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DIELECTRIC PERMITTIVITY OF GLACIER ICE MEASURED *IN SITU* BY RADAR WIDE-ANGLE REFLECTION*

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ABSTRACT. Values of relative permittivity measured by the wide-angle reflection technique on the Ross Ice Shelf show substantial variations between sites, from 3.09 to 2.89, with estimated errors of ±0.03. The largest values, closest to those normally measured in the laboratory, are found nearest to the grounded ice sheet; values decrease generally in the direction of thinner ice that has been longer on the ice shelf. We believe the variation reflects some real physical phenomenon in the ice shelf, either a true variation in the permittivity of the ice or a complication of the ray-path geometry, but are not able to offer a satisfactory model at present. We hope an explanation will be forthcoming when actual ice core samples from the deep shelf ice are available for examination.

RÉSUMÉ. *Permittivité diélectrique de glace de glacier mesurée* in situ *par reflexion aux grands angles d'ondes radar.* Les valeurs de la permittivité relative mesurée par la technique de réflexion aux grands angles dans le cas du Ross Ice Shelf montrent des variations substantielles entre les sites, de 3,09 à 2,89, avec une erreur standard estimée à ±0,03. La valeur la plus élevée, proche de celle mesurée en laboratoire, est trouvée à proximité de l'endroit où le couvert de glace est rattaché à la rive; les valeurs décroissent généralement dans la direction de la glace plus mince qui a formé plus longtemps le couvert de glace. Nous pensons que ces variations correspondent à quelque phénomène physique se produisant réellement dans la glace mais nous ne pouvons pas, actuellement, proposer de modèle satisfaisant. Nous espérons aboutir à une interprétation dès lors que des échantillons prélevés à grande profondeur dans le couvert de glace seront disponibles pour étude.

ZUSAMMENFASSUNG. *Die Dielekrtizitätskonstante von Gletschereis* — in situ *gemessen durch Radar-Weitwinkel-reflexion.* Werte der relativen Dielektrizitätskonstanten, die mit der Weitwinkeltechnik auf dem Ross Ice Shelf gemessen wurden, zeigen beträchtliche Unterschiede zwischen einzelnen Stellen: von 3,09 bis 2,89, mit einer abgeschätzten Standardabweichung von ±0,03. Die grössten Werte, die den im Labor gewöhnlich gemessenen am nächsten kommen, werden in nächster Nähe zum auf Grund aufsitzenden Eisschild gefunden; die Werte nehmen im allgemeinen ab in Richtung dünneren Eises, das länger auf dem Schelfeis gewesen ist. Wir glauben, die Abweichung spiegelt eine echte physikalische Erscheinung im Schelfeis wieder, aber wir sind gegenwärtig nicht in der Lage ein zufriedenstellendes Modell anzubieten. Wir hoffen, dass sich eine Erklärung finden lässt, wenn wirkliche Bohrkernproben aus dem tiefen Schelfeis für die Untersuchung zur Verfügung stehen.

INTRODUCTION

Since the advent of radio-echo sounding, several field measurements of the velocity of propagation of radio waves through ice have been made, yielding values for the average relative permittivity ϵ of the ice. These measurements have been made using four techniques: travel times from a transmitter in a bore hole to a receiver on the surface (Robin, 1975), direct comparison of bottom reflection travel times with bore-hole depth (Pearce and Walker, 1967), comparison of bottom-reflection travel times with seismically determined depth (Clough and Bentley, 1970; Drewry, 1975) and wide-angle velocity measurements.

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Wide-angle velocity measurements have been made by several investigators (Jiracek and Bentley, 1971; Robin and others, 1969; Bogorodsky and others, 1970; Clough and Bentley, 1970; Autenboer and Decleir, 1970) with values of ϵ obtained that are in general agreement with laboratory determinations, but that do show considerable variation. Some differences are due to errors in measurement; nonetheless, the results suggest the possibility of real variations in ϵ for ice from different regions. Table I summarizes previous results, including earlier work on the Ross Ice Shelf (Jiracek and Bentley, 1971). We have re-evaluated Jiracek and Bentley's (1971) data on the McMurdo Ice Shelf to exclude the effects of the lateral wave (see Clough, 1976).

In this paper we describe several wide-angle measurements made on the Ross Ice Shelf during 1973–77. Great care was taken to minimize the experimental error in the velocity determinations. The measured velocities do show a distinct regional variation, but the interpretation in terms of the relative permittivity is uncertain.

TABLE I. SUMMARY OF PREVIOUS WIDE-ANGLE REFLECTION EXPERIMENTS

Location	*Measured velocity* m/μs	*"Stripped" velocity* m/μs	ϵ	*Thickness* m	*Frequency* MHz	*c.r.p.*	*Reference*
Roosevelt Island Dome	174.8±1.0	173.3±1.3†	3.00±0.05	825±5	30	no	Jiracek and Bentley (1971)
Ross Ice Shelf West	174.9±0.5	173.0±0.8†	3.01±0.03	512±2	30	no	Jiracek and Bentley (1971)
McMurdo Ice Shelf							
Station 203	177.6±2.0‡	173.1±2.3†	3.00±0.08	155±3	30	no	Jiracek and Bentley (1971)
Station 204	178.3±1.2‡	174.7±1.5†	2.95±0.05	187±3	30	no	Jiracek and Bentley (1971)
Skelton Glacier	168.5±1.0	*	3.17±0.04	772±5	30	no	Jiracek and Bentley (1971)
Dronning Maud Land							
Station 840	172.0±1.0	171.0±2.0	3.12±0.05	1 550	35	yes	Clough and Bentley (1970)
Tuto East	166 ±4	*	3.26±0.17	306	35	yes	Robin and others (1969)
Barnes Ice Cap	171.4±0.8	*	3.07±0.03	270	35	yes	Clough and Bentley (1970)
Fimbul Ice Shelf	176.0±1.0	172.4±1.0†	3.03±0.04	310	35	no	Personal communication from H. Decleir in 1977
Molodezhnaya	167		3.23		213	yes	Bogorodskiy and others (1970)
Molodezhnaya	156		3.70		213	yes	Bogorodskiy and others (1970)
Molodezhnaya	158		3.60		213	yes	Bogorodskiy and others (1970)
Molodezhnaya	160		3.52		213	yes	Bogorodskiy and others (1970)
Molodezhnaya	189		2.52		213	yes	Bogorodskiy and others (1970)

