disjoint family of size less than $\mathfrak{c}$ can be extended to a Cohen indestructible MAD family is equivalent to $\mathfrak{b} = \mathfrak{c}$ (this is part of a joint work with Michael Hrušák, Ariet Ramos, and Carlos Martínez). A MAD family $\mathcal{A}$ is *Shelah–Steprāns* if for every $X \subseteq [\omega]^{<\omega} \setminus \{\emptyset\}$ either there is $A \in \mathcal{I}(\mathcal{A})$ such that $s \cap A \neq \emptyset$ for every $s \in X$ or there is $B \in \mathcal{I}(\mathcal{A})$ that contains infinitely many elements of X (where $\mathcal{I}(\mathcal{A})$ denotes the ideal generated by $\mathcal{A}$). This concept was introduced by Raghavan which is connected to the notion of "strongly separable" introduced by Shelah and Steprāns. We prove that Shelah–Steprāns MAD families have very strong indestructibility properties: Shelah–Steprāns MAD families are indestructible for "many" definable forcings that does not add dominating reals (this statement will be formalized in the fourth chapter). According to the author's best knowledge, this is the strongest notion (in terms of indestructibility) that has been considered in the literature so far. In spite of their strong indestructibility, Shelah–Steprāns MAD families can be destroyed by a ccc forcing that does not add unsplit or dominating reals. We also consider some strong combinatorial properties of MAD families and show the relationships between them (This is part of a joint work with Michael Hrušák, Dilip Raghavan, and Joerg Brendle).

The fifth chapter is one of the most important chapters in the thesis. A MAD family $\mathcal{A}$ is called *$+$-Ramsey* if every tree that branches into $\mathcal{I}(\mathcal{A})$-positive sets has an $\mathcal{I}(\mathcal{A})$-positive branch. Michael Hrušák's first published question is the following: Is there a $+$-Ramsey MAD family? It was previously known that such families can consistently exist. However, there was no construction of such families using only the axioms of ZFC. We solve this problem by constructing such a family without any extra assumptions.

In the fourth and fifth chapters, we introduce several notions of MAD families, in the sixth chapter we prove several implications and non implications between them. We construct (under CH) several MAD families with different properties.

In the seventh chapter we build models without P-points. We show that there are no P-points after adding Silver reals either iteratively or by the side by side product. These results have some important consequences: The first one is that is its possible to get rid of P-points using only definable forcings. This answers a question of Michael Hrušák. We can also use our results to build models with no P-points and with arbitrarily large continuum, which was also an open question. These results were obtained with David Chodounský.

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KLEIDSON ÊGLICIO CARVALHO DA SILVA OLIVEIRA, ***Paraconsistent Logic Programming in Three and Four-Valued Logics***, University of Campinas, Brazil, 2017. Supervised by Marcelo Esteban Coniglio. MSC: 03B53. Keywords: Logic programming, Paraconsistent logic, Many-valued logics.

Abstract

From the interaction among areas such as Computer Science, Formal Logic, and Automated Deduction arises an important new subject called Logic Programming. This has been used continuously in the theoretical study and practical applications in various fields of Artificial Intelligence. After the emergence of a wide variety of non-classical logics and the understanding of the limitations presented by first-order classical logic, it became necessary to consider logic programming based on other types of reasoning in addition to classical reasoning. A type of reasoning that has been well studied is the paraconsistent, that is, the reasoning that tolerates contradictions. However, although there are many paraconsistent logics with different types of semantics, their application to logic programming is more delicate than it first appears, requiring an in-depth study of what can or cannot be transferred directly from classical first-order logic to other types of logic.

Based on studies of Tarcisio Rodrigues on the foundations of Paraconsistent Logic Programming (2010) for some Logics of Formal Inconsistency (**LFI**s), this thesis intends to resume the research of Rodrigues and place it in the specific context of **LFI**s with three- and four-valued semantics. This kind of logics are interesting from the computational point of view, as presented by Luiz Silvestrini in his Ph.D. thesis entitled “A new approach to the concept of quase-truth” (2011), and by Marcelo Coniglio and Martín Figallo in the article “Hilbert-style presentations of two logics associated to tetravalent modal algebras” [***Studia Logica*** (2012)]. Based on original techniques, this study aims to define well-founded systems of paraconsistent logic programming based on well-known logics, in contrast to the *ad hoc* approaches to this question found in the literature.

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ANA CLAUDIA DE JESUS GOLZIO, ***Non-Deterministic Matrices: Theory and Applications to Algebraic Semantics***, University of Campinas, Brazil, 2017. Supervised by Marcelo Esteban Coniglio. MSC: 03G27. Keywords: Algebraic logic, Non-classical logic, Multialgebras, Non-deterministic semantics, Universal algebra, Category theory.

Abstract

We call multioperation any operation that return for even argument a set of values instead of a single value. Through multioperations we can define an algebraic structure equipped with at least one multioperation. This kind of structure is called multialgebra. The study of them began in 1934 with the publication of a paper of Marty. In the realm of Logic, multialgebras were considered by Avron and his collaborators under the name

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