* "Stripping" correction negligible.
† Correction applied by present authors.
‡ Data corrected to remove lateral wave.

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Basic principles

The principle of the wide-angle experiment is simple. The transmitter and receiver are separated by increasing intervals along the surface, and variations in travel times t are measured. Simple geometry yields

$$t^2 = \frac{x^2+4h^2}{V^2}, \tag{1}$$

where V is the mean wave velocity in ice, x is the horizontal distance between antennas, and h is the ice thickness. When absorption loss is small, the permittivity is then given simply by

$$V = c/\sqrt{\epsilon}, \tag{2}$$

where c is the speed *in vacuo* (see, e.g. Lorrain and Corson, 1970). For measurements in ice in the 30–300 MHz range, neglecting loss factors changes the magnitude of ϵ by less than 0.001%.

There are several possible sources of error to consider. If the ice bottom is sloping, velocities obtained are not accurate unless corrections are made. If one antenna is fixed in location, a bottom slope of only 1° would produce an error of ± 1.5 m/μs, *c.* 1% in ice 1 000 m thick. By moving the transmitter and receiver equal distances in opposite directions with respect to a fixed center ("common reflection point", or "c.r.p.", measurement), the effect of bottom slope is reduced to second order so that a slope of 10° would produce an error in velocity of less than 1%. Slopes of this magnitude or greater can easily be measured by vertical sounding profiles, so corrections can be applied to the calculated velocities. In most instances, an area of nearly flat bottom can be selected, the reflecting point is essentially fixed, and corrections are not needed.

The direct wave through the air from the transmitter to the receiver is used to trigger the oscilloscope sweep, and as a reference for all time measurements, so to avoid error the velocity of propagation of this wave must be known. Some investigators have stated that antennas must be elevated above the surface (by approximately one-quarter wave length) to insure that the direct wave travels at the velocity of propagation in air, despite the introduction of uncertainties into the geometry that can also effect the velocity determination (Jiracek, 1967; Robin and others, 1969). Such elevation is, in fact, not necessary. For an infinitesimal dipole lying on the surface of a homogeneous half-space, the expression (first-order approximation) for the horizontal component of the electric field E_y is

$$E_y = \frac{\mathrm{i}\mu_0 \omega I\,\mathrm{d}l}{2\pi(k_2{}^2-k_1{}^2)\,x^2}\left[\mathrm{i}k_2 \exp(\mathrm{i}k_2 x) - \mathrm{i}k_1 \exp(\mathrm{i}k_1 x)\right], \tag{3}$$

where μ_0 is the permeability of free space, ω is angular frequency, I is the current in the infinitesimal dipole of length $\mathrm{d}l$, and k_1 and k_2 are the wave numbers in free space and the lower medium, respectively (Baños, 1966, p. 46). The expression shows that there are two separate waves, one traveling in medium 1 (air) with velocity ω/k_1, and a second wave traveling in the lower medium (ice). On solid ice, the two waves produce an interference pattern as discussed by Hermance (1970) and Annan (1973), but on a snow surface where the density increases, and the velocity thus decreases, with depth, the second one disappears due to downward refraction of the wave. Only one wave is left, traveling along the surface with velocity c. During July 1977, two of the authors (K. C. Jezek and J. W. Clough) made measurements in Greenland that compare a signal transmitted on a 120 m cable with known delay against the wave in air. These results confirm that the air wave near the surface propagates with velocity c.

A third possible source of error is ray-path curvature. Since the ice column possesses a velocity gradient, ray-paths will be curved. In order to estimate the magnitude of the deviation from straight-ray geometry, ray-tracing calculations were performed.

Figure 1 shows the results of ray tracing using a general density model of the ice shelf. The rays for 10° intervals in angle of incidence up to the horizontal grazing ray (90°) are shown. It can be seen that rays deviate little from straight-line paths.

For example, the travel time for the grazing ray in a 425 m thick ice shelf is 0.03 μs less than the travel time for a straight ray at the same range. The velocity obtained from the t^2–x^2 plot will differ from the actual average velocity by less than 0.5%.

All the effects of the velocity gradient can be made negligible by a "stripping" correction. This is discussed in the section on data analysis.

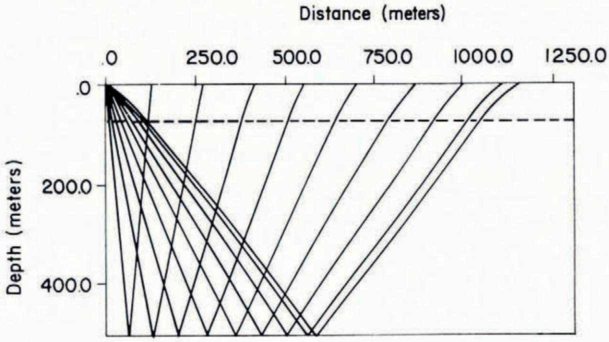

Fig. 1. Ray geometry for a wide-angle reflection measurement. The dashed line shows the approximate depth to which "stripping" corrections were applied.

Field measurements

The field measurements were made during the three field seasons of the Ross Ice Shelf Geophysical and Glaciological Survey (RIGGS), part of the Ross Ice Shelf Project. Three different radar systems were used: two SPRI-II systems (Evans and Smith, 1969), operating at 35 and 50 MHz, each having a pulse width of 0.2 μs at the 3 dB points, and a third, operating at 150 MHz, and having a pulse width of 0.1 μs at the 3 dB points, constructed by the Electrical Engineering Department, University of Wisconsin—Madison. Figure 2 shows a map of the ice shelf with the locations of stations at which wide-angle measurements were made. The orientation of each profile and flow directions near each station are also shown. Thirteen c.r.p. profiles were completed at ten locations. There were three in the vicinity of Q13—two along the same line using the 35 MHz and 150 MHz systems, respectively, and the other 4 km away carried out along a 40° azimuth relative to the other two. At C-16 two profiles were completed in perpendicular directions with centers offset by about 1 km.

Distances between antennas were measured with steel tapes except at H11 and M10 where seismic cables with measured take-out intervals were used. Errors should not exceed 0.1%. The antennas were folded dipoles and were always positioned perpendicular to the profile line. Signals displayed on the oscilloscope screen were recorded on Polaroid film. A 1 MHz signal was photographed periodically for calibration.

The bottom of the ice shelf is not always as well suited for wide-angle reflection measurements as one might expect. Although the bottom is essentially horizontal and produces a strong reflection, there are frequently internal reflections near the bottom which tend to obscure the bottom reflection. These internal reflections are produced by horizontal layering and bottom crevasses which extend into the underside of the shelf. At J9, for example, the bottom crevassing interfered to the point of making the data almost unusable (Clough and others, 1975). Figure 3 (1) and (2) show the effect of near-bottom layering at Q13 on the 150 MHz and 35 MHz systems, respectively. Note the greater sensitivity of the 150 MHz

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system to disturbance. Figure 3(3) also shows an example of the confusion caused by bottom crevassing.

Comparison of the amplitude displays of the wide-angle reflections with continuous, intensity modulated profiles (an example is in Figure 4), together with continuity of bottom reflected signals, allowed the true bottom signal to be identified.

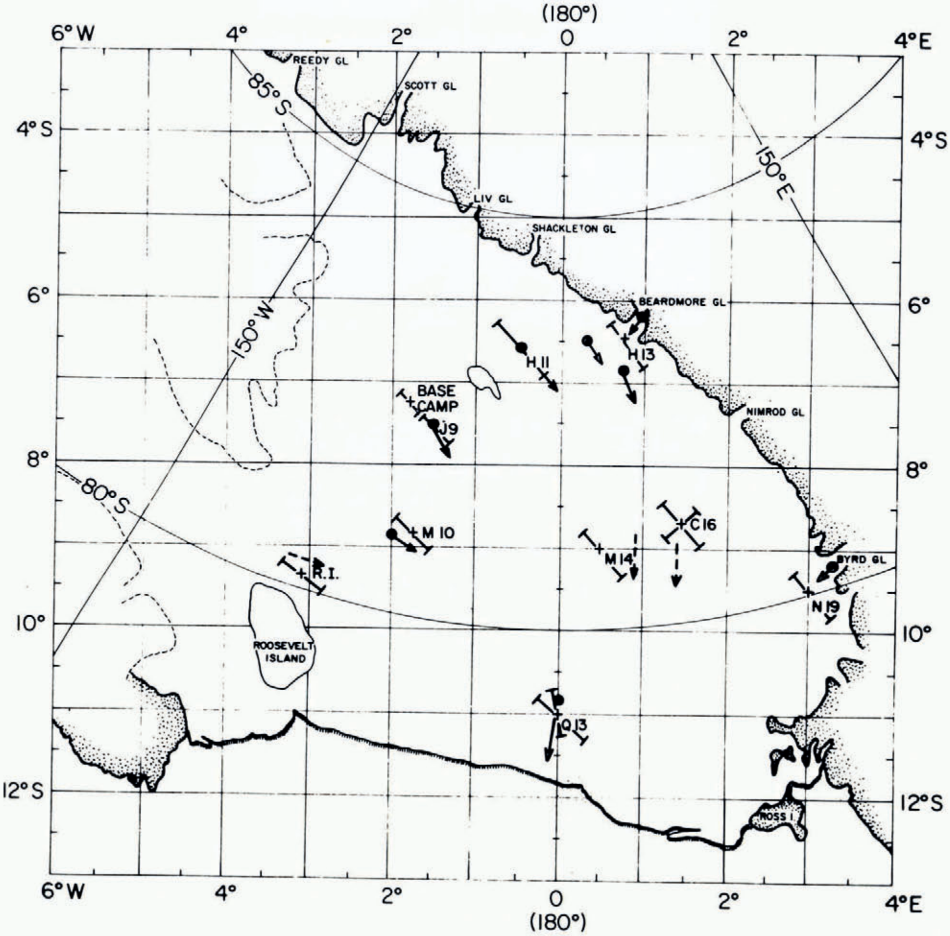

Fig. 2. A map of the Ross Ice Shelf showing: + station locations of RIGGS wide-angle experiments, —— orientation of wide-angle experiments, ●→ measured flow directions (Thomas, 1976; Swithinbank, 1963; Dorrer and others, 1969), - - -→ inferred flow direction (Robin, 1975[a]).

Data analysis

Travel times were measured on the oscilloscope photographs by measuring between base-line intercepts of straight lines fitted to the rise of the transmitted pulse and the rise of the reflected pulse. This method was found to give the most reproducible results with time errors on individual photographs of ± 0.02 μs, corresponding to a velocity error of about 0.3 m/μs.

Ray-path curvature and the average velocity through the ice column are affected by the higher velocities in the upper low-density firn zone. In order to eliminate the effects of the firn layers, a "stripping" correction was applied to all the data. To accomplish this, plots of

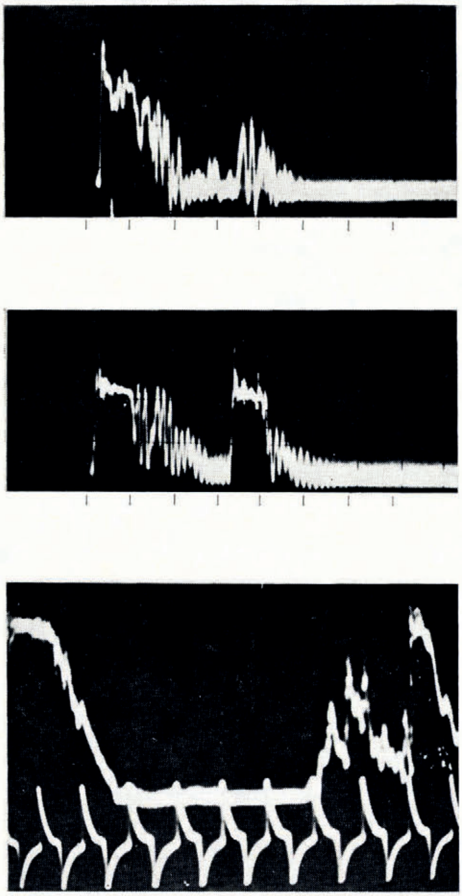

Fig. 3. Three examples of oscilloscope amplitude display: (1) Station Q13, 150 MHz system, 200 m antenna separation, no attenuation; (2) Station Q13, 35 MHz system, 200 m antenna separation, no attenuation; (3) Station H13, 35 MHz system, 30 m antenna separation, 20 dB attenuation. The interval between tick marks below (1) and (2) is 1 μs. Below the wave train in (3) is a 1 MHz calibration signal.

density versus depth were constructed using densities calculated from seismic wave velocities and, where available, core data. Figure 5 shows the good correlation between densities calculated from seismic data and those measured on cores at J9. Also included are the density–depth data for Q13—note that they differ little from densities at J9 despite widely different positions on the ice shelf. The density–depth relations were converted to velocity–depth using

$$n = 1+0.85\rho, \tag{4}$$

where n is the index of refraction and ρ is the density (Robin and others, 1969). Ray-tracing techniques were used to calculate the travel time and range to the maximum depth of seismic wave penetration, generally 55–75 m. These were then subtracted from the total measured t and x, respectively, giving "stripped" values appropriate to the deeper ice of nearly constant density.

When plots of t^2 against x^2 are made using either the uncorrected or "stripped" values, good fits to straight lines are generally obtained except at the shortest and longest distances. A typical data set is shown in Figure 6. The deviation at large distance arises simply from

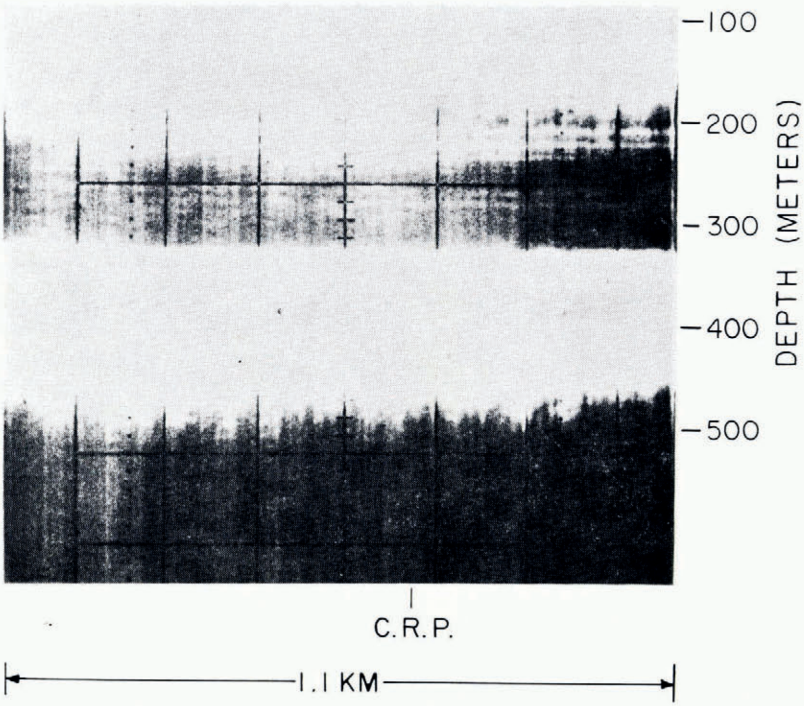

Fig. 4. Intensity modulated profile carried out at Q13 using the 35 MHz radar. The c.r.p. of the Q13 2 and 3 data sets is shown.

including arrivals beyond the limiting distance corresponding to grazing incidence in the firn at the boundary with the air. These arrivals appear to involve some sort of head-wave propagation and are inappropriate to analysis by plotting t^2 against x^2.

The difficulty at short distances is more difficult to explain, and is one that has been observed in many investigations (e.g. by Robin and others, 1969; Clough and Bentley, 1970; Bogorodskiy and others, 1970; Autenboer and Decleir, 1971; Jiracek and Bentley, 1971). It is seen more clearly on a travel–time plot as a region of apparent decrease in reflection time with increasing distance (Fig. 7). We believe it results from low-amplitude cycles in the rise of the transmitted pulse too weak to be observed at propagation path lengths greater than about 80 m. They never appear on the reflected signals, but are large enough to cause the initiation of the direct wave to appear increasingly early as the antennas are brought closer together. This interpretation has recently been supported by experiments during July 1977 on the Greenland ice sheet where photographs of the transmitted pulse show the attenuation and loss of the initial, low-amplitude cycles (Fig. 8).

Using data points only between 80 m and grazing incidence, velocities have been calculated from the plots of t^2 against x^2. Autenboer and Decleir (1971) have commented that it is statistically preferable to fit travel times directly to hyperbolas instead, so velocities were calculated in that manner as well. Velocities found using the two methods varied randomly with a maximum difference of 0.4 m/μs.

The results of the 13 wide-angle velocity profiles are summarized in Table II. Average velocities through the entire ice shelf and velocities found after "stripping", differing from the uncorrected values by about 2 m/μs, are both given, along with values of ϵ calculated from the "stripped" velocities. No corrections for temperature or frequency have been applied. Mean

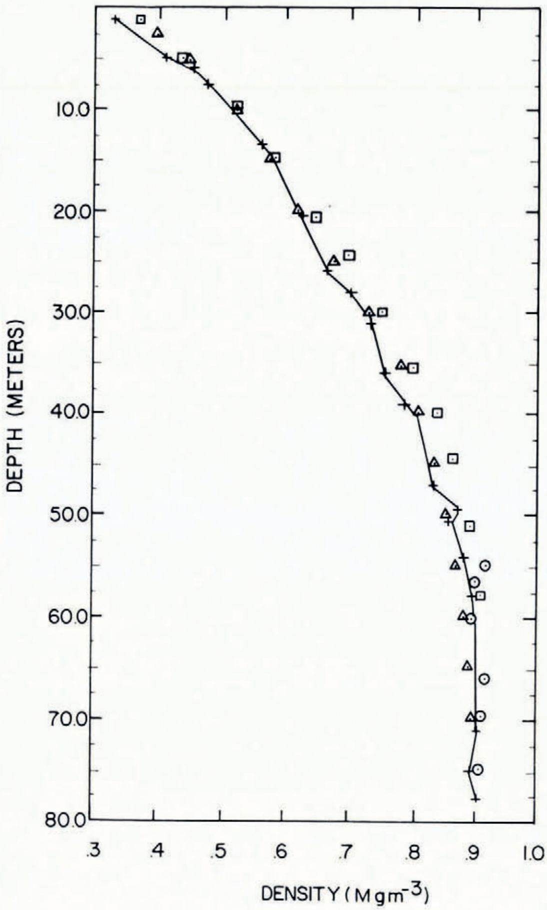

Fig. 5. Depth–density data for the Ross Ice Shelf: + measured densities at J9 (personal communication from C. C. Langway, jr. in 1977), △ densities derived from seismic short refraction experiments at J9 (Robertson, unpublished), □ densities derived from seismic short-refraction experiments at Q13 (personal communication from D. Albert, 1977), ⊙ maximum seismic densities found at other RIGGS stations.

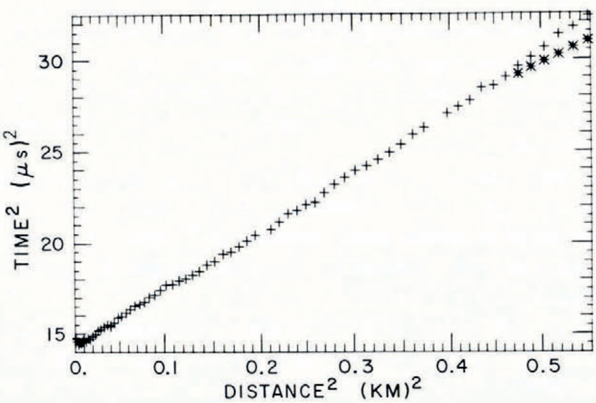

Fig. 6. t^2 plotted against x^2 for data set 1 at station Q13: + reflection data, ✱ lateral wave data.

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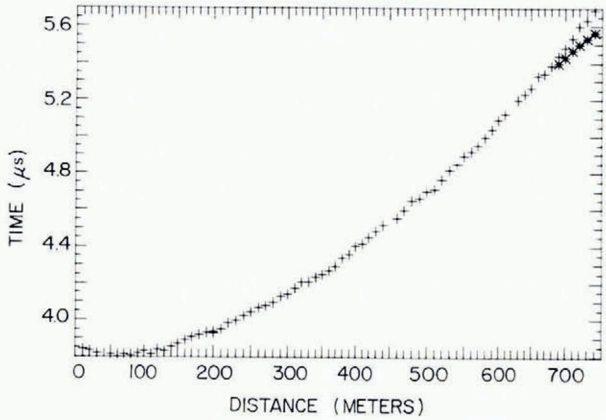

Fig. 7. t plotted against x for data set 1 at station Q13: + *reflection data,* ✱ *lateral wave data.*

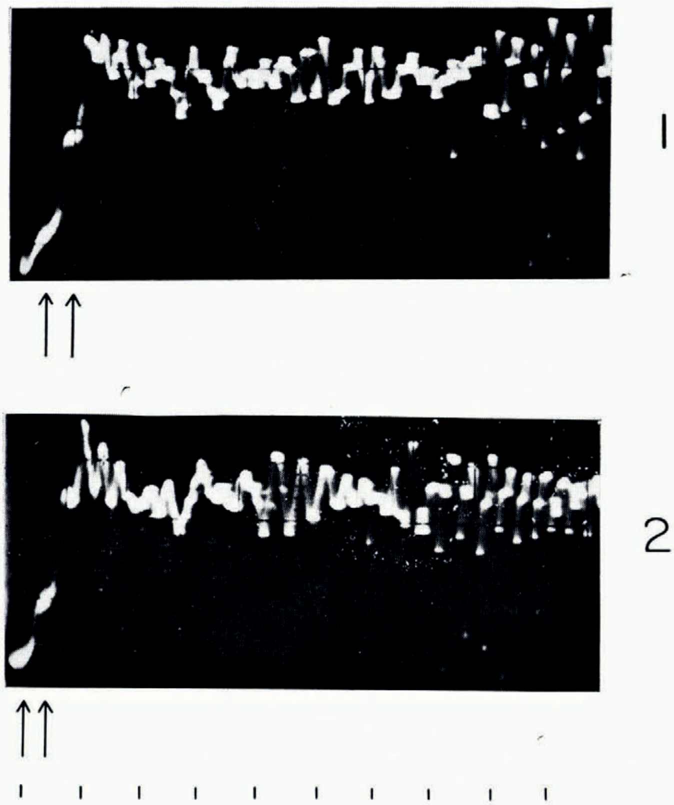

Fig. 8. Two amplitude displays of the 35 MHz transmitted pulse obtained at Camp Century, Greenland. The antenna separations are 30 m and 60 m for (1) and (2), respectively. The tick marks are spaced every 0.1 μs. Note the change in amplitude of the cycles in the rise of the pulse (indicated by the arrows) as antenna separation is increased.

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annual surface temperatures vary from place to place only over a 5° range (Thomas, 1976; Crary and others, 1962); average temperatures in the ice shelf will differ by only about half that. Since $\partial\epsilon/\partial T$ is only about 10^{-3} deg^{-1} (Hobbs, 1974, table 2.1), corrections are negligible. The dependence of ϵ on frequency is not well known since measured values of ϵ do not show a consistent trend (Hobbs, 1974, table 2.1). Johari and Charette (1975) do show a surprisingly large decrease of 0.01 in ϵ between 35 and 60 MHz, but there is little experimental or theoretical basis for extrapolation to higher frequencies. Since appropriate corrections would be only marginally significant at 50 MHz, and are of unknown magnitude at 150 MHz, none have been applied.

TABLE II. SUMMARY OF RESULTS OBTAINED DURING THE ROSS ICE SHELF GEOPHYSICAL AND GLACIOLOGICAL SURVEY

Station	*Measured average velocity through ice shelf* m/µs	*"Stripped" velocity* m/µs	*Average density in "stripped" ice column* Mg/m³	ϵ	*Thickness* m	*Number of data points*	*Frequency* MHz
H13	172.2±0.3	170.7±0.4	0.916	3.09±0.02	773±2	27	35
H11	173.3±0.6	171.5±0.8	0.914	3.06±0.03	627±3	24	35
R.I.	172.9±0.3	170.9±0.5	0.916	3.08±0.02	623±1	61	35
N19	175.3±0.5	173.8±0.6	0.916	2.98±0.02	568±4	19	35
Base Camp	175.7±0.4	173.5±0.4	0.915	2.99±0.01	476±3	51	35
J9	175.6±2.1	173.6±2.3	0.915	2.99±0.08	420±5	78	50
M10	176.6±0.9	175.0±1.0	0.916	2.94±0.03	396±2	12	35
C-16							
1	177.7±0.3	175.9±0.5	0.914	2.91±0.02	387±1	65	50
2	178.5±0.2	177.0±0.2	0.914	2.87±0.01	368±1	66	50
M14	178.1±0.7	176.2±1.0	0.916	2.90±0.03	349±2	14	35
Q13							
1	177.2±0.4	175.0±0.4	0.911	2.94±0.01	335±1	60	35
2	177.9±0.5	176.2±0.7	0.911	2.90±0.02	327±1	73	35
3	176.4±1.7	173.9±2.2	0.911	2.98±0.08	330±3	22	150

The data listed in Table II include error estimates calculated from linear regression fits to the slope and intercept on the plot of t^2 against x^2. They reflect a high precision in the data. The reproducibility is nearly as high, as can be seen by comparing the two values at C-16, and the three at Q13. We believe a reasonable estimate of the standard error in measured velocity is ±1 m/µs (except at J9 where the scatter of the data points was large). This corresponds to about ±0.03 for the values of ϵ and about ±5 m for the thickness.

DISCUSSION

The values of ϵ listed in Tables I and II show a large variation. Even omitting the particularly widely spread values from Molodezhnaya, the range is from 2.89 (average at C-16) to 3.26 (Tuto East). The range of our own measurements on the Ross Ice Shelf is 2.89 to 3.09 (H13). This may be compared with the value, reported by Johari and Charette (1975) from laboratory measurements, of 3.183 at −15°, 3% higher than our highest values and 9% higher than our lowest. The discrepancies show clearly in Figure 9, where our values of ϵ are plotted against density for comparison with two published regression lines of ϵ on ρ (Robin and others, 1969; Glen and Paren, 1975). Even our highest values fall below other measurements at a density of about 0.915 Mg/m³, and the rest fall far too low to be explained by the small variations in average density.

At the present time we do not know how to explain the differences. V. V. Bogorodskiy and B. A. Fedorov (personal communication in 1977) believe, based on two or three direct comparisons with reflection times where the ice thickness is independently known from

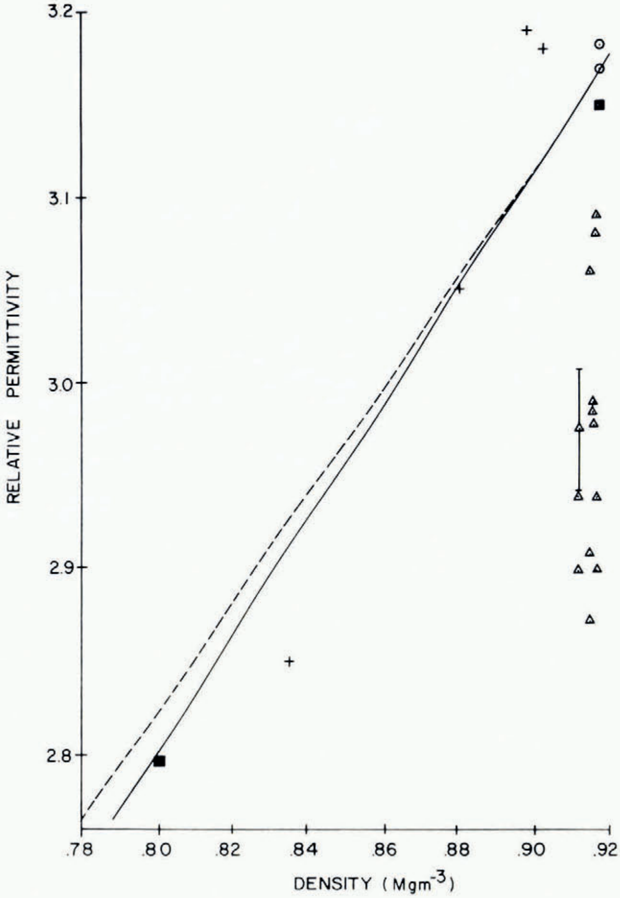

Fig. 9. Plot of ϵ against density. The open triangles are RIGGS wide-angle data. The error bar shown on one triangle applies to all the wide-angle data. The other points represent data from Cumming (1952) : ■; Westphal (see Jiracek, 1967) : +; and Johari and Charette (1975) : ○. The solid line represents a form of the Looyenga equation derived by Glen and Paren (1975). The dashed line corresponds to the empirical formula of Robin and others (1969).

drilling, that there is some inherent error in the wide-angle technique that has not been recognized. The only hypothetical model that we have been able to devise (also suggested by Bogorodskiy) is that the reflecting surface gradually migrates upward as the angle of incidence increases. As much as 15 to 20 m of migration would be needed—that perhaps might occur if there is a zone of salty ice frozen on to the base of the shelf.

Some support for that idea comes from a plot of the geographical distribution of ϵ (Fig. 10). Values decrease as the distance down-stream from the grounding line becomes greater, consistent with a model of bottom freezing, and then (perhaps) increase again toward the ice front in a region where bottom melting is likely to occur.

However, there is very little evidence from the actual oscilloscope photographs to support such a migration, nor is it easy to see how any migration could occur gradually rather than in a very few finite jumps. Furthermore, the reflection coefficient at any boundary should vary only gradually out to angles of incidence greater than those included in our experiments. Thus this model is difficult to support quantitatively.

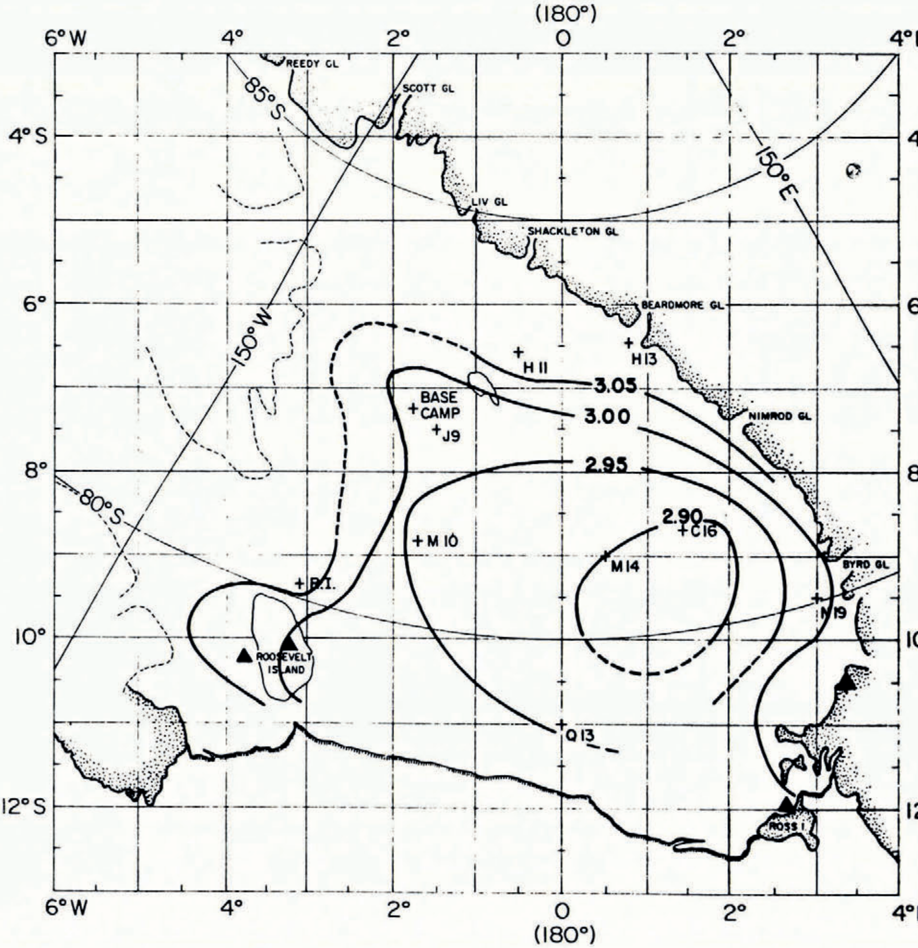

Fig. 10. Contour map of ϵ on the Ross Ice Shelf: + *RIGGS stations,* ▲ *Jiracek and Bentley (1971).*

If the variations in the measured values of ϵ do not arise from some systematic error in the wide-angle reflection technique, then they must represent real variations in the permittivity of the solid shelf ice.

This interpretation is supported (rather weakly) by the only directly relevant laboratory measurement of ϵ available, those by Westphal on ice cores from Little America V station (see Jiracek, 1967). While his values of ϵ (3.16 corrected to density 0.917 Mg/m^3) are higher than ours, they are all lower by about 0.1 than his for Arctic glacial ice. This is about the magnitude difference we observe. The measurement on the Fimbul Ice Shelf, on the other side of the continent, is also low (Table I). H. Decleir (personal communication in 1977), using Wiener's formula (Evans, 1965) to eliminate the contributions of the near-surface low-density material from the calculation of ϵ, obtains values of ϵ ranging from 3.11 to 3.04 for values of *Formzahl* ranging from 2 to ∞ for the Fimbul Ice Shelf data. Although the *Formzahl* of the data used to derive Equation (4) (Robin and others, 1969; Evans, 1965) is within this range, application of Equation (4) and ray-tracing yields a slightly lower result than the use of Wiener's formula. The use of either method still produces values of ϵ much lower than laboratory results.

Again we have no satisfactory model to explain such low values. If there are impurities in the ice, we would expect ϵ to be increased rather than decreased. Nor is there reason to believe that the explanation lies in crystal anisotropy, since any difference in ϵ parallel and perpendicular to the crystallographic *c*-axis that may exist must, from laboratory measurements, be less than 1% (Johari and Charette, 1975; see also Hargreaves, 1978).

There is a clear correlation between ϵ and ice thickness (Fig. 11), which could reflect some cause-and-effect relationship, or simply the fact that ice thicknesses decrease with distance from the grounding line.

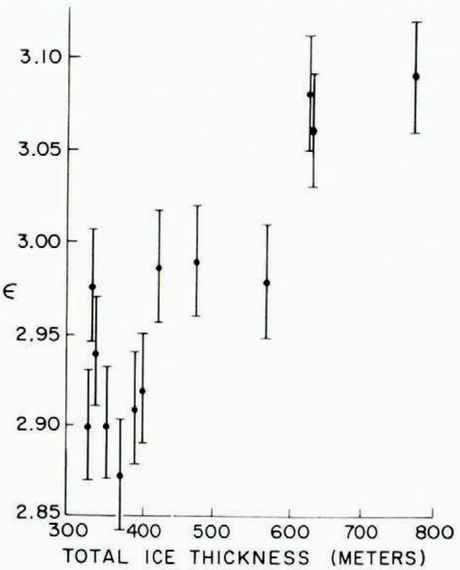

Fig. 11. Variation of ϵ with total ice thickness for wide-angle experiments completed during RIGGS.

Whatever the explanation for the variations in the measured values of ϵ, we are confident that they arise, not simply from ordinary experimental errors, but from some real physical characteristics of the ice shelf. We hope experiments on deep ice cores from the ice shelf will help to determine whether those characteristics relate to propagation paths or to real permittivity differences in Antarctic shelf ice.

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DISCUSSION

G. P. Johari: What is the magnitude of errors in your measurement?

K. Jezek: The magnitude of the error on the velocity, derived from statistical analysis, was in some cases as low as ± 0.1 m μs^{-1}. However from comparison of separate experiments at the same location and making qualitative judgements on the data, the error for most experiments is chosen to be about ± 1 m μs^{-1}. This converts to an error of ± 0.03 for the relative permittivity.

M. E. R. Walford: You have assumed a model in which the reflection is specular and horizontal. Can you justify this? In other words do preliminary echo-sounding profiles show constant echo delay time and no spatial fading?

Jezek: Vertical sounding profiles were made along the length of all the wide-angle lines. We have not specifically studied the character of the reflected pulse, but generally the pulse shape seems constant over the wide-angle lines.

C. R. Bentley: Continuous vertical sounding is carried out along the profiles to ensure negligible bottom slopes, as well as using the common-reflection-point techniques. The amount of error that would be needed is indicated by the 30 m upward migration of the reflecting surface mentioned in the paper; such an error could hardly occur. We are sure that our experimental results are valid in terms of accurate velocity determinations, but we do not have a satisfactory explanation. If there is a true variation of ϵ in the shelf ice, it is certainly unexpectedly large (although perhaps not any more surprising than the high d.c. conductivity in polar ice when it was first observed). On the other hand, we have no satisfactory model in terms of ray-path modification. As a result, we are presently at a loss for a satisfactory explanation for our observations, but we are convinced that they are telling us something real about the ice shelf.

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